Wave transformation in an inhomogeneous unstable plasma

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(Submitted March 22, 1973)
Zh. Eksp. Teor. Fiz. **65**, 1431–1447 (October 1973)

Linear transformation of beam space-charge waves and of magnetoactive-plasma oscillations by density inhomogeneities is investigated. It is shown that amplification of plasma oscillations by a beam occurs either in the presence of a nonlinear longitudinal plasma density gradient or at the expense of transverse inhomogeneity. Anisotropy of transverse wave emission by a charge in a magnetoactive plasma with a smooth longitudinal density inhomogeneity is observed.

INTRODUCTION

Linear transformation of waves in a stable plasma, due to inhomogeneity effects, has by now been sufficiently well studied (see, e.g.,[1-3]). Its practical significance is connected principally with plasma heating occurring when the electromagnetic waves are transformed into slow longitudinal waves. On the other hand, the use of plasma to generate electromagnetic radiation entails the use of nonequilibrium energy (beam or current). The slow plasma waves that are usually produced in this case by the instability are then transformed into transverse oscillations. The possibility of transforming waves in inhomogeneous systems for the purpose of producing plasma radiation generators has been under discussion in the literature for quite a long time, and even in the first papers it was pointed out that the transition to the radiating regime can greatly influence the very instability of the plasma, and in particular suppress instability[4,5]. Recent experiments have confirmed the earlier assumptions[3,6,7] and have also revealed many other curious properties[8,9], such as the anisotropy of the emission of transverse waves in a plasma-beam discharge[8].

The development of the theory has followed the path of investigation of the transformation in active media with strongly inhomogeneous parameters[10]; as to smooth inhomogeneities in the presence of intersection points of the oscillations modes, the source of the imbalance, say a beam, was regarded as a given generator, i.e., in a non-self-consistent manner[11,12]. At the same time, computer experiments on wave propagation in a plasma with inhomogeneity along the magnetic field[13], which have revealed, in particular, that the conversion of the beam modes into plasma modes is smaller than expected, have shown that it is necessary to eliminate this appreciable gap. The present paper is devoted to a rigorous analytic solution of the problem of mutual conversion of plasma oscillations and beam space-charge waves.

This enables us to understand the role of the longitudinal and transverse inhomogeneities in the transformation process, the causes of the significant decrease in the efficiency of conversion of beam modes into transverse waves by a longitudinal inhomogeneity, and also the connection between the anisotropy of the transition radiation and the emission of transverse waves in a plasma-beam discharge in the case of a magnetized plasma.

In addition, we discuss briefly the features of emission from charges whose field moves along the inhomogeneity with superluminal velocity, and the consequences of reversing the signs of the dissipative terms. Thus, in a number of cases it turns out that reversal of the sign of the dissipation is equivalent to conversion of longitudinal waves into transverse ones, and consequently we should expect generation of transverse waves in media with negative dissipation. We note that the properties of such media have been actively discussed in the literature in recent years, as for example in Starr's book[14].

1. TRANSFORMATION OF WAVES BY A LONGITUDINAL PLASMA-DENSITY GRADIENT. LONGITUDINAL PROPAGATION.

1. We investigate the transformation of plasma oscillations and space-charge waves (beam modes) in a beam-plasma system situated in a strong ($\omega_{He} \gg \omega$, ω_{pe}) magnetic field by a longitudinal plasma-density gradient $N_0(z)$. At $v_0 \ll c$, the initial equations for the high-frequency ($\omega \sim \omega_{pe}$) oscillations are

$$\begin{gathered}
i\omega\delta V_z=-\frac{e}{m}\delta E_z-v_{Te}^2\frac{d}{dz}\frac{\delta N}{N_0},\\
\left(i\omega+v_0\frac{d}{dz}\right)\delta v_z=-\frac{e}{m}\delta E_z,\\
i\omega\delta N+\delta V_z\frac{d}{dz}N_0=0,\quad c\,\mathrm{rot}\,\delta\mathbf{H}=i\omega\delta\mathbf{E}-4\pi j\mathbf{e}_z,\\
c\,\mathrm{rot}\delta\mathbf{E}+i\omega\delta\mathbf{H}=0,\quad \mathrm{div}\delta\mathbf{E}+4\pi e(\delta n+\delta N)=0,
\end{gathered}\tag{1.1}$$

where δN, δV_z, δn, and δv_z are the perturbations of the density and of the velocity of the particles in the plasma and in the beam, respectively

$$v_{Te}^2=T_e/m,\quad j=e(N_0\delta V_z+n_0\delta v_z+v_0\delta n);$$

all the perturbations are proportional to $\exp(i\omega t - ik_\perp y)$.

From (1.1) we obtain for the perturbations δN, δE_z, and δH_x three coupled second-order equations, with which we shall work subsequently:

$$\begin{gathered}
\left(i\omega+v_0\frac{d}{dz}\right)^2(\varepsilon\delta E_z-\alpha_1\delta H_x)+\omega_b^2\delta E_z=\\
=\frac{v_{Te}^2}{\omega^2}\left(i\omega+v_0\frac{d}{dz}\right)^2N_0\frac{d}{dz}4\pi e\frac{\delta N}{N_0},\\
v_{Te}^2\frac{d}{dz}N_0\frac{d}{dz}\frac{\delta N}{N_0}+\omega^2\delta N+\frac{d}{dz}\left(\frac{\omega_{pe}^2}{4\pi e}\delta E_z\right)=0,\\
\left(\frac{d^2}{dz^2}+\frac{\omega^2}{c^2}\right)\delta H_x=\alpha_1\frac{\omega^2}{c^2}\delta E_z;\\
\varepsilon(z)=1-\omega_{pe}^2(z)/\omega^2,\quad \alpha_1=ck_\perp/\omega,\quad \omega_b^2=4\pi e^2n_0/m.
\end{gathered}\tag{1.2}$$

The density n_0 and the beam velocity v_0 are assumed here to be constant. The system (1.2) describes propagation, in an inhomogeneous plasma, of beam space-charge waves, Langmuir waves distorted by the magnetic field, and also the so called "cold plasma modes,' which satisfy outside the region of intersection of the oscillation modes the dispersion equation

$$k_z^2+k_\perp^2/\varepsilon=\omega^2/c^2.$$

The general case of simultaneous transformation of three types of waves will be considered in Sec. 2, and we investigate initially the simplest purely longitudinal propagation, when $k_\perp = 0$. The transverse and the longitudinal waves can then be separated, with the transverse oscillations satisfying the vacuum wave equation. For Langmuir oscillations and beam modes we obtain from (1.2), in the case of weak inhomogeneity of the plasma density, the following equation:

$$\left(i\omega+v_0\frac{d}{dz}\right)^2\left(\omega^2\varepsilon+v_{Te}^2\frac{d^2}{dz^2}\right)\delta E_z+\omega_b^2\left(\omega^2+v_{Te}^2\frac{d^2}{dz^2}\right)\delta E_z=0. \quad (1.3)$$

For a fast beam ($v_0 \gg v_{Te}$) the interaction of the Langmuir oscillations and of the space-charge waves occur mainly near the resonance point $\epsilon = 0$, where $\epsilon(z)$ can be regarded as a linear function, $\epsilon(z) = - z/L$. The solution of (1.3) then takes the form of a Laplace contour integral:

$$\delta E_z=C\int ds\,(s\mp1)^{\pm i\gamma}\exp\left[-i\rho\left(\frac{s^3}{3\lambda^2}+\zeta s-\frac{\sigma}{s\mp1}\right)\right]; \quad (1.4)$$

$$\lambda=\left|\frac{v_0}{v_{Te}}\right|,\quad \rho=\frac{\omega L}{|v_0|}\gg1,\quad \sigma=\frac{\omega_b^2}{\omega^2}\left(1-\frac{v_{Te}^2}{v_0^2}\right),$$

$$\gamma=2\rho\left(\frac{\omega_b v_{Te}}{\omega v_0}\right)^2,\quad \zeta=\frac{z-z_0}{L},\quad z_0=\frac{\gamma|v_0|}{2\omega},$$

where C is a normalization constant and the signs + and − pertain to the cases when the beam moves along ($v_0 > 0$) or in opposition to ($v_0 < 0$) the plasma density gradient. The system of contours is shown in Fig. 1a for $v_0 > 0$ and in Fig. 1b for $v_0 < 0$. Thus, in each case we have six solutions, which are connected, if C is chosen the same in (1.4), by the relations

$$\bar B_{1,2}+B_{1,2}e^{2\pi\gamma}=0,\qquad A_1+A_2+A_3=V+B_{1,2}+\bar B_{1,2}.$$

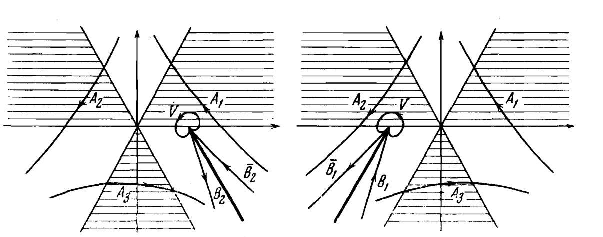
We investigate the asymptotic forms of the solutions (1.4) and construct the fundamental system of solutions by a method similar to that used in a number of papers[15-17].

We consider wave transformation when a beam moves in a direction of increasing plasma density. Using the dispersion equation

$$1=\frac{\omega_{pe}^2}{\omega^2-k_z^2v_{Te}^2}+\frac{\omega_b^2}{(\omega-k_zv_0)^2}, \quad (1.5)$$

we can easily show that when $n_0v_0^2/N_0v_{Te}^2 \ll 1$ the intersection of the oscillation modes occurs near $\epsilon = (v_{Te}/v_0)^2$. In the upper limiting case of a "powerful" beam (or weak thermal motion) $a \equiv \omega_b v_0/\omega v_{Te} \gg 1$, the modes intersect near $\epsilon = \omega_b^2/a\omega^2$.

Assume that a Langmuir oscillation is incident on the intersection region, which we shall henceforth call the "wave-interaction region." In the interaction



a b

FIG. 1

region, this oscillation is transformed into a slow beam mode that increases in the direction of beam motion, and in addition, a reflected Langmuir wave is generated in an above-the-barrier fashion. Since the plasma region where $\omega_{pe} > \omega$ is not transparent to Langmuir waves, the boundary conditions are satisfied by the solution $\delta E_z = A_1 + A_2$. Putting in (1.4) $C = - E_0(\rho/\pi\lambda)^{1/2}$, which corresponds to an energy flux $S_z = v_{Te}E_0^2/8\pi$, in the incident wave, we obtain the asymptotic form of the field to the left of the interaction region ($\zeta + \lambda^{-2} < 0$):

$$\delta E_z=E_0\left(\frac{\omega}{k_lv_{Te}}\right)^{1/2}[\exp(-i\varphi_+)+\exp(i\varphi_--\pi\gamma)]; \quad (1.6)$$

$$k_l=\frac{\omega}{v_{Te}}|\zeta|^{1/2},\quad \varphi_\pm=\frac{\pi}{4}+\int_{z_0}^{z}k_l(z')\,dz'\pm\gamma\ln(s_0\mp1)-\frac{\rho\sigma}{(s_0\mp1)};$$

here $s_0 = \lambda|\zeta|^{1/2}$ is the saddle point in (1.4).

From (1.6) we see that the Langmuir-wave reflection coefficient (with reflect to the energy flux) is equal to $R_{ll} = e^{-2\pi\gamma}$. Consequently, at $2\pi\gamma \gg 1$ the incident Langmuir wave is converted almost completely into a slow beam mode, i.e., the waves interact strongly in the intersection region. The conditions for the applicability of the asymptotic form (1.6), say at $a \ll 1$, are

$$\rho\gg\lambda^2,\qquad s_0-1\gg a^{2/3}.$$

We consider further the oscillation modes in the interaction region. At $a \ll 1$, changing over in the dispersion equation (1.5) to new variables

$$k_z=\frac{\omega}{v_0}(1+a^{2/3}X),\qquad \varepsilon=\frac{v_{Te}^2}{v_0^2}(1-3a^{2/3}Y),$$

we obtain the roots of this equation

$$X_n^{-1}=Y(1+D)^{-1/3}e^{-2i\pi n/3}-(1+D)^{1/3}e^{2i\pi n/3}, \quad (1.7)$$

where $D = (1 + Y^3)^{1/2}$. It follows from (1.7) that the beam is unstable in the region $Y > - 1$, the maximum of the increment

$$\varkappa=3^{1/2}2^{-4/3}\omega a^{2/3}/v_0$$

is reached at $Y = 0$, and the points where the oscillation modes intersect are

$$Y_1=-1,\qquad Y_{2,3}=e^{\pm i\pi/3}.$$

Let us investigate the solution in the interaction region $|Y| \sim 1$. The change δE_z in the interaction region is significant only in the case of a sufficiently weak inhomogeneity, when the dimension $La^{2/3}\lambda^{-2}$ of the interaction region is large in comparison with the reciprocal of the increment κ, i.e., at $q_1 \equiv \rho a^{4/3}\lambda^{-2} \gg 1$. In the region $(1 + Y)e^{i\pi} > q_1^{-2/3}$ the field of the incident Langmuir wave is determined by the contribution of the saddle point $s = 1 + a^{2/3}X_2$:

$$\delta E_z\approx E_0\left(\frac{\lambda/\pi}{1-X_2^{-3}}\right)^{1/2}\exp\left[\frac{2}{3}i\gamma\ln a-i\frac{\pi}{2}-\frac{i\rho}{3\lambda^2}-3iq_1X_2(2Y+X_2)-i\rho\zeta\right]$$

and it can be easily seen that at $|1 + Y| \ll 1$ it increases like $|1 + Y|^{-1/4}$, reaching at the point $Y = - 1$ the value

$$E_0\Gamma\left(\frac{1}{3}\right)\frac{(\rho\lambda)^{1/6}}{(3\pi)^{1/2}}a^{2/9}\exp\left(\frac{2}{3}i\gamma\ln a-i\frac{\pi}{3}-\frac{2i\rho}{3\lambda^2}+3iq_1-\frac{i}{3}q_1a^{2/3}\right).$$

In the same region we obtain for the perturbations of the velocity and of the density of the beam

$$\frac{\delta v_z}{v_0}\approx\frac{e\delta E_z}{im\omega v_0X_2a^{2/3}},\qquad \frac{\delta n}{n_0}\approx\frac{ie\delta E_z}{m\omega v_0X_2^2a^{4/3}}$$

To the right of the intersection point $Y = - 1$ the field increases, and at $1 + Y \ll 1$ the quantity

$$\delta E_z \sim \exp[8q_1(1+Y)^{3/2}]$$

reaches at Y = 0 the value

$$\delta E_z \sim \exp(3^{3/2}q_1/2^{5/3}),$$

after which the growth slows down:

$$\delta E_z \sim \exp[2q_1(3Y)^{1/2}]$$

as a result of the decrease of the increment

$$\varkappa = \omega a^{2/3}/(3Y)^{1/2}v_0.$$

In the case $a \gg 1$, when the resonance $\epsilon = 0$ is weakly smeared out, we make the substitution

$$k_z = \frac{\omega}{v_0}a^{1/3}X, \quad \varepsilon = -2\frac{\omega_b v_{Te}}{\omega v_0}Y$$

and obtain from (1.5) its roots:

$$X^2 = {}^1/_2[(1-Y)^{1/2} \pm i(1+Y)^{1/2}]^2.$$

We see therefore that the intersection occurs at the points $Y = \pm 1$ and is analogous to that investigated previously[18].

The instability region $Y > -1$ and the maximum of the increment

$$\varkappa = \frac{\omega}{v_0}\left(\frac{\omega_b v_0}{\omega v_{Te}}\right)^{1/2}$$

is reached at the point Y = 1. Near the interaction region, the solutions take the form

$$\delta E_z = E_0\left(\frac{\rho a}{\pi\lambda}\right)^{1/2}\exp\left(i\pi + \frac{i}{2}\gamma\ln a\right)\int dt\, t^{i\gamma}$$
$$\times \exp\left[-iq_2\left(\frac{t^3}{3} + 2Yt - \frac{1}{t}\right)\right]. \quad (1.8)$$

Here $q_2 = \rho a^{2/3}\lambda^{-2}$ and is assumed to be large.

In the region $e^{i\pi}(1+Y) > q_2^{-2/3}$, the asymptotic form of (1.8) is determined by the contribution of the saddle points $\pm t_0$:

$$\delta E_z = E_0(e^{i\varphi} + e^{-\pi\gamma - i\varphi})\frac{t_0^{1+i\gamma}}{[2(1+Y)]^{1/4}}\left(\frac{\lambda a^{1/3}}{1+t_0^2}\right)^{1/2}\exp\left(i\pi + \frac{i}{2}\gamma\ln a\right);$$
$$\varphi = \frac{3}{4}\pi + \frac{2^{3/2}}{3}q_2[(1-Y)^{3/2} + |1+Y|^{3/2}], \quad t_0 = \left(\frac{1-Y}{2}\right)^{1/2} + \left|\frac{1+Y}{2}\right|^{1/2}$$

At the intersection point Y = − 1 the field is equal to

$$\frac{E_0}{(2\pi)^{1/2}}\left(\frac{\rho\lambda}{6a}\right)^{1/4}\left[\exp\left(\frac{2}{3}i\pi + \frac{8}{3}iq_2\right)\right.$$
$$\left.+\exp\left(-\pi\gamma - \frac{2}{3}i\pi - \frac{8}{3}iq_2\right)\right]\exp\left(i\pi + \frac{i}{2}\gamma\ln a\right).$$

To the right of the point Y = − 1, the field increases as a result of the beam instability

$$\delta E_z \sim \exp[{}^4/_3\sqrt{2}q_2(1+Y)^{3/2}]$$

and reaches at Y = 1 the value

$$\delta E_z = E_0\Gamma\left(\frac{1}{3}\right)(2\pi)^{-1/2}\left(\frac{\rho\lambda}{6a}\right)^{1/2}\exp\left(\frac{8}{3}q_2 - \frac{\pi}{2}\gamma + \frac{i}{2}\gamma\ln a\right),$$

after which the instability increment decreases and the growth slows down.

We consider now the incidence of a slow space-charge wave, $k_z < \omega/v_0$, on the interaction region. The boundary conditions are satisfied by the solution $B_2 + V - A_3$. We put in formula (1.4)

$$C = \left(\frac{\omega_b L}{\pi v_0}\right)^{1/2}\frac{mv_0\omega_b}{e}\frac{n_\sim}{2n_0}.$$

The alternating component of the beam density at the plasma-vacuum boundary is then $n_\sim$. An investigation of this solution shows that slow space-charge waves generate in above-the-barrier fashion a Langmuir wave that propagates towards the plasma boundary. At the chosen normalization, the energy flux in the Langmuir wave is

$$S_z = \frac{m}{2}n_0 v_0^3\left(\frac{n_\sim}{2n_0}\right)^2\frac{\omega_b}{\omega}e^{-2\pi\gamma}.$$

In the instability region, the slow beam mode grows in analogy with the solution $A_1 + A_2$.

In the case of incidents of a fast beam mode $k_Z < \omega/v_0$ on the interaction region (the solution is $B_2 - A_3$), this mode is completely transformed into a Langmuir wave that is reflected back to the plasma boundary. This transition is adiabatic in character. To the right of the interaction region, the field attenuates in the interior of the plasma:

$$\delta E_z \sim H_p^{(1)}(2\rho(\sigma\xi)^{1/2}e^{i\pi/2}),$$

where $p = 1 + i\gamma$, $\xi = \zeta + (v_{Te}/v_0)^2$. It can be shown that the energy fluxes in the incident beam mode and in the reflected Langmuir wave are equal.

We note the special choice of boundary conditions corresponding to the solution V. Outside the interaction region, the principal terms in the asymptotic expression for V contain only beam modes and satisfy the abbreviated equation

$$\left(i\omega + v_0\frac{d}{dz}\right)^2\left(\varepsilon - \frac{v_{Te}^2}{v_0^2}\right)\delta E_z + \omega_b^2\delta E_z = 0.$$

3. We now consider the transformation of waves when a beam moves in the direction where the plasma density decreases. We confine ourselves here only to asymptotic forms of the solutions in the far zone, where the divergence of the oscillations modes is large enough.

For the solution $\overline{B}_1$ we put in (1.4)

$$C = E_0(\rho/\pi\lambda)^{1/2}e^{i\pi - \pi\gamma}.$$

To the right of the interaction region ($\xi > 0$), the asymptotic form of the solution $\overline{B}_1$ is the field of a space-charge wave that increases in the direction of beam motion,

$$\delta E_z = E_0\left(\frac{\pi}{\lambda}\rho\right)^{1/2}\left(\frac{\sigma}{\xi}\right)^{q/2}H_q^{(1)}(2\rho\sqrt{\sigma\xi}\,e^{i\pi/2})\exp\left(i\pi q + i\rho\xi - \frac{2i\rho}{3\lambda^2}\right). \quad (1.9)$$

Here $q = 1 - i\gamma$ and $H_q^{(1)}$ is a Hankel function.

To the left of the interaction region ($\xi < 0$) the solution $\overline{B}_1$ is a superposition of a Langmuir wave

$$\delta E_z = E_0\left(\frac{\omega}{k_l v_{Te}}\right)^{1/2}\exp(i\varphi_+),$$

that travels towards the plasma boundary, and a slow space-charge wave determined by an analytic continuation of (1.9) into the upper ζ half-plane. The latter circumstance is equivalent to introducing normal ($\nu > 0$) dissipation for the beam mode in the interaction region, and is connected with the expenditure of its energy to the generation of the Langmuir wave. It can be shown that the energy fluxes in both waves are equal in magnitude and opposite in direction. This can be easily understood if it is recognized that the system (1.2) has an integral, namely the total energy flux along the inhomogeneity, and the solution $\overline{B}_1$ attenuates in the interior of the plasma, so that it corresponds to a zero total energy flux.

Let us investigate the solution A_3. At $\zeta > 0$ the solution A_3 is a Langmuir wave that attenuates in the interior of the plasma: $\delta E_Z \sim \exp(-{}^2/_3\rho\lambda\zeta^{3/2})$. In the stability region $\xi < 0$ ($\arg \xi = \pi$) the field is superposition of an incident and of a reflected Langmuir wave, and also of space-charge waves generated in the region where the oscillation modes intersect:

$$\delta E_z = E_0 \left(\frac{\omega}{k_l v_{Te}}\right)^{1/2} [\exp(i\varphi_+ - \pi\gamma) + \exp(-i\varphi_-)] + \qquad (1.10)$$

$$+ iE_0 \left(\frac{\pi}{\lambda}\rho\right)^{1/2} \left(\frac{\sigma}{\xi}\right)^{q/2} [H_q^{(1)}(Z) + e^{-2i\pi q} H_q (Z)] e^{i\theta},$$

where

$$Z = 2\rho(\sigma\xi e^{i\pi})^{1/2}, \quad \theta = \rho(\xi - 2v_{Te}^2/3v_0^2).$$

It is seen from (1.10) that, in analogy with the case when the beam moves along the density gradient, the incident Langmuir wave is reflected, with a coefficient $R_{ll} = e^{-2\pi\gamma}$. As $\omega_b \to 0$, the amplitudes of the beam mode are proportional to n_0, and the solution A_3 goes over into one of the Airy functions. For $2\pi\gamma \gg 1$, the amplitude of the reflected Langmuir wave and of the slow beam wave are exponentially small. The incident Langmuir wave goes over adiabatically into a fast beam mode in this case.

We note also the solution $B_1 - A_3$, which attenuates the interior of the plasma like $H_q^{(1)}(Z)$, where $\xi > 0$, and is a superposition of the Langmuir wave and a fast beam mode to the left of the interaction region; it constitutes a special case of interference, in the interaction region, between the incident Langmuir wave and a beam mode that grows from the interior of the plasma, as the result of which the Langmuir wave is completely converted into a fast space-charge wave. Thus, the transformation processes described by the solution $B_2 - A_3$ when the beam moves along the density gradient and by the solution $B_1 - A_3$ are mutually reversible.

For the solution V, as before, the asymptotic formula contains only space-charge waves. Consequently, for a beam-mode combination specified by the solution V there is no transformation into Langmuir waves. This is easiest to understand from the following: If $V_{Te} = 0$, then all the solutions (with the exception of V) are singular at the resonance point $\epsilon = 0$, a situation corresponding to a finite absorption of the beam-mode energy in the resonance region at arbitrarily small dissipation. When the weak thermal motion is taken into account ($v_{Te} \neq 0$), generation of Langmuir waves takes place at the singular point, the absorption coefficients being equal to the transformation coefficients. For the solution V, the point $\epsilon = 0$ is not singular at $V_{Te} = 0$, so that there is no finite energy absorption in resonance region, and consequently there is no generation of Langmuir oscillations. The connection we obtained between the absorption and transformation will become manifest later on, and obviously reflects a certain general principle. It should be noted that the equivalence of thermal motion to dissipation was established earlier for a number of linear (see, e.g.,[2,3]) and nonlinear[12] processes.

2. WAVE TRANSFORMATION IN OBLIQUE PROPAGATION

1. In oblique propagation, when $k_\perp \neq 0$, all the oscillation modes become coupled with the plasma inhomogeneity, and the system (1.2) reduces in the case of a smooth density inhomogeneity to the following sixth-order equation

$$\left(i\omega + v_0\frac{d}{dz}\right)^2 \left[\left(\omega^2\varepsilon + v_{Te}^2\frac{d^2}{dz^2}\right)\left(\frac{\omega^2}{c^2} + \frac{d^2}{dz^2}\right) - k_\perp^2\left(\omega^2 + v_{Te}^2\frac{d^2}{dz^2}\right)\right]\delta H_x$$

$$+ \omega_b^2\left(\omega^2 + v_{Te}^2\frac{d^2}{dz^2}\right)\left(\frac{\omega^2}{c^2} + \frac{d^2}{dz^2}\right)\delta H_x = 0. \qquad (2.1)$$

In the case of a linear density gradient, when $\epsilon(z) = -z/L$, the solution of (2.1) takes the form of a Laplace contour integral. Using the relation

$$\alpha_1\beta^2\delta E_z(s) = (\beta^2 - s^2)\delta H_x(s)$$

between the kernels of the integral representations of the perturbations of the electric and magnetic fields, we obtain

$$\delta E_z(z) = C\int ds\,(s \mp 1)^{\pm i\gamma_2}\left(\frac{s-\beta}{s+\beta}\right)^{i\gamma_1}\exp\left[-i\rho\left(\frac{s^3}{3\lambda^2} + \zeta s - \frac{\sigma}{s \mp 1}\right)\right]. \qquad (2.2)$$

The parameters ρ, λ, σ, and α_1 coincide with those given in Sec. 1,

$$\beta = \left|\frac{v_0}{c}\right|, \quad \zeta = \frac{z - z_0}{L}, \quad z_0 = L\left(\frac{v_{Te}}{v_0}\right)^2\left(\frac{\omega_b^2}{\omega^2} - \frac{k_\perp^2 v_0^2}{\omega^2}\right),$$

$$\gamma_1 = \frac{ck_\perp^2 L}{2\omega}\left(1 - \frac{v_{Te}^2}{c^2}\right), \quad \gamma_2 = \frac{2\omega_b^2 v_{Te}^2 L}{\omega|v_0|^3}.$$

The meaning of the signs + and – and the value of C are analogous to those indicated in (1.4). The system of contours is indicated in Fig. 2a for $v_0 > 0$ and in Fig. 2b for $v_0 < 0$. If C is chosen the same, the solutions on the contours satisfy the relations

$$B_{1,2} + F_{2,1} + D_{2,1} = 0, \quad D_1 + D_2 + A_3 = (F_1 + F_2)\exp(2\pi\gamma_1),$$

$$\bar{B}_{1,2} + B_{1,2}\exp(2\pi\gamma_2) = 0, \quad A_1 + A_2 = V + D_{2,1} + F_{2,1} + \bar{B}_{1,2}.$$

We note that the quasistatic approximation for the electric field in the vicinity of the resonance point $\epsilon = 0$ follows from (2.2) in the limit as $c \to \infty$. In this case

$$[(s-\beta)/(s+\beta)]^{i\gamma_1} \to \exp(-i\rho\alpha_2^2/s); \quad \alpha_2 = \beta\alpha_1.$$

2. Let us investigate wave transformation when the beam moves in the direction of increasing plasma density, i.e., along the density gradient. It is convenient here and subsequently to characterize each solution by a scattering diagram in which the circle denotes the interaction region and the lines correspond to waves; the notation is the following: t—cold plasma molds, l—Langmuir waves, $b_{2,1}$—fast and slow beam modes (b_1 increases in the direction of beam motion in the instability region). Finally, the arrows indicate the wave propagation direction. Typical scattering diagrams are shown in Fig. 3. In order not to encumber the text with long formulas of like type, we introduce symbols for parts of the solutions:

$$U_{1,2} = \pm 2\beta E_0\Gamma(1 \pm i\gamma_1)\Psi(1 \pm i\gamma_1, 2; xe^{\pm i\pi/2})\exp\left(-i\rho\sigma \pm \frac{2i}{3\lambda^2}\rho\beta^3 \mp i\frac{x}{2}\right),$$

$$U_{3,4} = \pm i\pi E_0\left(\frac{\sigma}{\xi}\right)^{p/2} H_p^{(1,2)}(u)\exp\left(\frac{2i}{3\lambda^2}\rho - 2i\rho\alpha_2^2 - i\rho\xi \pm i\pi p\right), \qquad (2.3)$$

$$U_{5,6} = E_0\frac{(\pi\lambda/\rho)^{1/2}}{\zeta^{1/4}}\exp\left[\pm i\frac{\pi}{4} \pm \frac{2}{3}\rho\lambda\zeta^{3/2} \pm i\rho\frac{\alpha_2^2}{s_0} + i\gamma_2\ln(s_0 \pm 1) - \frac{i\rho\sigma}{1 \pm s_0}\right].$$

Here Ψ is a confluent hypergeometric function, $H_p^{(1,2)}$ is a Hankel function, E_0 is a normalization constant, $p = 1 + i\gamma_2$,

$$x = 2\rho\beta(\zeta + \sigma + \lambda^{-2}), \quad \xi = \zeta + \lambda^{-2} - (k_\perp v_0/\omega)^2,$$

$$u = 2\rho(\sigma\xi)^{1/2}e^{-i\pi/2}.$$

The quantity $s_0 = \lambda\zeta^{1/2}e^{-i\pi/2}$ is the saddle point in (2.2). For the sake of argument, we put $0 < \arg \zeta \leq \pi$ throughout.

Let us consider the incidence of a Langmuir wave

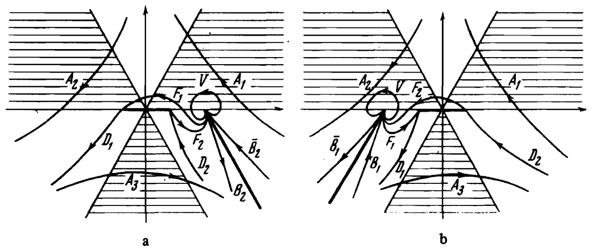


FIG. 2

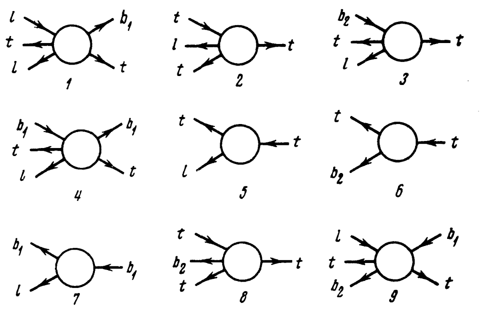


FIG. 3

on the interaction region. The boundary conditions are satisfied by the solution $\delta E_Z = A_1 + A_2 - \eta D_1$, where $\eta = 1 - e^{-2\pi\gamma_1}$. This solution corresponds to scattering diagram 1 in Fig. 3. To the left of the interaction region ($\xi < 0$), taking (2.3) into account, we obtain the amplitudes of the Langmuir waves:

$$\delta E_z^{(l)} = U_6 + U_5 \exp(-2\pi\gamma_1 - 2\pi\gamma_2)$$

and of the reflected cold-plasma mode

$$\delta E_z^{(t)} = \eta U_2 \exp(-\pi\gamma_2 - i\pi/4).$$

In the interaction region, the incident wave excites a slow beam mode $\delta E_Z^{(b)} = U_3 e^{3i\pi/4}$, which increases along the beam, and also a cold-plasma mode

$$\delta E_z^{(t)} = \eta U_1 \exp(-\pi\gamma_2 - i\pi/4),$$

that penetrates into the interior of the plasma ($\xi > 0$).

The reflection and transmission coefficients R and T, determined by the energy-flux ratio, are equal to

$$R_{ll} = \exp(-2\pi\gamma_2 - 4\pi\gamma_1), \quad R_{lt} = \eta \exp(-2\pi\gamma_1 - 2\pi\gamma_2),$$
$$T_{lt} = \eta \exp(-2\pi\gamma_2).$$

We see therefore that the reflection coefficient R_{ll} of the Langmuir wave decreases exponentially when the wave-vector component transverse to the inhomogeneity direction increases. The fraction of the incident-wave power consumed in the generation of the beam mode is equal to $1 - \exp(-2\pi\gamma_2)$.

When a cold plasma is incident on the interaction region from the side of lower densities, the solution is $D_2 + A_3$, corresponding to diagram 2. To the right of the region $\xi > 0$ the solution has the asymptotic form $\delta E_Z = U_1 \exp(-2\pi\gamma_1)$. At $\xi < 0$ there is added to the solution the Langmuir-wave field

$$\delta E_z^{(l)} = U_5 \exp(-2\pi\gamma_1 - 3i\pi/4).$$

The conversion coefficients are equal to

$$R_{tt} = \eta^2, \quad T_{tt} = \exp(-2\pi\gamma_1), \quad R_{tl} = \eta \exp(-2\pi\gamma_1).$$

We see that in this case there is no transformation of the incident wave into beam modes.

The transformation of the fast space-charge wave (phase velocity $v_{ph} > v_0$) is described by the solution $F_1 + D_1 \exp(-2\pi\gamma_1)$ with scattering diagram 3. At $\xi > 0$, the asymptotic value of the field is determined by the cold-plasma mode $\delta E_Z = \eta U_1$, and in the region $\xi < 0$ it takes the form

$$\delta E_z = \eta U_2 + U_4 e^{i\pi p} + U_5 \exp(i\pi/4 - 2\pi\gamma_1).$$

It follows therefore that

$$R_{b_2 l} = \exp(-4\pi\gamma_1), \quad T_{b_2 t} = \eta, \quad R_{b_2 t} = \eta \exp(-2\pi\gamma_1).$$

The transformation of the slow space-charge wave corresponds to the solution $V + F_1 + D_1 \exp(-2\pi\gamma_1)$ with diagram 4. To the right of the interaction region, the solution contains a growing beam mode, $\delta E_Z^{(b)} = U_3$, and a cold-plasma mode $\delta E_Z^{(t)} = \eta U_1 e^{i\pi p}$, while to the left it takes the form

$$\delta E_z = U_3 + \eta U_2 - U_5 \exp(i\pi/4 - 2\pi\gamma_1).$$

The conversion coefficients are

$$R_{b_1 t} = \eta \exp(-2\pi\gamma_1 - 2\pi\gamma_2), \quad R_{b_1 l} = \exp(-2\pi\gamma_2 - 4\pi\gamma_1),$$
$$T_{b_1 t} = \eta \exp(-2\pi\gamma_2).$$

Incidence of a mode t on the interaction region from the direction of the denser part of the plasma corresponds to the solution D_1 with diagram 5. The fields of the Langmuir wave and of the cold-plasma mode are respectively equal to $U_5 \exp(5i\pi/4)$ and U_2. From this we get, using (2.3),

$$T_{tl} = \eta, \quad T_{tt} = 1 - \eta.$$

As before (see Sec. 1), the solution V describes perturbations concentrated in the beam:

$$V \sim (\sigma/\xi)^{p/2} I_{-p}(2\rho\sqrt{\sigma\xi})\, e^{-i\rho\xi},$$

and passing through the interaction region without being transformed into other waves.

3. We now investigate wave transformation when a beam moves in a direction opposite to the plasma-density gradient ($v_0 < 0$). In this case we introduce in place of (2.3)

$$W_{1,2} = U_{1,2} e^{2i\rho\sigma},$$
$$W_{3,4} = \pm i\pi E_0 \left(\frac{\sigma}{\xi}\right)^{q/2} H_q^{(1,2)}(u) \exp\left(i\rho\xi + 2i\rho\alpha_2^2 \pm i\pi q - \frac{2i}{3\lambda^2}\rho\right),$$
$$W_{5,6} = E_0 \left(\frac{\pi}{\rho}\lambda\right)^{1/2} \zeta^{-1/4} \exp\left[\pm i\frac{\pi}{4} \pm \frac{2}{3}\rho\lambda\zeta^{3/2} - i\gamma_2 \ln(s_0 \mp 1) \pm i\rho\frac{\alpha_2^2}{s_0} + \frac{i\rho\sigma}{1 \mp s_0}\right] \tag{2.4}$$

where $q = 1 - i\gamma_2$. The remaining notation coincides with that introduced in (2.3).

We consider the incidence of a cold-plasma mode on the interaction region from the right. The solution is F_1 with diagram 6. Taking (2.4) into account, we obtain the asymptotic form of the field of the mode t, $\delta E_Z^{(t)} = W_2$, which is valid in the entire upper ζ half-plane. At $\xi < 0$ we have $\delta E_Z = W_2 - W_3$. Using (2.4) and the formulas for the transformation of the confluent hypergeometric function (see [19]) we obtain

$$T_{tb_2} = 1 - \exp(-2\pi\gamma_1), \quad T_{tt} = \exp(-2\pi\gamma_1).$$

It should be noted here that the circling around the singular point x = 0 of the function W_2 corresponds to introduction of normal dissipation in the vicinity of the point x = 0 for the incident cold-plasma mode. In this case, the effective dissipation mechanism is the generation of a fast beam mode. At the same time, for the beam mode, the expression $\delta E_z^{(b)} = W_3$ is valid in the lower half-plane, corresponding to introduction of anomalous dissipation and to the circling around the singular point u = 0 of the function W_3 from below.

The solution $\overline{B}_1$ describes the transformation of a slow beam mode, that grows from the interior of the plasma in the direction of the beam motion. It corresponds to the diagram 7. The wave fields are given by the expressions

$$\delta E_z^{(b)} = W_2 e^{i\pi q}, \quad \delta E_z^{(l)} = W_5 e^{i\pi/4}.$$

In this case an enhanced beam mode generates at the intersection point a Langmuir oscillation in such a way that the fluxes in both waves are equal in magnitude.

The transformation of the cold-plasma mode t incident on the interaction region from the direction of lower plasma densities is described by the solution $B_1 - D_2 - A_3$ with diagram 8. For the t-mode field we have $\delta E_z^{(t)} = W_1$. To this expression there is added at $\xi < 0$ a fast beam mode $\delta E_z^{(b)} = -W_3$. The conversion coefficients are equal to

$$R_{tt} = \eta^2, \quad T_{tt} = 1-\eta, \quad R_{tb_2} = \eta(1-\eta).$$

Let us consider, finally, the incidence of a Langmuir wave on the oscillation interaction region. In this case it is necessary to investigate the solution $D_2 + F_2 + \eta F_1$, corresponding to the diagram 9. To the right of the interaction region we have

$$\delta E_z = \eta W_1 + (1-\eta) W_4,$$

where W_4 attenuates in the interior of the plasma. At $\xi < 0$ the field is equal to

$$\delta E_z = \eta W_2 + W_6 e^{i\pi/4} + (1-\eta) W_4.$$

Comparing the expressions, we obtain the conversion coefficients for the incident wave:

$$R_{lb_2} = \exp(-4\pi\gamma_1), \quad R_{lt} = \eta(1-\eta), \quad T_{lt} = \eta.$$

The solution considered here is a special case of interference, in the interaction region, between an incident Langmuir wave and a beam mode that increases from the interior of the plasma, in which there is no reflected Langmuir oscillation.

We present also asymptotic formulas for the solution

$$\delta E_z = F_1 + F_2 + D_2 + D_1 \exp(-2\pi\gamma_1).$$

To the right of the interaction region ($\xi > 0$) we have

$$\delta E_z = \eta W_1 e^{-i\pi/4},$$

and to the left ($\xi < 0$) we have a superposition of an incident Langmuir wave and reflected waves:

$$\delta E_z = W_6 + W_5 \exp(-\pi\gamma_2 - 2\pi\gamma_1) + \eta W_2 \exp(-i\pi/4) + (W_3 + W_4)\exp(-i\pi/4 - 2\pi\gamma_1). \quad (2.5)$$

Wit the aid of (2.4) we obtain from (2.5)

$$R_{ll} = \exp(-2\pi\gamma_2 - 4\pi\gamma_1) = R_{lb_1}, \quad T_{lt} = \eta,$$
$$R_{lt} = \eta(1-\eta), \quad R_{lb_2} = \exp(-4\pi\gamma_1).$$

It is easy to establish that

$$1 + R_{lb_1} = R_{ll} + R_{lt} + T_{lt} + R_{lb_2}.$$

At $2\pi\gamma_1 \gg 1$, the incident Langmuir wave goes over adiabatically into the cold-plasma mode.

Let us dwell briefly on the case of a cold plasma, when $v_{Te} = 0$. Certain solutions have then a singularity at the point $\epsilon = 0$, and this singularity moves away from the real z axis when dissipation is introduced. When dissipation is taken into account we have

$$\varepsilon = 1 - \omega_{pe}^2(z)/\omega(\omega - i\nu),$$

where ν is the effective collision frequency, and for a linear density variation, $\omega_{pe}^2(z) = \omega_{pm}^2 Z/d$, we have

$$\varepsilon = 1 - \frac{z}{L_\omega} \equiv -\zeta, \quad \text{where} \quad L_\omega = \frac{d(\omega^2 + \nu^2)}{\omega_{pm}^2(1 + i\nu/\omega)}$$

is the complex inhomogeneity length. For the electric field we obtain the Laplace integral

$$\delta E_z = C \int ds \left(\frac{s-\beta}{s+\beta}\right)^{i\gamma} \exp\left[-i\rho\left(\zeta s \pm \frac{\mu}{1 \mp s}\right)\right]; \quad (2.6)$$
$$\gamma = \frac{ck_\perp^2}{2\omega} L_\omega, \quad \mu = \frac{\omega_b^2}{\omega^2}, \quad \rho = \frac{\omega}{|v_0|} L_\omega.$$

The signs + and − pertain to the case of beam motion with and against the density gradient.

The system of contours for the solutions (2.6) is analogous to that indicated in Fig. 2. On contours that terminate at infinity, the solutions are singular at the point $\zeta = 0$, which is a logarithmic branch point for these solutions. When moving in the complex ζ plane for the singular solutions, arg s satisfies the condition $\arg(\zeta s) = \text{const}$ as $|s| \to \infty$, where the value of the constant is fixed, e.g., as $\zeta \to \infty$ and $\arg \zeta = 0$. For normal dissipation, when $\nu > 0$, the asymptotic forms of the solutions (2.6), apart from terms corresponding to the Langmuir waves, coincide with the expressions obtained above. Of course, this holds if the scattering diagrams are of the same type. For the singular solutions in the vicinity of the point $\epsilon = 0$, energy absorption takes place, and at small ν the absorption coefficient is equal to the coefficient of transformation into a Langmuir wave. When thermal motion is taken into account, $|\epsilon_{eff}|$ has a lower bound

$$|\varepsilon_{eff}| \geq \frac{\nu_{eff}}{\omega} = \left(\frac{v_{Te}}{\omega L_\omega}\right)^{2/3}.$$

We can draw the following conclusions from the foregoing investigation: In a plasma with a longitudinal density gradient, the point of intersection of the beam-oscillation modes with the plasma oscillations lies closer to the plasma boundary than the region of beam instability. As a result, when the beam moves parallel to the density gradient, i.e., into the plasma, the plasma oscillations are excited (if the density has a monotonic variation) only as a result of the initiating beam modulation in the vicinity of the intersection point. Only the slow space-charge waves is amplified in the instability region. If the beam moves in the opposite direction, a beam space-charge wave grows from the interior of the plasma, and generates in the vicinity of the intersection point plasma oscillations that go over into Langmuir waves far from the intersection point. In the stability region, the energy flux in a Langmuir wave traveling towards the plasma boundary is equal to the flux in the slow space-charge wave. The generation of the transverse waves by the beam is anisotropic, since the transverse oscillations emerging to the vacuum are generated only when the modulated beam moves in the direction of increasing plasma density. The latter circumstance was pointed out earlier[12] for the case of an inhomogeneous plasma without a magnetic field.

4. The plasma oscillations can be amplified by the beam if its transverse dimensions are bounded. The

problem of the interaction of a bounded cylindrical beam with a homogeneous infinite plasma was considered by Bogdanov, Kislov and Chernov[20]. The amplification mechanism consists in the following: When the modulated beam moves through the plasma, a slow space-charge wave builds up in the beam, and is transformed on the beam boundary into plasma oscillations, as a result of the latter also grow along the beam. We note that the outflow of a slow space-charge wave having a negative energy from the beam leads in accordance with the general concepts concerning waves with negative energy[21,22], to amplification of the wave.

We shall now show that the increment κ of the spatial wave amplification is connected by a simple relation with the coefficient Q of transformation of the space-charge wave on the beam boundary. In planar geometry for a beam of density n_0 and a diameter 2a, the dispersion equation of the oscillations is

$$k_2 = ik_1 \operatorname{tg} k_2 a, \quad k_1 = k_z \tilde{\varepsilon}^{1/2}, \qquad (2.7)$$
$$k_2 = k_z[\tilde{\varepsilon} + \omega_b^2/(\omega - k_z v_0)^2]^{1/2},$$

where $\tilde{\epsilon} = (\omega_{pe}/\omega)^2 - 1$ and the perturbations are proportional to $\exp(i\omega t - ik_z z)$. The dispersion equation (2.7) corresponds to a symmetrical solution.

From (2.7) we obtain near the instability boundary $\tilde{\epsilon} \ll |\omega_b/(\omega - k_z v_0)|^2$

$$k_z = \frac{\omega}{v_0}\left[1 + \frac{\omega_b}{k_\perp v_0}\left(1 + \frac{i\omega\tilde{\varepsilon}^{1/2}}{k_\perp^2 a v_0}\right)\right], \quad k_\perp = \frac{\pi}{a}\left(n + \frac{1}{2}\right), \quad n = 0, 1, 2, \ldots, \qquad (2.8)$$

whence

$$\varkappa = \frac{\omega_b}{v_0}\left(\frac{\omega}{k_\perp v_0}\right)^2 \frac{\tilde{\varepsilon}^{1/2}}{k_\perp a}.$$

In the frequency region $\tilde{\epsilon} \gg |\omega_b/(\omega - k_z v_0)|^2$ we obtain from (2.7)

$$k_z = \frac{k_\perp}{\tilde{\varepsilon}^{1/2}}\left[1 + i(k_\perp a)^{-1} \ln (\omega - k_z v_0)\frac{2\tilde{\varepsilon}^{1/2}}{\omega_b}\right], \qquad (2.9)$$

and consequently

$$\varkappa = (a\tilde{\varepsilon}^{1/2})^{-1} \ln\left[2\frac{\tilde{\varepsilon}^{1/2}}{\omega_b}(\omega - k_z v_0)\right].$$

We now determine the connection between the increment κ and the coefficient Q. Let $\kappa \ll |k_z|$; on the one hand, during the time $\Delta t = 4a/v_\perp$ required for the beam mode to travel across the beam and back the mode moves along the beam a distance $\Delta z = v_\parallel \Delta t$ (where $v_\perp$ and $v_\parallel$ are the group-velocity components). Its amplitude is increased at the same time by a factor $e^{\kappa \Delta z}$. On the other hand, after two acts of transformation, the amplitude increases by a factor $Q = |(k_2 + k_1)/(k_2 - k_1)|^2$. Equating, we get

$$\varkappa = \frac{v_\perp}{4av_\parallel} \ln Q. \qquad (2.10)$$

Determining $v_\parallel$ and $v_\perp$ with the aid of (2.8) and (2.9) and substituting them in (2.10), we arrive at the expressions obtained above for κ. We note also that for a bounded beam the oscillations are unstable also in the frequency region $\omega < \omega_{pe}$, whereas for an unbounded beam the range of frequencies of the unstable oscillations is narrower, $\omega < (\omega_{pe}^2 - k_\perp^2 v_0^2)^{1/2}$. This circumstance is the consequence of the outflow of negative-energy waves from the beam into the plasma at $\epsilon < 0$.

3. RADIATION OF THE CHARGE IN A WEAKLY-INHOMOGENEOUS PLASMA

If we put $n_0 = 0$ in (2.2) and (2.6), choose the normalization constant in suitable manner, and place the origins of the contours at the points $s = \pm 1$, corresponding to the motion of the charge with and against the gradient, we obtain solutions that describe waves radiated by the charge in a weakly-inhomogeneous plasma. Thus, in a cold plasma the field of the charge is given by

$$\delta E_z(k_\perp, \omega; z) = \mp \frac{eL_\omega}{2\pi^2|v_0|}\left(\frac{1+\beta}{1-\beta}\right)^{i\gamma_1} \int_{\pm 1} ds \left(\frac{s+\beta}{s+\beta}\right)^{i\gamma_1} e^{-i\rho\zeta s \mp i\rho}, \qquad (3.1)$$

where the parameters β, γ_1, ρ, and ζ coincide with those introduced in formulas (1.4), (2.2), and (2.6). The boundary conditions of the radiation are satisfied by the solution $F_1 + De^{-2\pi\gamma}$ for Fig. 2a when the charge moves along the density gradient. In the case of opposite motion of the charge, they are satisfied by the solution $\overline{B}_1$. This choice of contours is made for $\zeta \to +\infty$. Variation of ζ produces a deformation of the contours, analogous to that indicated for the solutions (2.6).

It follows from the foregoing that the singularities of beam-mode transformation into plasma oscillations, which were considered in Sec. 2, should become manifest in the form of singularities in the generation of plasma oscillations by a moving charge. Thus, when the charge moves along the density gradient, transverse waves are present in addition to the field of moving charge. The spectral energy density of the radiation is accordingly

$$I_+(k_\perp, \omega) = \frac{e^2 L_\omega}{4\pi\omega}(1 - e^{-2\pi\gamma_1}), \quad I_-(k_\perp, \omega) = I_+(k_\perp, \omega) e^{-2\pi\gamma_1}. \qquad (3.2)$$

We see therefore that the backward radiation is exponentially small outside the cone

$$\theta < \arcsin\left(\frac{c}{\pi\omega L_\omega}\right)^{1/2}, \quad \sin\theta = \frac{ck_\perp}{\omega},$$

where θ is the angle at which the radiation emerges to the vacuum.

We note that in the case of a plasma without the magnetic field, the radiation cone is determined by the angle $\arcsin (c/\pi\omega L_\omega)^{1/3}$. It follows also from (3.2) that at $2\pi\gamma > 1$ the transverse waves are radiated predominantly forward. At $\gamma \ll 1$ the intensity of the radiation decreases as the result of the decreased dimension of the region of interaction between the charge and the plasma oscillations. When the charge moves against the plasma density gradient, no transverse waves are radiated. Allowance for the thermal motion shows that in this case, near the point of synchronism of the charge with the oscillations of a plasma situated in the region $\epsilon < 0$, a quasipotential plasma mode is generated, which goes over into a Langmuir wave far from the synchronism point.

Let us consider the purely formal case $\beta > 1$. If $\beta < 1$, transverse waves are radiated independently of the direction of charge motion only in vacuum. The synchronism point is located then beyond the opacity barrier, in the region $\epsilon > 0$, so that the radiation has no exponential smallness. Superluminal velocities of the source are possible when the charge moves at an angle to the inhomogeneity. Thus, in the case of an isotropic plasma, for a charge moving with velocity v_0 at an angle ϑ_c to the direction of the inhomogeneity, the effective source velocity along the inhomogeneity is

$$v_{eff}=v_0\left(1-\frac{k_\perp v_0}{\omega}\sin\vartheta_c\right)^{-1}\cos\vartheta_c.$$

The velocity $v_{eff}(\vartheta_c)$ has a maximum $v_0(1-k_\perp^2 v_0^2/\omega^2)^{-1/2}$ at $\vartheta_c = \arcsin(k_\perp v_0/\omega)$ and can exceed the velocity of light if

$$v_0>c(1+k_\perp^2c^2/\omega^2)^{-1/2}.$$

Other possibilities of producing superluminal motion of a source were discussed earlier by Ginzburg and Bolotovskiĭ[23]. We note that in a magnetoactive plasma the possibility of superluminal source motion indicated above leads, as can be shown, to a direct synchronization of the charge with the transverse waves. Although direct synchronization of the charge with the transverse waves is impossible in an isotropic plasma, yet the increase of v_{eff} can increase sharply the order of magnitude of the transition radiation on the smeared boundary, namely, the formula obtained by Galeev[24] for the intensity of the transition radiation now takes the form

$$I\sim\exp(-\omega L_\omega/v_{eff}).$$

The obtained anisotropy of the transverse waves radiated by a charge in a weakly inhomogeneous plasma situated in a strong magnetic field is analogous to that indicated earlier[12] for a plasma without a magnetic field. This anisotropy is preserved also when account is taken of the nonlinearity of the density gradient. For the density of a plasma (situated in a strong magnetic field)

$$\omega_{pe}^2(z)=\frac{1}{2}\omega_{pm}^2\left(1+\mathrm{th}\frac{z}{L}\right)$$

the analysis shows that the emission of transverse waves by a nonrelativistic charge ($v_0 \ll c$) moving in the direction of decreasing density is exponentially small (like $\exp(-\omega L/v_0)$), in analogy with the transition radiation investigated by Galeev[24]. In the case of $\nu < 0$, the singular point $\xi = 0$ of the solutions (3.1) is circled in the lower ξ half-plane, as the result of which the picture of the emission of the transverse waves by the charge is inverted, and the charge radiates transverse waves when it moves in the direction toward the lower density.

[1]V. L. Ginzburg, Rasprostranenie élektromagnitnykh voln v plazme (Propagation of Electromagnetic Waves in a Plasma), Nauka, 1967 [Gordon and Breech; Pergamon].

[2]V. E. Golant and A. D. Piliya, Usp. Fiz. Nauk 104, 413 (1971) [Sov. Phys.-Uspekhi 14, 413 (1972)].

[3]N. S. Erokhin and S. S. Moiseev, Usp. Fiz. Nauk 109, 225 (1973) [Sov. Phys.-Uspekhi 16, 64 (1973)].

[4]S. S. Moiseev, Trudy mezhdunarodnogo simpozuma po probleme mnogikh tel i fizike plazmy (Proc. Internat. Symposium on the Many-Body Problem and Plasma Physics), Nauka, 1967, p. 184.

[5]S. S. Moiseev, Prikl. Mat. Teor. Fiz., No. 3, 3 (1966).

[6]A. S. Bakai, A. K. Berezin et al. Proc. of the 4-th Intern. conf. on Plasma Physics and controlled Nuclear Fusion Research, Nuclear Fusion Supplement, 1972, p. 113.

[7]N. A. Manzyuk, S. S. Moiseev, A. M. Pozhkov, V. A. Suprunenko, E. A. Sukhomlin, and A. M. Ternopol, Zh. Tekh. Fiz. 43, 869 (1973) [Sov. Phys.-Tech. Phys. 18, 550 (1973)].

[8]A. K. Berezin, G. P. Berezina, N. S. Erokhin, S. S. Moiseev, and Ya. B. Fainberg, ZhETF Pis. Red. 14, 149 (1971) [Sov. Phys.-JETP Lett. 14, 97 (1971)].

[9]I. E. Sakharov and V. I. Fedorov, Zh. Tekh. Fiz. 41, 1539 (1971) [Sov. Phys.-Tech. Phys. 16, 1209 (1972)].

[10]S. S. Kalmykova and V. I. Kurilko, Prikl. Mat. Mekh. 33, 638 (1969) S. S. Kalmykova, Dokl. Akad. Nauk SSSR 208, 1062 (1973) [Sov. Phys.-Doklady 18, 149 (1973)].

[11]T. Jancarik, V. Kopecky, V. Piffl, T. Pohanka, T. Preinhaelter, M. Seidl, T. Ullschmied. Third Intern. Conf. on Plasma Physics and Controlled Nuclear Fusion Research, I.A.E.A., Vienna, 1969, vol. II, p. 733.

[12]N. S. Erokhin, V. I. Kurilko et al. Proc. of the 4-th Intern. Conf. on Plasma Physics and Controlled Nuclear Fusion Research, Nuclear Fusion Supplement, 1972, p. 135.

[13]S. S. Kalmykova, V. I. Kurilko, V. I. Kucherov, and A. P. Tolstoluzhskiĭ, Zh. Eksp. Teor. Fiz. 64, 471 (1973) [Sov. Phys.-JETP 37, 240 (1973)].

[14]V. P. Starr, Physics of Negative Viscosity Phenomena, McGraw, 1971.

[15]W. Wasow, Ann. of Math, 52, 350 (1950).

[16]A. L. Rabenstein, Arch. Rational Mech. and Anal., 1, 418 (1958).

[17]N. S. Erokhin, Ukr. Fiz. Zh. 14, 2059 (1969).

[18]G. M. Zaslavskiĭ, S. S. Moiseev, and R. Z. Sagdeev, Dokl. Akad. Nauk SSSR 158, 1295 (1964) [Sov. Phys.-Doklady 9, 863 (1965)].

[19]A. Erdelyi, Higher Transcendental Functions, McGraw Vol. 1, 1953.

[20]E. V. Bogdanov, V. Ya. Kislov, and Z. S. Chernov, Radiotekhnika i élektronika 5, 229 (1960).

[21]B. B. Kadomtsev, A. B. Mikhaĭlovskii, and A. V. Timofeev, Zh. Eksp. Teor. Fiz. 47, 2266 (1964) [Sov. Phys.-JETP 20, 1517 (1965)].

[22]V. M. Dikasov, L. I. Rudakov, and D. D. Ryutov, Zh. Eksp. Teor. Fiz. 48, 913 (1965) [Sov. Phys.-JETP 21, 608 (1965)].

[23]V. L. Ginzburg and B. M. Bolotovskiĭ, Usp. Fiz. Nauk 106, 577 (1972) [Sov. Phys.-Uspekhi 15, 184 (1972)].

[24]A. A. Galeev, Zh. Eksp. Teor. Fiz. 46, 1335 (1964) [Sov. Phys.-JETP 19, 904 (1964)].

Translated by J. G. Adashko
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