Effect of anomalies of the energy spectrum on relaxation phenomena in helium II

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The first viscosity coefficient and also the dispersion and absorption coefficients for first and second sounds in helium II at temperatures $< 1.7°K$, when the phonon-roton quasiparticle gas can be regarded as perfect, are calculated. The phonon part of the momentum dependence of energy of elementary excitations is assumed to be in the form $\epsilon = cp\,(1 + \gamma p^2 - \delta p^4)$,[8] where ϵ is the energy, p the phonon momentum, $\gamma = 2.6 \times 10^{37}$, $\delta = 2.5 \times 10^{75}$ CGSE. The existence in the roton part of the curve of a roton sound momentum $P_c = P_0 + \mu c$, where $P_0 = 1.9\ \mathrm{cm}^{-1}$, $\mu = 0.16 m_{\mathrm{He}^4}$ is the roton mass. The kinetic equation for quasiparticles is obtained by a method first developed by Landau and Khalatnikov[1] and developed further by Khalatnikov and Chernikova.[2] A comparison of the theory with the experiments is carried out.

INTRODUCTION

The viscosity and the dispersion of sound in liquid He^4 below the λ point was first calculated by Landau and Khalatnikov [1] and by Khalatnikov and Chernikova[2]. It was assumed in these papers that the phonon section of the energy spectrum is given by

$$\varepsilon = cp(1 - \gamma' p^2), \tag{1}$$

where γ' was estimated by Landau and Khalatnikov at 2.8×10^{37} cgs esu (p is the phonon momentum). The energy and momentum conservation laws allow for such a spectrum only collinear three-phonon processes, which add nothing to the kinetic phenomena. As a result, only four- and five-phonon processes had been considered. It turned out that the cross section for small-angle elastic scattering of a phonon by a phonon is anomalously large. This process therefore ensures rapid establishment of energy equilibrium for phonons moving in a specified direction.

It was shown that at temperatures <1.2°K the five-phonon process ensures establishment of equilibrium with respect to the number of phonons in a given direction, and the distribution function for such phonons is

$$n(\varepsilon) = [e^{\varepsilon/kT} - 1]^{-1}. \tag{2}$$

At temperatures >1.2°K, the five-phonon process time becomes comparable with the phonon-roton scattering time, and the distribution function takes the form

$$n(\varepsilon) = [e^{(\varepsilon+\alpha)/kT} - 1]^{-1}. \tag{3}$$

The chemical potential α and the temperature T in (2) and (3) are functions of the direction: $\alpha = \alpha(\cos\theta)$, $T = T(\cos\theta)$.

Equilibrium in a roton gas is ensured by large-angle roton-roton scattering, and therefore rotons are described by Boltzmann functions

$$N(\mathscr{E}) = \exp\left[-\frac{\mathscr{E} - \mathbf{P}(\mathbf{v}_n - \mathbf{v}_s)}{kT_r}\right]^{-1}; \tag{4}$$

here $\mathscr{E}$ is the roton energy, P is the roton momentum, and v_n and v_s are the velocities of the normal and superfluid components.

Landau and Khalatnikov obtained next[1] from the kinetic equation the nonequilibrium distribution, and then the viscosity coefficient, while Khalatnikov and Chernikova[2] added to the kinetic equation the equations for the conservation of the mass and of the superfluid motion, and obtained the dispersion and the absorption coefficient of sound from the condition of compatibility of these equations.

Recent papers[3-6] point to the possible existence of positive dispersion in the phonon part of the dispersion curve. Nonparallel three-phonon processes are then possible. We shall show below that the fastest process which establishes equilibrium both with respect to energy and with respect to the number of phonons. The fact that the roton has acoustic momentum will be taken into account in the roton part of the dispersion curve[7].

According to Jackle and Kehr[3] there exists a limiting momentum p_c such that

$$\left.\frac{\partial\varepsilon}{\partial p}\right|_{p<p_c} \geqslant c. \tag{5}$$

Taking the phonon part of the spectrum in the form[8]

$$\varepsilon = cp(1 + \gamma p^2 - \delta p^4),$$

we obtain p_c [3]:

$$p_c = (3\gamma / 5\delta)^{1/2} \approx 0.2\ \text{Å}^{-1}, \quad T_c = 3{,}86°\ K. \tag{6a}$$

An important role is played in kinetic phenomena by thermal phonons of energy

$$pc = xkT, \tag{6b}$$

where x is an integer. Therefore, starting with a certain temperature, the thermal-phonon momentum will exceed the limiting momentum p_c at a fixed x. Since the dispersion is normal at $p > p_c$, the equilibrium with respect to energy is established in this section by small-angle elastic scattering of the phonons by phonons. Calculation shows that we can use in the estimates the relaxation time calculated for this process by Khalatnikov[9]:

$$\frac{1}{t_{\mathrm{ph}}} = \frac{4{,}15(u+1)^2}{192\pi^3\gamma' c\rho^2}\left(\frac{kT}{\hbar c}\right)^7\left(1 + 14.7\frac{1}{x}\right), \quad x \gg 1;$$
$$u = \frac{\rho}{c}\frac{\partial c}{\partial \rho} = 2.84, \tag{7}$$

since the replacement of the spectrum (1) by (6) introduces in (7) an inessential factor on the order of unity.

We note in conclusion that it follows from momentum and energy conservation that a three-phonon process in which one of the phonons has normal dispersion is impossible. Therefore the phonon momenta in the three-phonon processes should satisfy the inequality $p \le p_c$ or, taking (6a) and (6b) into account,

$$xT \leqslant 3.8. \tag{8}$$

The values of x that enter in (8) will be shown later on.

INTERACTION OF EXCITATIONS

1. Absorption and Emission of Phonons by Phonons

The interaction Hamiltonian for this process[1] takes the form

$$V_3=\frac{1}{2}\mathbf{v}\rho'\mathbf{v}+\frac{1}{3!}\frac{\partial}{\partial\rho}\left(\frac{c^2}{\rho}\right)\rho'^3,\quad \rho_0=\rho+\rho'. \tag{9}$$

The matrix elements for the Fourier components of the density ρ_p' are

$$(\rho_{\mathbf{p}}')_{n_p,\,n_p+1}=\left[\frac{\rho p}{2c}(n_p+1)\right]^{1/2}e^{-i\omega t}$$

$$(\rho_{\mathbf{p}}'^*)_{n_p,\,n_p-1}=\left[\frac{\rho p}{2c}n_p\right]^{1/2}e^{i\omega t}, \tag{10}$$

$$\mathbf{v}_{\mathbf{p}}=\frac{c\mathbf{p}}{\rho p}\rho_{\mathbf{p}}',\qquad \omega=\frac{cp}{\hbar}.$$

Calculating the absorption and decay cross sections, we obtain

$$d\sigma(p_1,p_2)=\frac{\pi|\mathbf{p}_1||\mathbf{p}_2||\mathbf{p}_1\pm\mathbf{p}_2|(u+1)^2}{\hbar\rho}\delta(\varepsilon(|\mathbf{p}_1|)\pm\varepsilon(|\mathbf{p}_2|)-\varepsilon(|\mathbf{p}_1\pm\mathbf{p}_2|)), \tag{11}$$

where the plus sign corresponds to absorption and the minus sign to decay. We consider the case $|\mathbf{p}_1|\gg|\mathbf{p}_2|$, and then the two cross sections are equal.

The relaxation time for the parallel three-phonon process will be calculated approximately, assuming only the distribution functions for the phonons with the largest momenta to be perturbed[1]. Then

$$\frac{1}{\tau_{1\to2}}\approx\frac{1}{\tau_{2\to1}}\approx\frac{c}{(2\pi\hbar)^3}\int d\sigma(p_1,p_2)\left[\exp\left(\frac{cp_2}{kT}\right)-1\right]^{-1}d\mathbf{p}_2$$

$$=\frac{(u+1)^2\zeta(2)}{2\pi\hbar^4\rho}r^2\left(\frac{kT}{c}\right)^5 \tag{12}$$

$$\zeta(2)=1.63.$$

In the integral (12), the main contribution is made by phonons with momenta $p_2=2kT/c$, and the condition $|\mathbf{p}_1|\gg|\mathbf{p}_2|$ is satisfied, since we shall assume below that only phonons with momenta $p_1=6kT/c$ are perturbed. The extension of the integration in (13) to infinity means neglect of terms $\sim\exp(-T_c/T)\sim e^{-6}\ll1$ at a temperature $T<0.6$°K.

It is easy to verify that the "induced" three-phonon process is forbidden for thermal phonons, since the energy deficit is larger than the line width. Indeed,

$$\Delta E_3=|\varepsilon(|\mathbf{p}_1|)+\varepsilon(|\mathbf{p}_2|)-\varepsilon(|\mathbf{p}_1+\mathbf{p}_2|)|\approx cp_2 3\gamma p_1^2.$$

The line width is equal to

$$\Gamma=\hbar/2\tau_{1\to2}(p_2).$$

Substituting $p_1=6kT/c$ and $p_2=2kT/c$, we obtain

$$\Delta E_3>\hbar/2\tau(p_2).$$

Landau and Khalatnikov[1] have shown that the four-phonon collision integral converges without allowance for dispersion, and therefore the reversal of the sign of γ' in (1) reduces to the appearance of a three-phonon collision integral, the exact form of which we can write out, because the relaxation times for this process are

$$\frac{1}{\tau}\sim\frac{1}{\tau_{1\to2}}3\gamma p^2\ll\frac{1}{\tau_{1\to2}}$$

and equilibrium has been established.

2. Absorption and Emission of Phonons by Rotons

Just as in [1], the phonon-roton interaction energy is equal to

$$\mathscr{H}_{\mathrm{ph\,r}}=\frac{1}{2}(\mathbf{P}\mathbf{v}+\mathbf{v}\mathbf{P})+\left(\frac{\partial\Delta}{\partial\rho}+\frac{P_0}{\mu}\frac{\partial P_0}{\partial\rho}-\frac{P_0^2}{2\mu^2}\frac{\partial\mu}{\partial\rho}\right)\rho'$$

$$-\frac{1}{2}\left[(P\rho'+\rho'P)\left(\frac{1}{\mu}\frac{\partial P_0}{\partial\rho}-\frac{P_0}{\mu^2}\frac{\partial\mu}{\partial\rho}\right)+\frac{1}{2\mu^2}\frac{\partial\mu}{\partial\rho}(P^2\rho'+\rho'P^2).\right. \tag{13}$$

The matrix element is

$$(\mathscr{H}_{\mathrm{ph\,r}})_{\mathbf{p},\mathbf{P};\mathbf{p}+\mathbf{P}}=\frac{1}{2}(2\mathbf{P}+\mathbf{p})\mathbf{v}_{\mathbf{p}}+\left(\frac{P_0}{\mu}\frac{\partial P_0}{\partial\rho}-\frac{P_0^2}{2\mu^2}\frac{\partial\mu}{\partial\rho}\right)\rho_{\mathbf{p}}$$

$$-\frac{1}{2}\left[(|\mathbf{p}|+|\mathbf{P}+\mathbf{p}|)\left(-\frac{P_0}{\mu^2}\frac{\partial\mu}{\partial\rho}+\frac{1}{\mu}\frac{\partial\mu}{\partial\rho}\right)+\frac{1}{2\mu^2}\frac{\partial\mu}{\partial\rho}(p^2+|\mathbf{P}+\mathbf{p}|^2)\right]\rho_{\mathbf{p}}. \tag{14}$$

Estimates show that the term with $\sigma\Delta/\partial\rho$ in (14) can be neglected. The cross section for the absorption of a phonon by a roton is expressed, accurate to terms p/P, in the form

$$d\sigma(p,P)=\frac{2\pi}{\hbar c}(\mathscr{H}_{\mathrm{ph\,r}})^2_{\mathbf{p},\mathbf{P};\mathbf{p}+\mathbf{P}}\delta\left(pc-\frac{2p(P-P_0)\cos\theta+p^2\cos^2\theta}{2\mu}\right). \tag{15}$$

CALCULATION OF THE VISCOSITY COEFFICIENT

As already noted, all the changes introduced by the considered spectrum consist in the appearance of additional collision integrals in the right-hand side of the kinetic equation. We now write out these integrals.

The three-phonon collision integrals

$$I_3(n_1)=-\frac{c}{(2\pi\hbar)^3}\left\{\int d\sigma(p_1,p_2)[n_1n_2(n_3+1)-n_3(n_1+1)(n_2+1)]d\mathbf{p}_2\right.$$

$$\left.+\int d\sigma(p_1,p_2)[n_1(n_2+1)(n_3+1)-n_3n_2(n_1+1)]d\mathbf{p}_2\right\},$$

where $n_i=[\exp(\epsilon_i/kT)-1]^{-1}$, can be reduced to the form

$$I_3(n_1)=-\frac{108\beta P_2(\cos\theta)(u+1)^2\zeta(4)\gamma P_1^3n_1}{\pi\hbar^4\rho}\left(\frac{kT}{c}\right)^4. \tag{16}$$

We have used here the approximation[1] $\delta T/T=\beta P_2(\cos\theta)$, where $P_2(\cos\theta)$ is a Legendre polynomial.

In exactly the same way, we write out the collision integral for the absorption and emission of a phonon by a roton, recognizing that in the presence of a velocity gradient the deviation of the roton distribution function from equilibrium will be much less than the deviation of the phonon function, so that the roton distribution functions can be assumed to be at equilibrium[1]:

$$I_{\mathrm{ph\,r}}(n)=-\delta n(p)\frac{ce^{-\Delta/T}}{(2\pi\hbar)^3}\int_{P_c}^{\infty}\exp\left[-\frac{(P-P_0)^2}{2\mu T}\right]d\sigma(p,P)\,d\mathbf{P},$$

$$\delta n(p)=-n_0(n_0+1)(\alpha-pc\beta/kT). \tag{17}$$

Here $P_c=P_0+\mu c$ is the acoustic momentum of the roton. The lower limit of integration in (17) is the acoustic momentum P_c, but rotons with momenta $P_0\le P\le P_c$ can also emit and absorb phonons with momenta $p_{ph}\ge p'_{ph}$, where p'_{ph} is the minimal momentum at which this process is possible. For rotons with momenta $P\approx P_0$, the phonon momentum is $p'_{ph}\approx2\mu c$, and these processes can be neglected because of the additional factor $\sim\exp(-2\mu c^2/T)\sim\exp(-8.8/T)$.

Writing down the kinetic equation for the phonons in a spherical coordinate system with polar axis directed along the macroscopic velocity vector **u**

$$n_0(n_0+1)\frac{cp}{kT}\frac{du}{dx}\cos\theta\sin\theta\cos\varphi=I_{\mathrm{ph}}(n)+I_3(n) \tag{18}$$

and integrating (18) over all the energies of the phonons moving in a given direction, we obtain an equation for the determination of the unknown quantity β:

$$\frac{4\pi^4}{15}\frac{\partial u}{\partial x} = -\beta\left(\frac{1}{\tau_{\mathrm{ph}}} + \frac{1}{\tau'_{1\to2}}\right), \tag{19}$$

where $1/\tau_{\mathrm{ph}}$ and $1/\tau'_{1\to2}$ are respectively the rates of large-angle phonon-phonon scattering and the absorption of a phonon by a phonon:

$$1/\tau_{\mathrm{ph}} = \frac{3\cdot 13!\,(u+1)^4}{5\cdot 2^{15}(2\pi)^3\hbar^7\rho^2 c}\left(\frac{kT}{c}\right)^9,$$

$$1/\tau'_{1\to2} = \frac{108\cdot 6!\,\zeta(4)\zeta(6)\,(u+1)^2\gamma}{\pi\hbar^4\rho}\left(\frac{kT}{c}\right)^7.$$

In the integration of (18) over all the energies, the principal role was played by phonons with momenta $p_1 = 6kT/c$, and therefore the inequality (8) is satisfied in the region $T < 0.6°K$. Obtaining β and then δn from (19), we get the viscosity coefficient by comparing the two expressions for the momentum z-component transferred in the x direction:

$$\sigma_{xz} = \eta_{\mathrm{ph}}\frac{\partial u}{\partial x} = -(2\pi\hbar)^{-3}\int pc\delta n\cos\theta\sin 0\cos\varphi p^2\,dp\,dO.$$

This yields the viscosity coefficient at $T < 0.6°K$:

$$\eta_{\mathrm{ph}} = \frac{16\pi^2(kT)^4}{75c^3\hbar^3}\left(\frac{1}{\tau_{\mathrm{ph}}} + \frac{1}{\tau'_{1\to2}}\right)^{-1} = \frac{1.03\cdot 10^{-4}}{T^3(1+2.47T^2)}. \tag{20}$$

At temperatures $T > 0.6°K$ it is necessary to take the phonon-roton interactions into account. Proceeding as before, we obtain

$$\frac{\partial u}{\partial x}\frac{4\pi^4}{15} = -\beta\left(\frac{1}{\tau_{\mathrm{ph}}} + \frac{1}{\tau_{\mathrm{ph\,r}}} + \frac{5}{\tau'_{\mathrm{ph\,r}}}\right),$$

$$\frac{1}{\tau_{\mathrm{ph\,r}}} = \frac{2\cdot 7!N_p}{\pi c}\left[\frac{P_0(kT/c)^2}{\hbar^2\rho}\right]^2\left[\frac{2}{15} + \frac{33}{35^2}\left(\frac{P_0}{\mu c}\right)^2 + \frac{14A}{75}\left(\frac{P_0}{\mu c}\right) + A^2\right],$$

$$A = \frac{\rho^2}{P_0 c}\left[\frac{\partial^2\Delta}{\partial\rho^2} + \frac{1}{\mu}\left(\frac{\partial P_0}{\partial\rho}\right)^2\right]; \tag{21}$$

here $1/\tau_{\mathrm{ph\,r}}$ is the rate of elastic scattering of a phonon by a roton, and was obtained in [1] under the condition $p_{\mathrm{ph}} < 2\,\mu c$.

Inasmuch as the principal role in the elastic scattering of phonons by rotons is played by phonons with momenta $p \sim (7-8)\,kT/c$, this condition is violated at $T \gtrsim 1.1°K$. In this case resonant scattering is possible at small angles between the colliding particles ($\cos\theta_{\mathrm{res}} = (2\,\mu c/p)^{1/2}$). As already noted above, however, the cross section of such a process contains an additional factor $\sim\exp(-2\,\mu c^2/T)$ (after averaging over the phonon energies), and this effect can be neglected, assuming that the expression for $1/\tau_{\mathrm{ph\,r}}$ is valid also at small angles. The rate of absorption of phonons by the rotons $1/\tau'_{\mathrm{ph\,r}}$ is equal to

$$\frac{1}{\tau'_{\mathrm{ph\,r}}} \approx \frac{8P_0^4 kT\exp(-\Delta_c/T)}{15\pi\hbar^4 c\rho}g(T),$$

where $\Delta_c = \Delta_0 + c^2/2$, Δ_c is the roton energy at the acoustic point, and g(T) is a slowly varying function with an approximate value 2.

When (18) was integrated over all the rotons we have put $\mathbf{P} - \mathbf{P}_0 \approx p$ and $\cos^2\theta \approx \cos\theta$, so that the angle θ between the phonon and the roton is

$$\cos\theta \approx 2\mu c/3(P-P_0) \approx {}^2/_3.$$

The viscosity coefficient at $0.6°K < T < 1.2°K$ is

$$\eta_{\mathrm{ph}} = \frac{16\pi^2(kT)^4}{75c^3\hbar^3}\left(\frac{1}{\tau_{\mathrm{ph}}} + \frac{1}{\tau_{\mathrm{ph\,r}}} + \frac{5}{\tau'_{\mathrm{ph\,r}}}\right)^{-1} = 1.9\cdot 10^{-9}\frac{e^{\Delta/T}}{T^{1/2}}\varphi(T). \tag{22}$$

A plot of $\varphi(T)$ is shown in Fig. 1.

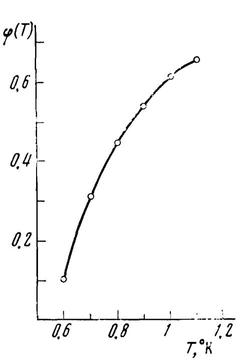

FIG. 1

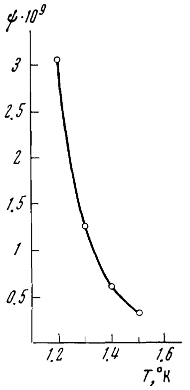

FIG. 2

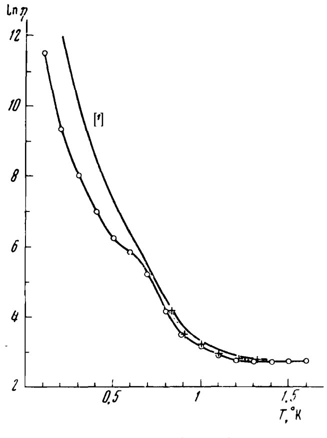

FIG. 3. Temperature dependence of the first-viscosity coefficient. The upper curve was obtained by Landau and Khalatnikov [1] and the lower in the present paper; the crosses mark the experimental data of Zinov'eva [10].

At $T > 1.2$ it is necessary to take the chemical potential α into account in the expressions for δn. Integrating the kinetic equation over all the energies and over all the phonons, we obtain the following system:

$$\begin{aligned}\frac{\partial u}{\partial x}\frac{4\pi^4}{15} &= \left(\frac{\alpha}{8} - \beta\right)\frac{1}{\tau_{\mathrm{ph\,r}}} + (\alpha - 5\beta)\frac{1}{\tau'_{\mathrm{ph\,r}}},\\ \frac{\partial u}{\partial x}\frac{36}{5} &= \left(\frac{\alpha}{56} - \frac{\beta}{8}\right)\frac{1}{\tau_{\mathrm{ph\,r}}} + \frac{\alpha}{\tau_{2\to3}}\frac{36}{5} + \left(\frac{\alpha}{4} - \beta\right)\frac{1}{\tau_{\mathrm{ph\,r}}}.\end{aligned} \tag{23}$$

We took into account the fact that in this region $1/\tau_{\mathrm{ph}} \ll 1/\tau_{\mathrm{ph\,r}}$. Here $1/\tau_{2\to3} = akT^{12}/3N_{\mathrm{ph}}$ is the rate of the five-phonon process, $a = 3.4\times10^{43}$, and $N_{\mathrm{ph}} = 2\zeta(3)4\pi(kT/2\pi\hbar c)^3$. Solving the system (23), we obtain

$$\eta_{\mathrm{ph}} = \frac{(kT)^4}{5\pi^2\hbar^3 c^3}\psi(T). \tag{24}$$

The function $\psi(T)$ has the dimension $[\sec^{-1}]$ and its plot is shown in Fig. 2.

Figure 3 shows the dependence of the viscosity coefficient on the temperature. The roton viscosity was assumed equal to 1.53×10^{-5} poise. The temperature dependence of Δ was taken into account by the interpolation formula [11]

$$\Delta(T) = 8.70 - 0.0289\,T^5.$$

The formation of a "shoulder" at temperatures 0.5–0.6°K is due to the contribution from the anomalous phonons (inequality (8)).

DISPERSION AND ABSORPTION OF SOUND

1. Temperature Region Below 0.6°K

It is easy to show, taking the inequality $p_1 \gg p_2$ into account, that the three-phonon collision integral reduces to the expression

$$\int J_3(p)\varepsilon p^2\,dp \approx -\frac{1}{\tau'_{1\to2}}[\nu(\cos\theta)-\nu_0-\nu_1]kT\left(\frac{kT}{c}\right)^3. \tag{25}$$

Formula (23) was obtained using the expression for δn from [2]

$$\delta_n=\frac{\partial n_0}{\partial\varepsilon}\left(\frac{\partial\varepsilon}{\partial\rho}\rho'+\varepsilon\nu(\cos\theta)\right).$$

Expression (23) corresponds exactly to the form of the four-phonon collision integral, so that we must make in all the equations obtained in [2] the substitution

$$\frac{1}{\tau_{\text{ph}}}\to\frac{1}{\tau}=\left(\frac{1}{\tau_{\text{ph}}}+\frac{1}{\tau'_{1\to2}}\right)\frac{15}{4\pi^4}.$$

At low frequencies, $\omega\tau<1$, the modified formulas take the form

$$\alpha_2=\frac{2\sqrt{3}}{5}\frac{\omega^2\tau}{c},\quad \alpha_1=\frac{3}{10}(u+1)^2\frac{\omega^2\tau}{c}\frac{\rho_n}{\rho}; \tag{26}$$

α_1 and α_2 are the absorption coefficients of the first and second sound in the hydrodynamic regime. The velocities of the first and second sound do not depend on τ in this limit, and therefore remain unchanged.

At high frequencies, $\omega\tau\gg1$, we have

$$u_1=u_{10}+\frac{3}{8}(u+1)^2c\frac{\rho_n}{\rho}\ln\left\{\frac{1+4\omega^2\tau^2}{1+[3\gamma(2\pi kT/c)^2\omega\tau]^2}\right\}, \tag{27}$$

$$\alpha_1=\frac{3}{4}(u+1)^2\frac{\rho_n}{\rho}\left\{\operatorname{arctg}(2\omega\tau)+\operatorname{arctg}\left[\frac{5}{7}\omega\tau\cdot3\gamma\left(2\pi\frac{kT}{c}\right)^2\right]\right\}. \tag{28}$$

Expression (28) was first proposed by Maris and Massey[4], but with a different relaxation time τ_{ph}.

2. Temperature Range from 0.6 to 1.2°K

We express the collision integral $J'_{\text{ph r}}$ in the form

$$\begin{aligned}J'_{\text{ph r}}&=-(2\pi\hbar)^{-3}c\int d\sigma'_{\text{phr}}(p,P)\,[n(p)N(P)\,(N(P+p)+1)\\&\quad-(n(p)+1)\,(N(P+p)+1)N(P)\,]d\mathbf{P}=(2\pi\hbar)^{-3}c\\&\quad\times\int d\sigma'_{\text{phr}}(p,P)n_0(p)N_0(P)cp[\nu(\cos\theta)+T_r'+W_r\cos\theta)\,]d\mathbf{P}.\end{aligned} \tag{29}$$

The deviation $\delta N(P)$ was taken in the form[2]

$$\delta N=\frac{\partial N_0}{\partial\mathscr{E}}\left(\frac{\partial\mathscr{E}}{\partial\rho}\rho'-\mathscr{E}\frac{T_r'}{T}-PW_r\cos\theta\right),$$

$$W_r=|\mathbf{v}_n-\mathbf{v}_s|.$$

Comparing the expression

$$\nu(\cos\theta)+T_r'+W_r\cos\theta$$

in (29) with the expression in the integral for elastic phonon-roton scattering obtained in [2]:

$$\nu(\cos\theta)-\nu_0+W_r\cos\theta+\beta(\nu_0+T_r'),$$

we note that the sum of integrals in the right-hand side of the kinetic equation

$$\int I_{\text{ph r}}\varepsilon p^2\,dp\Big/\int\frac{\partial n_0}{\partial\varepsilon}\varepsilon^2p^2\,dp+\int I_{\text{ph r}}'\varepsilon p^2\,dp\Big/\int\frac{\partial n_0}{\partial\varepsilon}\varepsilon^2p^2\,dp,$$

reduces to the form

$$-\frac{1}{\tau''_{\text{ph r}}}[\nu(\cos\theta)-\nu_0+\cos\theta W_r+\beta'(\nu_0+T_r')\,], \tag{30}$$

$$\frac{1}{\tau''_{\text{ph r}}}=\left(\frac{1}{\tau_{\text{ph r}}}+\frac{5}{\tau'_{\text{ph r}}}\right)\frac{15}{4\pi^4},$$

$$\beta'=\beta+(1-\beta)\frac{5\tau''_{\text{ph r}}}{\tau'_{\text{ph r}}},\quad\beta=\frac{3kT}{\mu c^2}.$$

Thus, allowance for the integral $J'_{\text{ph r}}$ consists of replacing the time $\tau_{\text{ph r}}$ by the time $\tau''_{\text{ph r}}$ and the parameter β by β'.

We now write down the new formulas in this temperature region. At low temperatures when $\omega\tau''_{\text{ph r}}\ll1$, $\omega\tau_{\text{ph}}\ll1$, and $0.6°\text{K}<T<0.9°\text{K}$ we have

$$\begin{aligned}\alpha_1=\frac{\omega^2\tau''_{\text{ph r}}}{c}\frac{\rho_{n\,\text{ph}}}{\rho}&\left\{\frac{2}{15}\frac{1}{1+\tau''_{\text{ph r}}/\tau_{\text{ph}}}+\frac{1}{6\beta'}\left(3u+1+\delta\frac{C_{\text{ph}}}{C}\right)^2\right.\\&\left.+\frac{1}{2}\delta^2\left(\frac{S_{\text{ph}}}{C}\right)^2\left(1-\frac{TS}{\rho_nc^2}\right)^2\right\},\end{aligned} \tag{31}$$

$$\begin{aligned}\alpha_2=\frac{\omega^2c^2\tau''_{\text{ph r}}}{u_{20}^2}\frac{\rho_s\rho_{n\,\text{ph}}}{\rho_n\rho}&\left\{\frac{2}{15}\frac{1}{1+\tau''_{\text{ph r}}/\tau_{\text{ph}}}+\frac{1}{2}\frac{\rho_n}{\rho_s}\frac{\rho c^2}{TC}\left(1-\frac{TS}{\rho_nc^2}\right)^2\right.\\&\left.+\frac{1}{6\beta'}\left(1-\frac{3S}{C}\right)\right\}.\end{aligned} \tag{32}$$

The velocities u_1 and u_2 remain unchanged.

At high frequencies, when $\omega\tau''_{\text{ph r}}\gg1$, $\omega\tau_{\text{ph}}\gg1$, and $0.6°\text{K}\le T\le0.9°\text{K}$ we obtain

$$u_1=u_{10}+c\frac{\rho_n}{\rho}\left\{\frac{3}{4}(u+1)^2\ln\frac{2\omega\tau''_{\text{ph r}}}{1+\tau''_{\text{ph r}}/\tau_{\text{ph}}}-3u-2\right\}, \tag{32'}$$

and the absorption coefficient is independent of $\tau''_{\text{ph r}}$ and β'.

For second sound, in the case $\omega\tau'_{\text{ph r}}\gg\beta'C_{\text{ph}}/C_r$ and $0.6°\text{K}\le T\le0.9°\text{K}$ we get

$$\alpha_2=\frac{1}{2}\frac{\beta'C_{\text{ph}}/C_r}{u_{2\infty}\tau''_{\text{ph r}}}\qquad u_{2\infty}=\left[\left(1-\frac{\rho_{nr}}{\rho}\right)\frac{TS_r^2}{\rho_{nr}C_r}\right] \tag{33}$$

and $0.9°\text{K}<T<1.2°\text{K}$ and $\omega\tau''_{\text{ph r}}\ll1$

$$\alpha_1=\frac{\omega^2\tau''_{\text{ph r}}}{c}\frac{\rho_{nr}}{\rho}\left[\frac{2}{15}+\frac{(3u+1)^2}{\beta'}\right], \tag{34}$$

while for α_2 the correct formula is (23) with the first term equal to 2/15 (since $\tau''_{\text{ph r}}<\tau_{\text{ph}}$ at $T>0.9°\text{K}$). At high frequencies, formulas (32) and (33) are valid.

3. Temperature Region Above 1.2°K

It can be shown, in exactly the same manner, that in this region all the equations remain the same as in [2] (with allowance for the replacement of $\tau_{\text{ph r}}$ by $\tau''_{\text{ph r}}$ and of β by β'), and the time $\tau_{2\to3}$ is transformed as follows:

$$\frac{1}{\tau_{2\to3}}\to\frac{1}{\tau'_{3\to2}}\approx\frac{1}{\tau_{3\to2}}+\frac{1}{25\tau'_{\text{ph r}}}.$$

The sound absorption coefficients in the low-frequency region $\omega\tau''_{\text{ph r}}\ll1$ take the form

$$\alpha_1=\frac{\omega^2\tau''_{\text{phr}}}{c}\frac{\rho_n}{\rho}\left[\frac{2}{15}+\frac{1}{6\tilde\beta}(3u+1)^2\right], \tag{35}$$

$$\alpha_2=\frac{\omega^2c^2\tau''_{\text{phr}}}{u_{20}^3}\frac{\rho_s}{\rho}\frac{\rho_{nr}}{\rho}\left\{\frac{2}{15}+\frac{1}{2}\frac{\rho_n}{\rho_s}\frac{\rho c^2}{TC}\left(1-\frac{TS}{\rho_nC^2}\right)^2+\frac{1}{6\tilde\beta}\left(1-3\frac{S}{C}\right)^2\right\}, \tag{36}$$

where

$$\begin{aligned}\frac{1}{\tilde\beta}\approx\left[\frac{1}{\beta'}+\left(\frac{27}{\pi^4}-\frac{1}{9}\right)\frac{\tau'_{3\to2}}{\tau_{\text{phr}}}\right]&\left\{1+\frac{1}{8}\left(\frac{\pi^4}{216}-1\right)^2\right.\\&\left.\times\left[\frac{\pi^4}{216}\left(\frac{1}{56}\frac{\pi^4}{216}+\frac{\tau''_{\text{phr}}}{\tau'_{3\to2}}\right)\right]^{-1}\right\}^{-1},\end{aligned}$$

$$\frac{1}{\tilde\tau_{\text{ph r}}}\approx\frac{1}{\tau''_{\text{ph r}}}\left\{1+\frac{1}{8}\left(\frac{\pi^4}{216}-1\right)^2\left[\frac{\pi^4}{216}\left(\frac{1}{56}\frac{\pi^4}{216}+\frac{\tau''_{\text{ph r}}}{\tau'_{3\to2}}\right)\right]^{-1}\right\}^{-1}.$$

Since the absorption of first sound is determined mainly by the second viscosity (the second terms in formulas (31), (34), and (35)), while the absorption of second sound is determined by the thermal conductivity (second terms in formulas (32) and (36)), the difference between the first-sound absorption coefficients calculated in this paper and by Khalatnikov and Chernikova[2] is more noticeable than the difference between the second-sound absorption coefficients.

Estimates show that the decrease of the first-sound absorption coefficient in comparison with the data of [2] amounts to ~15% at $T < 1.2°K$. Owing to the inequality (8), a step analogous to that of Fig. 3 appears at $T \approx 0.6°K$. The difference from the previously derived formulas in the $T > 1.2°K$ region is less noticeable (≈10%). In the high-frequency regime ($\omega\tau \gg 1$), the difference between the formula is not noticeable, since the time τ enters under the logarithm sign. The difference between the second-sound absorption coefficients is most strongly pronounced in the high-frequency region.

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[1] L. D. Landau and I. M. Khalatnikov, Zh. Eksp. Teor. Fiz. **19**, 637, 709 (1949).

[2] I. M. Khalatnikov and D. Chernikova, ibid. **49**, 1957 (1965); 50, 411 (1969) [Sov. Phys.-JETP **22**, 1336 (1966); **23**, 274 (1966)].

[3] J. Jackle and K. W. Kehr, Phys. Rev. Lett. **27**, 654 (1971).

[4] H. J. Maris and W. E. Massey, Phys. Rev. Lett. **25**, 220 (1970.

[5] N. E. Philips, C. G. Waterfield, and J. K. Haffer, Phys. Rev. Lett. **25**, 1260 (1970).

[6] R. B. Hallock, Phys. Rev. **A5**, 320 (1972).

[7] C. W. Dietrich, E. H. Graf, C. H. Huang, and L. Russell, Phys. Rev. **A5**, 1377 (1972).

[8] A. D. Woods and R. A. Cawley, Phys. Rev. Lett. **24**, 646 (1970).

[9] I. M. Khalatnikov, Teoriya svertekuchesti (Theory of Superfluidity), Nauka, 1971, p. 53.

[10] K. N. Zinov'eva, Zh. Eksp. Teor. Fiz. **31**, 31 (1956) [Sov. Phys.-JETP **4**, 36 (1957)].

[11] A. D. Singh, Can. J. Phys. **46**, 2607 (1968).

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