Possibility of propagation of dopplerons in metals with an open Fermi surface

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It is shown that electromagnetic waves whose spectrum is located near the Doppler-shifted cyclotron resonance frequencies can exist in metals with an open Fermi surface located in a magnetic field. The spectrum, polarization, damping, and excitation of such waves are investigated. The results of the theoretical investigation are qualitatively compared with recent experiments on the size effect in copper.

1. INTRODUCTION

In many recent papers[1-10] it is shown that there exists in metals unique electromagnetic waves called dopplerons. The spectra of these waves are localized near the cyclotron-resonance threshold which is shifted as a result of the Doppler effect

$$\omega = \Omega - k_z \bar{v}_z. \tag{1.1}$$

Here ω is the wave frequency, k_z is the projection of its wave vector on the direction of the constant magnetic field H, Ω is the cyclotron frequency, and $\bar{v}_z$ is the velocity component averaged over the period of the cyclotron rotation of the electrons in the magnetic field. The values of Ω and v_z correspond to that section (or point) of the Fermi surface where the average displacement per period along the vector H, equal to $u = 2\pi\bar{v}_z/\Omega$, is extremal. The first to point out the possibility, in principle, of the existence of a dopperlon were McGroddy, Stanford, and Stern[1] and Overhauser and Rodriguez[2]. They considered the propagation of the doppleron in an alkali metal along a magnetic field, and have shown that the magnetic-field interval in which this wave exists is exceedingly small and amounts to only about half of one per cent of the threshold field. The same papers[1,2], and later on the articles by Chambers and Skobov[3] and by Falk, Gerson, and Carolan[4], considered also other Fermi-surface models, for which the region of the existence of the doppleron turned out to be much broader. The reason is that the singularities of the Fourier components of the conductivity turned out to be stronger in the models considered in[3,4] than in an alkali metal. Subsequently, Konstantinov, Skobov, Fisher, et al.[5,6] have shown, using cadmium as an example, that the role of the doppleron can be greatly enhanced in a compensated metal in which the electron and hole densities are equal and there is no Hall effect in a strong magnetic field. The question of dopplerons in cadmium was investigated also by Naberezhnykh et al.[7,8]. Finally, Konstantinov and Skobov[9,10] have predicted the existence of doppleron solutions of Maxwell's equations for a wave propagating in an alkali metal almost perpendicularly to the direction of the magnetic field.

It should be noted that the properties of the doppleron as a electromagnetic eigenwave in a metal are closely connected with the singularities of anomalous electromagnetic-field penetration of the trajectory type[11]. Thus, dopplerons in a magnetic field normal to the surface of the metal[1-8] go over below threshold into quasi-harmonic oscillations of the electromagnetic field, with an amplitude that varies slowly with distance (the so-called mechanism of focusing ineffective electrons[12]). A doppleron in an oblique magnetic field[9,10] supplements the picture of the trajectory penetration due to the focusing of the effective electrons from the vicinity of the limiting point[13]. It is shown in[13] that a drift-type anomalous penetration of the electromagnetic field takes place also in a magnetic field parallel to the surface of the metal. In this case the drift motion of electrons into the interior of the sample is due to the existence of an open Fermi surface. Unlike all the other cases of anomalous penetration, the amplitude of the spatial harmonic that penetrates into the metal turned out to be independent of the coordinate. A similar "focusing of ineffective electrons" on an open Fermi surface, when the magnetic field is parallel to the boundary of the metal, was observed experimentally in cadmium[14] and copper[15]. The harmonic character of the dependence of the amplitude of the transmitted wave on the coordinate (or the magnetic field) points to a close connection between the considered mechanism of the trajectory penetration and a wave solution of the doppleron type.

In the present paper we investigate the possibility of propagation of a weakly damped wave in the vicinity of the threshold of the Doppler-shifted resonance (1.11), due to electrons by the open Fermi surface. It is shown that under definite requirements imposed on the Fermi surface, such solutions do indeed exist and the penetrating harmonics of the field correspond to roots of the dispersion equation for the eigenwave in the metal. In other words, the anomalous penetration is in this case not of the trajectory type, but constitutes an assembly of dopplerons, i.e., of electromagnetic eigenwave due to drift motion of "ineffective" electrons on open periodic orbits.

2. MODEL OF THE FERMI SURFACE

As the model of the open Fermi surface we choose the figure of revolution shown in Fig. 1. It is obtained by rotating a periodic system of interconnected circular arcs about the axis p_x. The centers of the individual circles lie on the axis of this corrugated cylinder. The radius of the circle will be designated p_0, and the period of the surface will be designated b, with $b < 2p_0$. The axis p_z is chosen along the projection of the momentum on the direction of the magnetic field H parallel to the z axis. The intersections of the Fermi surface with the planes $|p_z| < p_0 r$ (where $r = 1 - b^2/4p_0^2)^{1/2}$) are open periodic curves; the intersec-

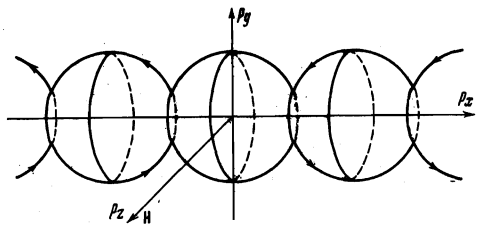

FIG. 1. Model of open Fermi at surface.

tions with $|p_z| > p_0 r$, on the other hand, break up into a periodically-repeating system of circles that are not interconnected. If we choose the origin at the center of one of the spheres, then the electron dispersion law within the confines of one period is given by

$$\varepsilon(\mathbf{p}) = (p_x^2 + p_y^2 + p_z^2)/2m = \varepsilon_0, \quad 2|p_x| < b, \tag{2.1}$$

where m is the effective mass of the electron. The electrons belonging to both sections of the Fermi surface execute in coordinate space a finite periodic motion in the direction of the axis, and their motion in the y direction is infinite. Let us determine the period of the corresponding finite and periodic motion on the open orbits in momentum space. According to[16] we have

$$T = \frac{c}{eH}\int_{-b/2}^{b/2}\frac{dp_x}{v_y} = \frac{mc}{eH}\int_{-b/2}^{b/2}\frac{dp_x}{(p_0^2 - p_x^2 - p_z^2)^{1/2}} = 2\frac{mc}{eH}\arcsin\frac{b}{2(p_0^2 - p_z^2)^{1/2}}. \tag{2.2}$$

Here c is the velocity of light, e is the absolute value of the electron charge, and $\mathbf{v} = \partial\varepsilon/\partial\mathbf{p}$ is the electron velocity. The cyclotron mass μ is obviously equal to

$$\mu = m\alpha, \quad \alpha = \pi^{-1}\arcsin[b/2(p_0^2 - p_z^2)^{1/2}]. \tag{2.3}$$

We introduce the dimensionless time (phase) of electron motion along an open periodic orbit, $d\tau = 2\pi dt/T$ (τ varies from $-\pi$ to π within the limits of one period). Then the electron velocity components v_x and v_y can be expressed in terms of τ in the form

$$v_x = v_0(1 - p_z^2/p_0^2)^{1/2}\sin(\alpha\tau), \qquad v_y = v_0(1 - p_z^2/p_0^2)^{1/2}\cos(\alpha\tau), \quad v_0 = p_0/m. \tag{2.4}$$

It is seen from (2.4) that

$$\bar{v}_x = (2\pi)^{-1}\int_{-\pi}^{\pi} v_x(\tau)\,d\tau = 0, \quad \bar{v}_y = b/2\pi\mu. \tag{2.5}$$

These formulas show that an electron moving along an open periodic orbit executes infinite motion along the y axis. It is easy to see that the displacement of these electrons within a period T in the x direction does not depend on p_z and is equal to

$$u = \bar{v}_y T = cb/eH. \tag{2.6}$$

We note that the transverse velocity components v_x and v_y in the model chosen by us experience jumps at the kink points $|p_x| = b/2, 3b/2, \ldots$ The cyclotron mass on the closed orbits of the considered open Fermi surface ($|p_z| > p_0 r$) coincides with m, and $v_x = v_y = 0$.

It should be pointed out that this Fermi-surface model is the simplest one for the analysis of the dopplerons properties. In particular, it is much more convenient for the calculation of the conductivity-tensor components than a Fermi surface made up by rotating a sinusoid about the p_x axis. The reason is that in the chosen model there are no hyperbolic limiting points in the p_z direction, at which the cyclotron mass beomes infinite. On the other hand, an open surface of this type is realized in metals of the first group (copper, silver, gold) in the zeroth approximation in the pseudopotential.

In the subsequent calculations we shall assume that the thickness of the layer of the open orbits is small, i.e.,

$$r = (1 - b^2/4p_0^2)^{1/2} \ll 1. \tag{2.7}$$

In addition, analysis shows that for a doppleron solution to exist it is necessary to have in some cases, besides a corrugated cylinder, also at least one more electron group. We assume for this group a Fermi-sphere model with radius p_1 and effective mass m_1.

3. CONDUCTIVITY TENSOR

We calculate first the contribution made to the Fourier component of the conductivity tensor σ_{ik} by the open periodic orbits on the corrugated cylinder. According to[17], the general expression for σ_{ik} is

$$\sigma_{ik}^{(0)} = \frac{2e^2}{(2\pi\hbar)^3}\int_{-p_0 r}^{p_0 r} dp_z \frac{\mu}{\Omega}\int_{-\pi}^{\pi} d\tau v_i(\tau)\int_{-\infty}^{\tau} d\tau' v_k(\tau') \times \exp[\gamma(\tau' - \tau)]\cos\left[\frac{k}{\Omega}\int_{\tau}^{\tau'} d\tau'' v_y(\tau'')\right], \quad \gamma = \frac{\nu - i\omega}{\Omega}, \tag{3.1}$$

where ν is the frequency of the collisions between the electrons and the scatterers. We are interested in dopplerons having an extraordinary polarization ($E_z = 0$). We therefore confine ourselves to calculation of the tensor components $\sigma_{\mu\nu}$ that are transverse with respect to the vector $\mathbf{H}$ ($\mu, \nu = x, y$). We obtain an asymptotic expression for $\sigma_{\mu\nu}$ in the limiting case of strong magnetic fields, when

$$|\gamma| = |(\nu - i\omega)/\Omega| \ll 1. \tag{3.2}$$

It is convenient to transform the expression for $\sigma_{xx}^{(0)}$ with the aid of a Fourier expansion of the quantity

$$v_x(\tau)\exp\left\{-ik\Omega^{-1}\int_0^{\tau} d\tau'[v_y(\tau') - \bar{v}_y]\right\} = \sum_{n=-\infty}^{\infty} w_{xn}\exp(in\tau) \tag{3.3}$$

The Fourier coefficients w_{xn} are given by

$$w_{xn} = (2\pi)^{-1}\int_{-\pi}^{\pi} d\tau v_x(\tau)\exp\left\{-in\tau - ik\Omega^{-1}\int_0^{\tau} d\tau'[v_y(\tau') - \bar{v}_y]\right\}. \tag{3.4}$$

If we express the cosine in (3.1) in terms of the half-sum of two exponentials and take formula (3.3) into account, then the integration with respect to τ and τ' is elementary, and

$$\sigma_{xx}^{(0)} = \frac{2\pi e^2}{(2\pi\hbar)^3}\int dp_z \frac{\mu}{\Omega}\sum_{n=-\infty}^{\infty}\left[\frac{1}{\gamma + i(q-n)} + \frac{1}{\gamma - i(q-n)}\right]|w_{xn}|^2. \tag{3.5}$$

It is seen from this expression that in the limiting case $|\gamma| \to 0$ the conductivity has singularities at

$$q = ku/2\pi = n, \quad n = 0, \pm 1, \pm 2, \ldots. \tag{3.6}$$

As will be shown below, the spectrum of different dopplerons becomes localized near one of these singularities. Therefore the quantity k in (3.4) can be replaced with the aid of (3.6), after which the term $-in\tau$ in the argument of the exponential cancels out the term with $\bar{v}_y$. If we substitute in (3.4) the explicit expressions (2.4) for $v_x(\tau)$ and $v_y(\tau)$, and also take the inequality (2.7) into account, then we can easily verify that

$$w_{xn} \approx -iv_0 J_1(\pi n), \tag{3.7}$$

where J is a Bessel function. In the evaluation of the integral with respect to p_z in (3.5) we can neglect the dependence of μ/Ω on p_z, by virtue of (2.7), and take

this ratio outside the integral sign at $p_z = 0$. We then obtain for $\sigma_{xx}^{(0)}$ the following asymptotic expression, which is valid near the singularity (3.6):

$$\sigma_{xx}^{(0)}=\frac{3N_0e^2J_1^2(\pi n)}{2\mu_0(\nu-i\omega)}\varphi(\Delta),\quad 2\varphi(\Delta)=f(\Delta)+f(-\Delta),$$
$$f(\Delta)=\int_0^r d\xi\left[1-\frac{2}{\pi}(r^2-\xi^2)^{-1/2}-\frac{\Omega_0\Delta}{\omega+i\nu}\right]^{-1},\quad \Delta=q-n, \tag{3.8}$$

where $N_0 = p_0^3/3\pi^2\hbar^3$ is the characteristic value of the summary concentration of the electrons on the corrugated cylinder, and μ_0 and Ω_0 are the corresponding values of μ and Ω at $p_z = 0$.

In the calculation of the two other transverse components, $\sigma_{xy}^{(0)} = -\sigma_{yx}^{(0)}$ and $\sigma_{yy}^{(0)}$, it is necessary first to integrate by parts the integrals with respect to τ and τ' in (3.1) with respect to the frequencies as many times as the factor $v_y(\tau)$ is encountered. We can then easily obtain, using the approximations (2.7), (3.2), and $|\Delta| \ll 1$, the following asymptotic expressions:

$$\sigma_{xy}^{(0)}=-\frac{3N_0ec}{\pi nH}J_0(\pi n)J_1(\pi n)\varphi(\Delta), \tag{3.9}$$

$$\sigma_{yy}^{(0)}=-\frac{6N_0e^2(\nu-i\omega)}{\pi^2n^2\mu_0\Omega_0^2}J_0^2(\pi n)\varphi(\Delta). \tag{3.10}$$

We have neglected in (3.10) a relatively small term (of the order of r), which results in the integration by parts from the term outside the integral sign.

A characteristic feature of the model considered by us is that the conductivity $\sigma_{\mu\nu}^{(0)}$ does not contain the small parameter r if the quantity $|1 - \Omega_0\Delta/(\omega + i\nu)|$ is of the order of r. In other words, the smallness of the number of electrons moving on the open periodic orbits in p-space can be offset by the proximity to resonance, when $k\bar{v}_y = \Omega_0 + \omega + i\nu$.

We proceed now to calculate the contribution made to $\sigma_{\mu\nu}$ by the closed sections of the open Fermi surface. Since the thickness of the layer of the open periodic orbits is small, we can assume in the calculation of the contribution from the closed orbits that the parameter r is equal to zero, and we can use the known expressions for the conductivity-tensor components in the case of a spherical Fermi surface. It must be borne in mind here that the period of the circular orbit and the cyclotron mass on the closed section are twice as large as $2\pi/\Omega_0$ and μ_0, respectively. Consequently,

$$\sigma_{\mu\nu}^{(c)}=\frac{3N_0e^2}{4\mu_0}\sum_{s=-\infty}^{\infty}\int_0^{\pi}d\theta\frac{\sin\theta w_\mu(s,\theta)w_\nu^*(s,\theta)}{\nu+i(s\Omega_0/2-\omega)}, \tag{3.11}$$

where

$$w_x=-i\sin\theta J_s'(\varkappa_0\sin\theta),\quad w_y=s\varkappa_0^{-1}J_s(\varkappa_0\sin\theta),$$
$$\varkappa_0=kv_0/2\Omega_0\approx\pi n.$$

In the approximation (3.2) for $\sigma_{xx}^{(c)}$ it suffices to retain one term of the sum with s = 0. Then

$$\sigma_{xx}^{(c)}=\frac{3N_0e^2}{2\mu_0(\nu-i\omega)}a_{xx},\quad a_{xx}=\int_0^{\pi/2}d\theta\sin^3\theta J_1^2(\pi n\sin\theta). \tag{3.12}$$

Since $w_y(0, \theta) \equiv 0$, it is necessary to take into account all the terms in the sum over s in the remaining elements of the transverse-conductivity tensor. Expanding them in powers of $(\nu - i\omega)/\Omega_0$, we obtain after simple calculations

$$\sigma_{xy}^{(c)}=-\frac{3N_0ec}{\pi nH}a_{xy},\quad a_{xy}=\int_0^{\pi/2}d\theta\sin^2\theta J_0(\pi n\sin\theta)J_1(\pi n\sin\theta), \tag{3.13}$$

$$\sigma_{yy}^{(c)}=\frac{6N_0e^2(\nu-i\omega)}{\pi^2n^2\mu_0\Omega_0^2}a_{yy},\quad a_{yy}=1-\int_0^{\pi/2}d\theta\sin\theta J_0^2(\pi n\sin\theta). \tag{3.14}$$

We present the numerical values of the tensor $a_{\mu\nu}$ for n = 1:

$$a_{xx}\approx 0{,}106,\ a_{xy}\approx -0{,}016,\ a_{yy}\approx 0{,}880. \tag{3.15}$$

Thus, the total contribution to the transverse conductivity from the closed and open orbits on the corrugated cylinder is described by the following expressions:

$$\sigma_{xx}^{(0)}+\sigma_{xx}^{(c)}=\frac{3N_0e^2}{2\mu_0(\nu-i\omega)}[\varphi(\Delta)J_1^2(\pi n)+a_{xx}],$$
$$\sigma_{xy}^{(0)}+\sigma_{xy}^{(c)}=-\frac{3N_0ec}{\pi nH}[\varphi(\Delta)J_0(\pi n)J_1(\pi n)+a_{xy}], \tag{3.16}$$
$$\sigma_{yy}^{(0)}+\sigma_{yy}^{(c)}=\frac{6N_0e^2(\nu-i\omega)}{\pi^2n^2\mu_0\Omega_0^2}[-\varphi(\Delta)J_0^2(\pi n)+a_{yy}].$$

It was indicated at the end of the last section that besides the open Fermi surface we consider also a spherical surface with Fermi radius p_1 and an effective mass m_1. The contribution to the conductivity tensor from this surface was in essence described by formulas (3.12)—(3.14), in which it is necessary to replace N_0 by $N_1 = p_1^{3/3}\pi^2\hbar^3$, μ_0 by $m_1/2$, Ω_0 by $2\Omega_1$, ν by ν_1, and all the letters π by $\pi p_1/p_0$. The quantities with index 1 pertain to the additional Fermi sphere. The final expressions for the Fourier components of the transverse conductivity are

$$\sigma_{xx}=\frac{3N_0e^2}{2\mu_0(\nu-i\omega)}\left[\varphi(\Delta)J_1^2(\pi n)+a_{xx}+\frac{2\mu_0}{m_1}\left(\frac{p_1}{p_0}\right)^3b_{xx}\right],$$
$$\sigma_{xy}=-\frac{3N_0e^2}{\pi n\mu_0\Omega_0}\left[\varphi(\Delta)J_0(\pi n)J_1(\pi n)+a_{xy}+\left(\frac{p_1}{p_0}\right)^2b_{xy}\right], \tag{3.17}$$
$$\sigma_{yy}=-\frac{6N_0e^2(\nu-i\omega)}{\pi^2n^2\mu_0\Omega_0^2}\left[\varphi(\Delta)J_0^2(\pi n)-a_{yy}-\frac{m_1}{2\mu_0}\frac{p_1}{p_0}b_{yy}\right],$$

where $b_{\mu\nu}$ differ from $a_{\mu\nu}$ in that the arguments of the Bessel functions in (3.12)—(3.14) contain the additional factor p_1/p_0.

The function $\varphi(\Delta)$, which describes the resonant contribution of the open periodic orbits, is an even function of its argument. At small r it can therefore be approximated by the expression

$$\varphi(\rho)\approx\frac{\pi}{4}\int_0^{\pi/2}d\theta\frac{\cos\theta}{\rho-\cos\theta},\quad \rho=\frac{\pi}{4r}\left[1-\left(\frac{\Omega_0\Delta}{\omega+i\nu}\right)^2\right] \tag{3.18}$$

This function is real for all real values of ρ, with the exception of the interval $0 < \rho < 1$, in which a nonzero imaginary part arises

$$\operatorname{Im}\varphi(\rho)=-(\pi^2/4)\rho(1-\rho^2)^{-1/2}, \tag{3.19}$$

and describes collisionless absorption of the wave by the electrons on the open periodic orbits. Outside this interval, there is no collisionless absorption, i.e., the function, $\varphi(\rho)$ is real and its explicit expression is determined by the formulas:

$$\varphi(\rho)=\frac{\pi}{2}\frac{\rho}{(\rho^2-1)^{1/2}}\operatorname{arctg}\frac{\rho+1}{(\rho^2-1)^{1/2}}-\frac{\pi^2}{8},\quad |\rho|>1,$$
$$\varphi(\rho)=\frac{\pi}{8}\frac{\rho}{(1-\rho^2)^{1/2}}\ln\left|\frac{1-(1-\rho^2)^{1/2}}{1+(1-\rho^2)^{1/2}}\right|-\frac{\pi^2}{8},\quad |\rho|<1. \tag{3.20}$$

A plot of the function $\varphi(\rho)$ is shown in Fig. 2. We see that the minimum value of the real function $\varphi(\rho)$ in the region where collisionless damping exists is attained at $\rho = 0$ and is equal to $-\pi^2/8$. The derivative φ' at this point is equal to $-\infty$. At the point $\rho = 1$ there is an infinite discontinuity (of the logarithmic type) of Re $\varphi(\rho)$. In the region from zero to unity there is a

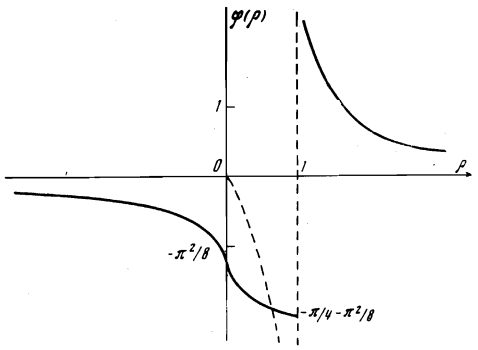

FIG. 2. Plot of the function $\varphi(\rho)$. The solid and dashed lines show Re $\varphi(\rho)$ and Im $\varphi(\rho)$, respectively.

nonzero imaginary part of the function $\varphi(\rho)$, describing collisionless absorption of the wave.

4. SPECTRUM AND POLARIZATION OF THE DOPPLERON

Let us find the dispersion equation for a doppleron having extraordinary polarization ($E_z = 0$). Maxwell's equation for the component E_z is separated from the equation for E_x and E_y because the off-diagonal components σ_{xz} and σ_{yz} are identically equal to zero in the geometry considered by us. Therefore Maxwell's equations for E_x and E_y can take the form

$$k^2c^2E_x = 4\pi i\omega(\sigma_{xx}E_x + \sigma_{xy}E_y), \tag{4.1}$$

$$\sigma_{yx}E_x + \sigma_{yy}E_y = 0. \tag{4.2}$$

The dispersion equation describing the doppleron spectrum can therefore be expressed in the form

$$k^2c^2\sigma_{yy} = 4\pi i\omega(\sigma_{xx}\sigma_{yy} + \sigma_{xy}^2). \tag{4.3}$$

A characteristic feature of this equation is that the terms quadratic in $\varphi(\rho)$ in the right-hand side of (4.3) cancel each other, and we are left only with terms linear in $\varphi(\rho)$ with allowance for (3.17), we obtain

$$\varphi(\rho) = \frac{A_{xy}^2 + (A_{xx} + n^2\zeta)A_{yy}}{(A_{xx} + n^2\zeta)J_0^2(\pi n) - A_{yy}J_1^2(\pi n) - 2A_{xy}J_0(\pi n)J_1(\pi n)}, \tag{4.4}$$

where

$$A_{xx} = a_{xx} + \frac{2\mu_0}{m_1}\left(\frac{p_1}{p_0}\right)^3 b_{xx}, \quad A_{xy} = a_{xy} + \left(\frac{p_1}{p_0}\right)^2 b_{xy}, \quad A_{yy} = a_{yy} + \frac{m_1}{2\mu_0}\frac{p_1}{p_0}b_{yy},$$

$$\zeta = \frac{\pi^2}{3}\frac{H^2/8\pi}{N_0\varepsilon_0}\left(1 + i\frac{\nu}{\omega}\right). \tag{4.5}$$

The parameter ζ characterizing the solenoidal part of the electric field of the wave is a small quantity to the extent that the ratio of the magnetic pressure to the kinetic pressure is small. Therefore, if we neglect ζ, then the dispersion equation (4.4) describes potential electromagnetic oscillations.

We analyze first the spectrum of such potential oscillations ($|\zeta| \to 0$) with $n = 1$ in the absence of the additional Fermi sphere ($p_1 = 0$). Then, taking (3.15) into account, we reduce the dispersion equation (4.4) to the form

$$\varphi(\rho) = \frac{a_{xy}^2 + a_{xx}a_{yy}}{a_{xx}J_0^2(\pi) - a_{yy}J_1^2(\pi) - 2a_{xy}J_0(\pi)J_1(\pi)} \approx -1{,}6 < -\frac{\pi^2}{8}. \tag{4.6}$$

From (4.6) we get a somewhat unexpected result, namely, without the additional electron groups the solution of the dispersion equation falls in the region $0 < \rho < 1$, where collisionless absorption takes place. In other words, there is no weakly-damped doppleron with $n = 1$ in this case. On the other hand, if we take into account that the contribution made to the component $A_{\mu\nu}$ by the additional Fermi sphere, then we can easily choose the parameters p_1 and m_1 in such a way that the right-hand side of the dispersion equation (4.6) turns out to be larger than $-\pi^2/8$. In this case there is no collisionless absorption. Thus, for example, at $p_1 = p_0$ and $2\mu_0/m_1 > 7.5$, the right-hand side of (4.6) is generally positive, i.e., there exists a doppleron for which $\rho > 1$. This means that the spectrum of the doppleron is localized in the region where its damping is due only to collisions of electrons with scatterers

$$\omega = \pm\frac{k\bar{v}_y - \Omega_0}{1 - 2r\rho_1/\pi} - i\nu. \tag{4.7}$$

The quantity $\rho_1 = \text{const}$ is the solution of the dispersion equation (4.6).

It is easy to determine the polarization of the doppleron from Eq. (4.2). Substituting in it the relations (3.17), we obtain

$$\frac{E_x}{E_y} = \frac{2(\nu - i\omega)}{\pi n\Omega_0}\frac{\varphi(\rho)J_0^2(\pi n) - A_{yy}}{\varphi(\rho)J_0(\pi n)J_1(\pi n) + A_{xy}}. \tag{4.8}$$

Owing to the small factor $(\nu - i\omega)/\Omega_0$ in the right-hand side of (4.8), the wave turns out to be polarized mainly along the y axis.

We consider now the solutions of the dispersion equation (4.4) for multiple dopplerons with $n > 1$. To simplify the analysis, we investigate the limiting case of large n, when the Bessel functions in (3.12)—(3.14) and in (4.4) can be replaced by their asymptotic forms at large values of the argument. Then

$$a_{xx} \approx (4\pi n)^{-1}, \quad a_{xy} \approx -\pi^{-2}(2n)^{-3/2}, \quad a_{yy} \approx 1;$$
$$J_0(\pi n) = -J_1(\pi n) \approx (-1)^n(\pi^2 n)^{-1/2}.$$

We retain in the dispersion equation (4.4), only that part of the components of the tensor $A_{\mu\nu}$ which is due to the contribution of the open Fermi surface. Neglecting small terms, we represent (4.4) in the form:

$$\varphi(\rho) \approx -\frac{\pi}{4}\frac{1 + 4\pi n^3\zeta}{1 - n^2\zeta}. \tag{4.9}$$

We consider the case of high frequencies $\omega \gg \nu$ and not too large values of n, for which $n^2|\zeta| \ll 1$. Equation (4.9) has a real solution only if the right-hand side of (4.9) is larger than $-\pi^2/8$, i.e.,

$$n < \left(\frac{\pi - 2}{8\pi|\zeta|}\right)^{1/3}. \tag{4.10}$$

For larger numbers n, the solution of (4.9) leads to complex values of ρ, i.e., to a relatively strong collisionless damping of the dopplerons. Since the right-hand side of (4.10) decreases with increasing magnetic field, the number of dopplerons that do not experience collisionless damping decreases with increasing H.

If $n^2|\zeta| > 1$, then the right-hand side of (4.9) is equal to $\pi^2 n$, i.e., it is always positive and large. The corresponding solutions (4.9) are real and are located near $\rho = 1$. However, as we shall see in the next section, the amplitude of such harmonics excited in the metal is inversely proportional to $|d\varphi/d\rho|$. This means that such dopplerons will be weakly excited by an external electromagnetic wave.

5. DOPPLERON EXCITATION

In this section we estimate the amplitudes of waves propagating inside the metal at distances that are large in comparison with the thickness of the skin layer. Generally speaking, an accurate calculation of the am-

plitudes of the different dopplerons, with allowance for the real character of the reflection of the electrons from the surface of the metal, is a very difficult problem. The difficulties are due to the change in the character of motion of the electrons near the interface, as a result of which the connection between the Fourier components of the current and of the field becomes nonlocal. We shall therefore use the model of the so-called "antispecular" reflection[18], in which the distribution of the electromagnetic field in a metal can be obtained in explicit form by using only the Fourier components of the conductivity tensor in an unbounded metal. Physically, the model of antispecular reflection is equivalent to excitation of an electromagnetic field in an unbounded metal with the aid of an infinitely thin "current sheet" located at y = 0. Maxwell's equations differ in this case from (4.1) and (4.2) in that in the left-hand side of the first equation there appears an additional term $2c^2\mathscr{E}_x'(0)$, proportional to the given surface current. Here

$$\mathscr{E}_x(y)=\pi^{-1}\int_0^\infty dk\cos(ky)E_x(k) \tag{5.1}$$

describes the spatial distribution of the electric field in a metal. The Fourier component $E_x(k)$ is given by

$$E_x(k)=-2\mathscr{E}_x'(0)\left[k^2-4\pi i\omega c^{-2}(\sigma_{xx}+\sigma_{xy}^2/\sigma_{yy})\right]^{-1}. \tag{5.2}$$

In the interval (5.1), in addition to the contribution made from the doppleron poles to (5.2), there is also a contribution from the poles $E_x(k)$ at large values of k ($ku \gg 1$). The latter is a rapidly damped (over distances on the order of the thickness δ_a of the anomalous skin layer) component of the electric field. At large distances $y \gg \delta_a$ this component can be neglected and it is necessary to take into account only those poles of $E_x(k)$ which are due to dopplerons. We rewrite (5.1) by introducing a new integration variable $q = qu/2\pi$

$$T(y)=\frac{\mathscr{E}_x(y)}{i\mathscr{E}_x'(0)}=i\frac{u\zeta}{\pi^2}\int_0^\infty dq\frac{\cos(2\pi qy/u)}{q^2\zeta+(S_{xx}-S_{xy}^2/S_{yy})} \tag{5.3}$$

where $S_{\mu\nu}$ stand for the corresponding expressions in the square brackets in (3.17); the quantity ζ is defined in (4.5). In the integral with respect to q, the oscillating cosine is represented in the form of a half-sum of two exponentials. The solution of the dispersion equation (4.4) with respect to the wave number k is double-valued, one of the values lying in the upper half-plane of the complex variable q, and the other in the lower half-plane. Accordingly, it is necessary to take into account only one of the two doppleron poles with given number n in each of the integrals containing a definite exponential. As a result, the summary contribution from these poles is described by the formula

$$T(y)=-\frac{u\zeta}{4\pi}\sum_n\left\{\left[2n\zeta-\frac{\pi\Omega_0C_n}{2r(\omega+i\nu)}\frac{d\varphi}{d\rho_n}\right]^{-1}\exp\left(i\frac{2\pi yn}{u}\right)\right.$$
$$\left.+\left[2n\zeta+\frac{\pi\Omega_0C_n}{2r(\omega+i\nu)}\frac{d\varphi}{d\rho_n}\right]^{-1}\exp\left(-i\frac{2\pi yn}{u}\right)\right\}\exp\left[-\frac{y}{l}\left(1-\frac{2r\rho_n}{\pi}\right)\right]. \tag{5.4}$$

where the constants C_n are given by

$$C_n=\left[\frac{J_0^2(\pi n)(n^2\zeta+A_{xx})-J_1^2(\pi n)A_{yy}-2J_0(\pi n)J_1(\pi n)A_{xy}}{J_0(\pi n)A_{xy}+J_1(\pi n)A_{yy}}\right]^2,$$

$l = l/(1 - i\omega\tau)^{-1}$ is the effective mean free path of the electrons in the high-frequency field, and l and τ are the length and time of the free path of the electrons on the open periodic orbits. The sum in (5.4) contains those dopplerons which do not experience collisionless damping. In other words, the distribution of the field is a superposition of weakly-damped dopplerons with one frequency and with different wave numbers, such that their imaginary part ρ_n is equal to zero.

It was indicated at the end of the preceding section that when the magnetic field is varied the multiple dopplerons (even with relatively small numbers, see (4.10)) acquire a large imaginary part ρ_n. This means that with increasing H there should be a decrease in the number of terms in the sum (5.4). This leads in turn to a change in the form of the oscillating dependence of the amplitude of the electromagnetic field inside the metal. If the sum of (5.4) contains in the region of weaker magnetic fields several terms with approximately equal amplitudes, then their interference causes the electromagnetic field in a metal to be a periodic system of narrow spikes separated by relatively broad minima. With decreasing number of interfering dopplerons, the field distribution in the metal becomes more nearly harmonic. It is precisely this variation of the form of the oscillations of the amplitude of the size effect which was observed by Perrin, Weisbuch, and Libchaber[15] in measurements of the surface impedance of a copper plate. The experimental conditions were as follows: normal to the surface of the plane $n \parallel [110]$, vector $H \parallel [\bar{1}12]$, electric field vector $E \parallel [1\bar{1}1]$; plate thickness $y = d = 2.45 \times 10^{-2}$ cm, and frequency $\omega/2\pi = 4.45$ MHz. In this case the electron trajectories passing through the "necks" of the Fermi surfaces of copper are open, almost periodic, and very similar to the trajectories shown in Fig. 1. The first dozen of oscillations in fields approximately up to 10 kOe have patently anharmonic forms (narrow maxima and broad minima with fine structure) whereas the subsequent oscillations become more harmonic (the fine structure disappears and the minima and maxima have practically the same shape). We assume that the size-effect curve given in Fig. 4 of[15] can be explained by using the mechanism proposed in the present article for excitation of dopplerons on open electron orbits.

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81