Tensile strength of liquid hydrogen

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The cavitation strength of liquid hydrogen due to heterosphase fluctuations in the presence of spherical vapor nuclei is considered. It is shown that the highest possible cavitation strength of liquid hydrogen at the triple point temperature is 56 atm. The tensile strength of the liquid decreases with increase of size of the nuclei. The problem of a superfluid state occurring in liquid hydrogen at the result of supercooling and static tension [1] is discussed.

Ginzburg and Sobyanin[1] predict the possibility of realization of the superfluid state in liquid hydrogen by reducing the freezing point T_f below the Bose condensation temperature down to $T \approx 6°K$. They suggest that such reduction can be achieved by static stretching. According to the estimates given in[1], this can be achieved by establishing a tensile stress of 200 atm in liquid hydrogen. In view of this, it would be interesting to consider the tensile (cavitation) strength of liquid hydrogen.

In contrast to normal liquids existing at high temperatures, the cryogenic liquids (including hydrogen) can be quite pure because one can ensure that the concentrations and size of the cavitation nuclei (in the form of solid particles or gas bubbles) are sufficiently small. Only helium or neon can dissolve in liquid hydrogen but their concentrations can be made quite small. Nevertheless, it is impossible to remove all the cavitation nuclei from liquid hydrogen and this is also true of other cryogenic liquids. Vapor bubbles, generated by high-energy particles, are present in such liquids because it is not possible to absorb completely the cosmic-ray background. Moreover, cryogenic liquids may contain vapor bubbles formed by local heat sources due to imperfections of the cryogenic systems. Therefore, the tensile strength of liquid hydrogen should be considered in the presence of cavitation nuclei. This is the subject of the present paper.

We shall define the cavitation rupture of a liquid as the formation of spherical vapor cavities of critical size r_c given by the condition

$$p_c' = p_c + 2\sigma / r_c, \tag{1}$$

where p_c is the threshold pressure for the cavitation in a liquid, p_c' is the vapor pressure in a cavity of critical size, and σ is the surface tension of the liquid. Such a critical cavity may form as a result of heterophase fluctuations,[2] which can be described formally as transitions across a potential barrier whose height is governed by the minimum work W_c that has to be done to form a cavity of critical size r_c. In general, the formation of supercritical cavities in a liquid may be attributed to a flux of cavities J along the phase axis r, described by the transport equation given in[3] and[4]. In the case of a steady-state flux

$$J = C \exp(-W_c / \varkappa T), \tag{2}$$

where C is the pre-exponential function governed by the parameters of the liquid and the kinetics of formation of cavities; κ is the Boltzmann constant; T is the temperature of the liquid.

We shall assume that in the case of heterophase fluctuations the formation of critical vapor cavities occurs at vapor nuclei of radius $r_0 < r_c$, which are formed by external energy sources. Then the minimum work W_c is a function of r_0 and is given by[5]

$$W_c = 4\pi\sigma(r_c^2 - r_0^2) - {}^4/_3\pi\Delta p_c(1 - \rho'/\rho)(r_c^3 - r_0^3), \tag{3}$$

where $\Delta p_c = p_s - p_c$ is the cavitation strength of the liquid; p_s is the pressure along the phase equilibrium curve; ρ' and ρ are, respectively, the density of the vapor and its liquid; $r_c = 2\sigma/\Delta p_c(1 - \rho'/\rho)$. The function C introduced in Eq. (3) depends on the various parameters of the liquid, the temperature T, and the value of Δp_c. In general, the determination of C is a difficult task, which is considered in detail by Kagan in[4].

Equations (2) and (3) yield the relationship between the cavitation strength Δp_c and the radius of a nucleus r_0:

$$\frac{4\pi}{3}\left(1 - \frac{\rho'}{\rho}\right)\Delta p_c r_0^3 - 4\pi\sigma r_0^2 + \frac{16\pi\sigma^3}{3(\Delta p_c)^2}\left(1 - \frac{\rho'}{\rho}\right)^{-2} - \varkappa T \ln\frac{C}{J} = 0. \tag{4}$$

The solution of Eq. (4) gives the dependence of r_0 on Δp_c:

$$r_0 = \frac{\sigma}{\Delta p_c}\left(1 - \frac{\rho'}{\rho}\right)^{-1}\left(1 \mp 2\cos\frac{\varphi}{3}\right), \tag{5}$$

$$\varphi = \arccos\left[\pm 1 \mp \frac{3}{8\pi}\frac{\varkappa T}{\sigma^3}\left(1 - \frac{\rho'}{\rho}\right)^2 (\Delta p_c)^2 \ln\frac{C}{J}\right], \tag{6}$$

where $\pi < \varphi < 3\pi/2$. The upper and lower signs in Eqs. (5) and (6) can be found from the conditions

$$\Delta p_0/\sqrt{2} < \Delta p_c < \Delta p_0, \qquad \Delta p_c < \Delta p_0/\sqrt{2}, \tag{7}$$

$$\Delta p_0 = \left(\frac{16\pi\sigma^3}{3\varkappa T \ln(C/J)}\right)^{1/2}\left(1 - \frac{\rho'}{\rho}\right)^{-1}. \tag{8}$$

The formula (8) represents the cavitation strength of a pure liquid without nuclei (this strength is governed by heterophase fluctuations). This strength was calculated first by Zel'dovich[3] to within ρ'/ρ. It should be noted that Eq. (8) follows from Eqs. (5) and (6) if $r_0 \to 0$.

Figure 1 gives the values of the cavitation strength of pure hydrogen (the calculations were made for the para-hydrogen form). The continuous curves represent the calculations based on Eqs. (5) and (6) with $J = 1$ $\text{cm}^{-3} \cdot \text{sec}^{-1}$. The case of a pure liquid without nuclei corresponds to the curve labeled $r_0 = 0$. The dashed ordinate corresponds to the temperature T at which $p_s = 1.0$ atm on the phase equilibrium curve. It is worth noting the tendency for Δp_c to fall with increasing T. As $T \to T_c$ the cavitation strength vanishes, $\Delta p_c \to 0$, which is mainly due to $\sigma \to 0$. A characteristic feature of our results is a considerable fall in Δp_c with increasing size of the nuclei r_0. It follows from our calculations that the presence of nuclei of 5×10^{-7}

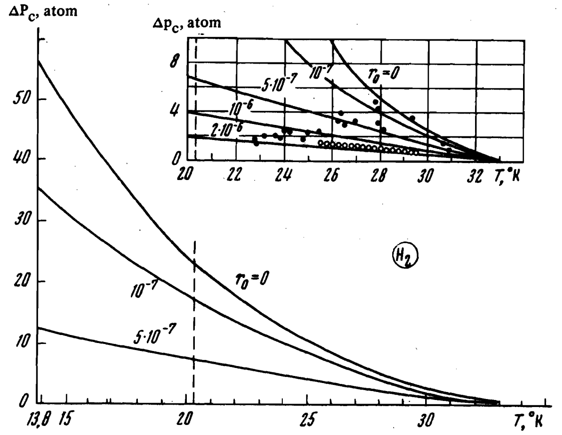

FIG. 1. Dependences of the cavitation strength of liquid hydrogen Δp_c on the temperature T, plotted for different values of the radius of nuclei r_0. The continuous curves represent calculations. Open circles represent calculations. Open circles correspond to the limit of sensitivity of liquid-hydrogen bubble chambers, [6] and the black dots are the experimentally determined values of the strength. [7]

– 10^{-6} cm size in liquid hydrogen reduces the cavitation strength to several atmospheres. This is the size of vapor cavities which are generated in liquid hydrogen by ionizing particles present in the cosmic-ray background.[6] The experimental results of Hord et al.[7] indicate that under real conditions the cavitation strength of liquid hydrogen does not exceed several atmospheres and this corresponds to the presence of various nuclei. The maximum possible tensile strength of nuclei-free liquid hydrogen is about 56 atm at $T_f = 13.8^\circ K$.

We shall now estimate the minimum supercooling temperature of liquid nitrogen in the presence of tensile forces close to the value of the cavitation strength. We shall do this by the simultaneous solution of the system (8) and (9) for the phase equilibrium curve separating liquid and solid nitrogen:

$$\Delta p = \left(\frac{dp}{dT}\right)_f (T_f - T), \tag{9}$$

where $\Delta p = p_f - p$ is the difference between the pressure p_f at the freezing point $T_f = 13.8^\circ K$ and the pressure p along the phase equilibrium curve; $(dp/dT)_f$ is the slope of the phase equilibrium curve at T_f; $(dp/dT)_f = 29$ atm/deg K.[8] Equation (8) must be modified to allow for the temperature dependence of the surface tension $\sigma = a - bT$, where a = 5.01 dyn/cm and b = 0.128 dyn $\cdot$ cm^{-1} $\cdot$ (deg K)$^{-1}$.[9] The solution of the system (8)–(9) shows that the minimum temperature is $T_{min} = 11.8^\circ K$ and the corresponding pressure is $\Delta p = 58$ atm. Further reduction in pressure should result in the rupture of liquid hydrogen and further cooling should result in its freezing.

The reported calculations are concerned with the static stretching of the liquid for $J = 1\ \text{cm}^{-3}\cdot\text{sec}^{-1}$. It follows from Eqs. (5)–(8) that the cavitation strength increases with increasing flux J. Figure 2 gives the values of Δp_c of liquid hydrogen for different values of J. It is evident from this figure that for small dimensions of the nuclei a change in J by a factor of 10^9 increases the strength by 5–10%. In the case of larger nuclei, the influence of J on the cavitation strength is negligible.

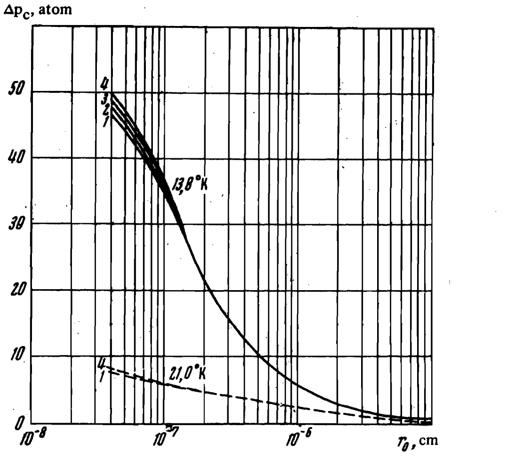

FIG. 2. Dependences of the cavitation strength of liquid hydrogen on the radius of nuclei r_0 for different values of J (cm^{-3} $\cdot$ sec^{-1}): 1–10^{-3}; 2–1; 3–10^3; 4–10^6.

Liquid hydrogen thus has a low tensile strength and this would make it difficult to realize the superfluid state by the suggested method. Moreover, the presence of cavitation nuclei in real liquids would reduce the strength.

If the elementary volume of a liquid being stretched is reduced, the probability of the presence of cavitation nuclei is lowered and this increases the tensile strength of a liquid. In view of this, the maximum tensile strength of liquid hydrogen would be expected under real conditions in the case of irradiation with a high-power high-frequency focused ultrasonic field. Moreover, we may expect the tensile strength of liquid nitrogen to increase for thin films and this is the way in which one could try to realize the predicted superfluid state of liquid hydrogen.

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