Coherent light emission by atoms excited by fast particles in an electromagnetic field

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It is shown that directivity of the coherent light emission in a system of atoms excited by fast particles results from the electromagnetic wave field. The direction of the coherent waves is rigidly related to the frequency of the radiation, velocity of the particle, and characteristics of the electromagnetic wave. Resonance between the wave frequency and frequency of transition between two atomic excited states is investigated. The possibility of employing coherent emission for measuring the energy of fast particles is discussed.

1. INTRODUCTION

It is known that from the microscopic point of view the source of the Cerenkov radiation is not the fast particle itself, but the atoms it excites in the medium. If the particle velocity exceeds that of light in matter, then the space-time distribution of the excited atoms is such that there exists a direction of coherent emission of the excited atoms. For this direction it is the amplitudes that are summed, and not the probabilities of emission of individual atoms. If the particle velocity is lower than that of light in the medium, then there are simply no directions of coherent emission, and the emission process proceeds incoherently and independently for each atom.

If the matter excited by the particle is acted upon by an electromagnetic field, then the field changes the space-time distribution of the atom excitations. Depending on the property of the external field, the existing coherent-emission directions can change, and new such directions can arise.

We note that the presence of an external field can lead also to an appreciable increase of the coherent-emission intensity in comparison with the Cerenkov radiation. This is possible because only a small fraction of the excitation energy goes to Cerenkov radiation.

The action of an electromagnetic wave on an excited atom can be reduced to the absorption of quanta of the wave. This enables us to consider the kinematics of coherent emission in the field of the wave. Assume that the particle excites an atom and that the particle momentum is changed thereby from $\mathbf{p}$ to $\mathbf{p} - \mathbf{q}$, after which quanta with momenta $\mathbf{k}_1$ and $\mathbf{k}_2$ are absorbed from the field and a quantum with momentum $\mathbf{k}$ is coherently emitted. Coherent participation of the atoms of the medium in such a process is ensured by the fact that the entire process leaves the atom in the same state as prior to interaction with the particle. In this case the energy and momentum transferred to the atom should be equal to zero:

$$\mathbf{q}+\mathbf{k}_1+\mathbf{k}_2=\mathbf{k},$$
$$(p^2+m^2)^{1/2}-((\mathbf{p}-\mathbf{q})^2+m^2)^{1/2}+\omega_1+\omega_2=\omega. \qquad (1.1)$$

Eliminating $\mathbf{q}$ from (1.1) we obtain a condition relating the emission direction of the quantum $\mathbf{k}$ with its frequency and with the field characteristics:

$$\frac{\mathbf{k}(\mathbf{p}+\mathbf{k}_1+\mathbf{k}_2)}{\omega(E+\omega_1+\omega_2)}=1 - \frac{2E(\omega_1+\omega_2)-2\mathbf{p}(\mathbf{k}_1+\mathbf{k}_2)+(\omega_1+\omega_2)^2-(\mathbf{k}_1+\mathbf{k}_2)^2+\omega^2-k^2}{2\omega(E+\omega_1+\omega_2)}. \qquad (1.2)$$

In the optical frequency range we can neglect ω/E, ω_1/E, and ω_2/E, and obtain for the quantum emission angle ϑ the condition

$$\cos\vartheta=\frac{1}{v\sqrt{\varepsilon}}\left(1-\frac{\omega_1+\omega_2-\mathbf{k}_1\mathbf{v}-\mathbf{k}_2\mathbf{v}}{\omega}\right). \qquad (1.3)$$

When the frequencies of the absorbed quanta vanish, Eq. (1.3) goes over into the known formula for the Cerenkov-radiation emission angle[1]. Relations (1.2) and (1.3) can be used to measure the energy of the fast particles[2].

2. MACROSCOPIC ANALYSIS OF COHERENT EMISSION IN THE FIELD OF A WAVE

In the case of coherent emission in the field of a wave, the medium is acted upon by two fields, the field $\mathbf{E}_0(\mathbf{r}, t)$ of the exciting wave and the particle field. The effect considered is therefore the result of a nonlinear interaction of the fields in the medium.

Effects of this kind are treated in nonlinear optics[3-5] by methods of nonlinear macroscopic electrodynamics. The nonlinear properties of the medium are described in this case phenomenologically, by introducing nonlinear susceptibilities of various orders. For fields that are not too strong, the dependence of the nonlinear polarization of the medium on the total field $\mathbf{E}^t(\mathbf{r}, t)$ in a homogeneous, isotropic, and stationary medium is given by

$$P_i^{NL}(\mathbf{r},\omega)=\int d\omega'\int d\omega''\chi(\omega,\omega',\omega'')E_i^t(\mathbf{r},\omega-\omega') \times E_j^t(\mathbf{r},\omega'-\omega'')E_j^t(\mathbf{r},\omega''), \qquad (2.1)$$

where $\chi(\omega, \omega', \omega'')$ is the nonlinear susceptibility of third order (the values of χ for a number of substances were measured in[6,7]). We choose now the exciting field $\mathbf{E}_0(\mathbf{r}, t)$ in the form

$$\mathbf{E}_0(\mathbf{r},t)=\mathbf{e}\{E_{01}\cos(\mathbf{k}_1\mathbf{r}-\omega_1t+\varphi_1)+E_{02}\cos(\mathbf{k}_2\mathbf{r}-\omega_2t+\varphi_2)\}, \qquad (2.2)$$

where $\mathbf{e}$ is a unit vector in the field direction, and represent the total field as a sum of $\mathbf{E}_0(\mathbf{r}, t)$ and the remainder $\mathbf{E}(\mathbf{r}, t)$

$$\mathbf{E}^t(\mathbf{r},t)=\mathbf{E}_0(\mathbf{r},t)+\mathbf{E}(\mathbf{r},t)$$

$\mathbf{E}(\mathbf{r}, t)$ includes the particle field as well as the coherent-emission field resulting from the interaction. If we consider a frequency range that does not coincide with the frequencies of the exciting field (2.2), then the linear part of the polarization of matter in this region is determined only by the field $\mathbf{E}(\mathbf{r}, t)$, and is accounted for by introducing the usual linear dielectric constant $\epsilon_0(\omega)$ of the medium. The field $\mathbf{E}_0(\mathbf{r}, t)$ produces in this frequency region the additional polarization (2.1).

Let $E_0 \gg E$, so that we can confine ourselves in (2.1) to the terms linear in E. The induction then remains linear in E regardless of the presence of the field E_0, We can therefore assume that in the considered approximation the action of the wave field $E_0(\mathbf{r}, t)$ reduces to a certain change in the dielectric properties of the medium. It is easy to obtain from (2.1) and (2.2) an expression for the electric induction

$$D_i(\mathbf{r},\omega)=\varepsilon_{ij}E_j(\mathbf{r},\omega)$$
$$+\sum_{\alpha,\beta=1,2}\sum_{\xi,\eta=\pm1}Q_{ij}(\omega,\alpha,\beta,\xi,\eta)\exp\{i\xi\mathbf{k}_\alpha\mathbf{r}-i\eta\mathbf{k}_\beta\mathbf{r}\}E_j(\mathbf{r},\omega-\xi\omega_\alpha+\eta\omega_\beta); \tag{2.3}$$

we have used here the notation

$$\varkappa=2\pi\sum_{\alpha=1,2}\sum_{\xi=\pm1}E_{0\alpha}{}^2\chi(\omega,\omega-\xi\omega_\alpha,\omega), \tag{2.4}$$
$$\varepsilon_{ij}=\varepsilon\delta_{ij}+\varkappa e_ie_j=(\varepsilon_0+{}^1/_2\varkappa)\delta_{ij}+\varkappa e_ie_j,$$
$$Q_{ij}(\omega,\alpha,\beta,\xi,\eta)=\pi(1-\delta_{\alpha\beta}\delta_{\xi\eta})(\delta_{ij}+2e_ie_j)$$
$$\times E_{0\alpha}E_{0\beta}\chi(\omega,\omega-\xi\omega_\alpha,\omega-\xi\omega_\alpha+\eta\omega_\beta)\exp\{i\xi\varphi_\alpha-i\eta\varphi_\beta\}.$$

It follows from (2.3) that the wave field $E_0(\mathbf{r}, t)$ makes the medium anisotropic, inhomogeneous, and nonstationary.

For the Fourier component of the field

$$\mathbf{E}(\mathbf{k},\omega)=(2\pi)^{-3}\int d^3r\,e^{-i\mathbf{k}\mathbf{r}}\mathbf{E}(\mathbf{r},\omega)$$

Maxwell's equations lead to

$$(k^2\delta_{ij}-k_ik_j-\omega^2\varepsilon_{ij})E_j(\mathbf{k},\omega)=4\pi ie\omega v_i\delta(\omega-\mathbf{kv})(2\pi)^{-3}$$
$$+\omega^2\sum_{\alpha,\beta=1,2}\sum_{\xi,\eta=\pm1}Q_{ij}(\omega,\alpha,\beta,\xi,\eta)E_j(k-\xi k_\alpha+\eta k_\beta,\omega-\xi\omega_\alpha+\eta\omega_\beta). \tag{2.5}$$

It is convenient to introduce an auxiliary quantity $E_i^0(\mathbf{k}, \omega)$, which is the solution of (2.5) as $Q \to 0$, i.e., the self-field of a charge moving with velocity v in a medium with dielectric constant (2.4):

$$E_i{}^0(\mathbf{k},\omega)=E_i{}^0(\mathbf{k})\delta(\omega-\mathbf{kv})=\frac{4\pi ie\delta(\omega-\mathbf{kv})}{(k^2-\omega^2\varepsilon)\varepsilon}(2\pi)^{-3}$$
$$\times\left\{\varepsilon\omega v_i-k_i+\frac{\varkappa}{\varepsilon}\frac{(k_i(\mathbf{ke})-e_i\omega^2\varepsilon)((\mathbf{ke})-(\mathbf{ve})\omega\varepsilon)}{k^2+(\mathbf{ek})^2\varkappa/\varepsilon-\omega^2(\varepsilon+\varkappa)}\right\} \tag{2.6}$$

Using (2.6), we can rewrite (2.5) in the form

$$E_i(\mathbf{k},\omega)=E_i{}^0(\mathbf{k},\omega)$$
$$\times\sum_{\alpha,\beta=1,2}\sum_{\xi,\eta=\pm1}\Phi_{is}(\mathbf{k},\omega,\alpha,\beta,\xi,\eta)E_s(\mathbf{k}-\xi\mathbf{k}_\alpha+\eta\mathbf{k}_\beta,\omega-\xi\omega_\alpha+\eta\omega_\beta). \tag{2.7}$$

Equation (2.7) can be solved by successive approximations, using the smallness of the ratio of the external exciting field to the internal one, i.e., the smallness of the quantity

$$\Phi_{is}(\mathbf{k},\omega,\alpha,\beta,\xi,\eta)=Q_{js}(\omega,\alpha,\beta,\xi,\eta)(k^2-\omega^2\varepsilon)^{-1}\Big\{\omega^2\delta_{ij}$$
$$-\frac{k_ik_j(k^2-\omega^2(\varepsilon+\varkappa))-\omega^4\varkappa\varepsilon e_ie_j+\omega^2\varkappa(\mathbf{ek})(e_ik_j+e_jk_i)}{\varepsilon[k^2+(\mathbf{ek})^2\varkappa/\varepsilon-\omega^2(\varepsilon+\varkappa)]}\Big\}$$

In the zeroth order we can neglect Φ_{is} in the right-hand side of (2.7), and then the solution (2.7) coincides with (2.6). In first order, we replace E_s by E_s^0 in the right-hand side of (2.7):

$$E_i^{(1)}(\mathbf{k},\omega)=E_i{}^0(\mathbf{k},\omega)$$
$$+\sum_{\alpha,\beta=1,2}\sum_{\xi,\eta=\pm1}\Phi_{is}(\mathbf{k},\omega,\alpha,\beta,\xi,\eta)E_s{}^0(\mathbf{k}-\xi\mathbf{k}_\alpha+\eta\mathbf{k}_\beta,\omega-\xi\omega_\alpha+\eta\omega_\beta).$$

The first-approximation solution (2.8) is a sum of terms, each of which can be conveniently investigated independently. By way of example we give the most interesting terms with $\xi = -\eta = 1$, $\alpha = 1$, $\beta = 2$, and $\alpha = 2$, $\beta = 1$, corresponding to absorption of two different quanta of the exciting field. We confine ourselves to media in which no Cerenkov radiation occurs, i.e., the zeroth-approximation solution (2.6) does not contain an emission field, and consequently $E_i^0(\mathbf{k}, \omega)$ has no poles on the real axis. The first-order emission field is then determined by the poles of the quantity

$$\Phi_{ij}(\mathbf{k},\omega)=\Phi_{ij}(\mathbf{k},\omega,1,2,1,-1)+\Phi_{ij}(\mathbf{k},\omega,2,1,1,-1).$$

Taking into account the presence of infinitesimally small absorption in the medium, we can determine the field at large distances from the known asymptotic formula

$$\int\frac{d^3pf(\mathbf{p})e^{i\mathbf{pR}}}{p^2-k^2-i0}\approx2\pi^2\frac{e^{ikR}}{R}f\left(k\frac{\mathbf{R}}{R}\right), \tag{2.9}$$

which is valid when $kR \gg 1$.

The anisotropy of the properties of the medium, in an external field leads to the existance of ordinary and extraordinary waves, as in uniaxial crystals. The field of the coherent emission of the ordinary waves can be obtained in the form

$$E_i^{\rm ord}(\mathbf{R},\omega)=2\pi^2\omega^2\frac{\exp\{i\omega\sqrt{\varepsilon}R\}}{R}\left(\delta_{ij}-\frac{n_in_j+e_ie_j-(\mathbf{en})(e_in_j+e_jn_i)}{1-(\mathbf{ne})^2}\right)$$
$$\times Q_{js}(\omega)E_s{}^0(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\delta(\omega-\omega_1-\omega_2-\mathbf{vk}+\mathbf{vk}_1+\mathbf{vk}_2);$$
$$Q_{js}(\omega)=Q_{js}(\omega,1,2,1,-1)+Q_{js}(\omega,2,1,1,-1),\quad\mathbf{k}=\mathbf{n}\omega\sqrt{\varepsilon}, \tag{2.10}$$

$\mathbf{n} = \mathbf{R}/R$ is a unit vector of direction to the observation point. The field of the coherent-emission of frequency ω is given by the expression

$$\cos\vartheta=\frac{1}{v\sqrt{\varepsilon}}\left(1-\frac{\omega_1+\omega_2-\mathbf{vk}_1-\mathbf{vk}_2}{\omega}\right), \tag{2.11}$$

which coincides with formula (1.2), which was obtained from kinematic considerations.

The field of coherent emission of extraordinary waves can be obtained in the form

$$E_i^{\rm extr}(\mathbf{R},\omega)=2\pi^2\omega^2\frac{\exp\{iR\omega[\varepsilon+\varkappa-\varkappa(\mathbf{ne})^2]^{1/2}\}}{R\sqrt{\varepsilon}[\varepsilon+\varkappa-\varkappa(\mathbf{ne})^2]^{1/2}}\frac{1-(\mathbf{ne})^2(\varkappa/\varepsilon+\varkappa)}{1-(\mathbf{ne})^2}$$
$$\times\{q_iq_j(\mathbf{eq})^2(\varepsilon+\varkappa)^2+\varepsilon e_ie_j-(\mathbf{eq})(\varepsilon+\varkappa)(e_iq_j+e_jq_i)\}Q_{js}(\omega)$$
$$\times E_s{}^0(\mathbf{q}\omega\sqrt{\varepsilon+\varkappa}-\mathbf{k}_1-\mathbf{k}_2)\delta(\omega-\omega_1-\omega_2-(\mathbf{qv})\omega\sqrt{\varepsilon+\varkappa}+\mathbf{vk}_1+\mathbf{vk}_2), \tag{2.12}$$

where

$$\mathbf{q}=\left(\mathbf{n}-\mathbf{e}(\mathbf{en})\frac{\varkappa}{\varepsilon+\varkappa}\right)\left(1-(\mathbf{en})^2\frac{\varkappa}{\varepsilon+\varkappa}\right)^{-1/2}. \tag{2.13}$$

The coherent emission of extraordinary waves of frequency ω can be registered in an observation point situated in a direction determined from the condition

$$\frac{\mathbf{n}(\mathbf{v}-\mathbf{e}(\mathbf{ev})\varkappa/(\varepsilon+\varkappa))}{[1-(\mathbf{ne})^2\varkappa/(\varepsilon+\varkappa)]^{1/2}}=\frac{1}{\sqrt{\varepsilon+\varkappa}}\left(1-\frac{\omega_1+\omega_2-\mathbf{vk}_1-\mathbf{vk}_2}{\omega}\right) \tag{2.14}$$

Formula (2.14) differ significantly from the kinematic formula (12) in which no account is taken of the anisotropy of the medium in the field. At small κ/ϵ, however, the difference between the ordinary and extraordinary waves is small, and in the limits as $\kappa \to 0$ expressions (2.14) and (2.11) coincide. If we neglect the ratio κ/ϵ, then the coherent-emission field is given by

$$E_i(R,\omega)=2\pi^2\omega^2\frac{\exp\{i\omega\sqrt{\varepsilon}R\}}{R}(\delta_{ij}-n_in_j)Q_{js}(\omega)$$
$$\times E_s{}^0(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\delta(\omega-\omega_1-\omega_2-\mathbf{vk}+\mathbf{vk}_1+\mathbf{vk}_2), \tag{2.15}$$

and the emission direction is the same as for the ordinary waves.

The energy radiated by coherent emission into a solid angle $d\Omega$ in the direction $\mathbf{n}$ and in the frequency interval $d\omega$ after a long interaction time T can be easily obtained from (2.10), (2.12), or (2.15). For example, the ordinary waves carry away an energy

$$d\mathscr{E}=\omega^4\,d\omega\,d\Omega\,T2\pi^3\sqrt{\varepsilon}\Big\{E_s{}^{0*}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)E_\sigma{}^0(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\cdot$$

$$\times Q_{ei}(\omega)Q_{sj}^{*}(\omega)\left[\delta_{ij}-\frac{n_in_j+e_ie_j-(\mathbf{en})(e_in_j+e_jn_i)}{1-(\mathbf{ne})^2}\right]\}$$
$$\times\delta(\omega-\omega_1-\omega_2-\mathbf{kv}+\mathbf{k}_1\mathbf{v}+\mathbf{k}_2\mathbf{v}). \quad (2.16)$$

In the limiting case $\kappa \ll \epsilon$, the difference between the ordinary and extraordinary waves vanishes, and the energy carried away is

$$d\mathscr{E}=\omega^4 d\omega d\Omega T\cdot 2\pi^3\sqrt{\varepsilon}\{E_s^{0*}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)E_e^0(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\cdot$$
$$\times Q_{ei}(\omega)Q_{sj}^{*}(\omega)(\delta_{ij}-n_in_j)\}\delta(\omega-\omega_1-\omega_2-\mathbf{kv}+\mathbf{k}_1\mathbf{v}+\mathbf{k}_2\mathbf{v}). \quad (2.17)$$

3. COHERENT EMISSION IN A RESONANT EXTERNAL FIELD

The exciting fields alters the populations of the atomic levels more strongly if its frequency coincides with the transition frequency of the bound electron in the atom.

In order that the external field produce weak excitation of the atoms in the absence of the particle, it is convenient to choose field frequencies that resonate not with the transitions from the ground state, but with transitions between two excited states. The field then strongly redistributes the populations of the atoms already excited by the particle, but has little effect on the unexcited atoms. We therefore choose the external field in the form (2.2), and the frequencies are equal to

$$\omega_1=E_2-E_1+\varepsilon_1=\omega_{21}+\varepsilon_1,\quad \omega_2=E_3-E_2+\varepsilon_2=\omega_{32}+\varepsilon_2$$
$$(\varepsilon_i\ll\omega_i);$$

here $E_3 > E_2 > E_1$ are the energies of the excited states of the atoms.

The absence of sufficiently detailed experimental data on the behavior of the nonlinear susceptibility $\chi(\omega, \omega', \omega'')$ at resonant frequencies makes it impossible to estimate the effect at present. We therefore estimate theoretically the effect in the resonant case by using a microscopic treatment and expressing the quantity $Q_{ij}(\omega)$, which enters directly in the final result, in terms of the microscopic characteristics of the medium and the linear dielectric constant.

We obtain the nonlinear part of the polarization of the medium

$$\mathbf{P}^{NL}(\mathbf{k},\omega)=(2\pi)^{-3}\int d^3r\, e^{-i\mathbf{kr}}\mathbf{P}^{NL}(\mathbf{r},\omega),$$

part of which is $Q_{ij}(\omega)E_j(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2, \omega-\omega_1-\omega_2)$.

If we introduce the dipole moment induced by the field in an atom located at the point $\mathbf{R}$:

$$\mathbf{d}(\mathbf{R},t)=\int d^3k\int d\omega\,\mathbf{d}(\mathbf{k},\omega)\exp\{i\mathbf{kR}-i\omega t\},$$

then the nonlinear polarization can be connected with the linear part $\mathbf{d}$, following[4,5],

$$\mathbf{P}^{NL}(\mathbf{k},\omega)=n_0\zeta\mathbf{d}^{NL}(\mathbf{k},\omega),$$

where the coefficient ζ, which takes into account the deviation of the field acting on the atom from the mean value, is given by[4,7]

$$\zeta=\prod_{i=1}^{4}{}^1/_3(\varepsilon_0(\omega_i)+2)$$

and is the cause of the unusually large polarizability of a medium with large refractive index. The nonlinear part of the induced dipole moment can be expressed in terms of the coefficients $c_s(\mathbf{R}, t)$ of the expansion of the wave function of the atom in the field in terms of the states of the isolated atom:

$$\mathbf{d}^{NL}(\mathbf{k},\omega)=\sum_s\{\mathbf{d}_{0s}c_s^{NL}(\mathbf{k},\omega-\omega_{s0})+\mathbf{d}_{s0}c_s^{NL*}(-\mathbf{k},-\omega+\omega_{s0})\},$$
$$c_s(\mathbf{k},\omega)=(2\pi)^{-4}\int d^3R\int dt\,c_s(\mathbf{R},t)\exp\{i\omega t-i\mathbf{kR}\}, \quad (3.1)$$

where account is taken of the fact that the excitation of the atoms is weak, $c_s \ll c_0 \approx 1$.

As is well known, in a long wave field the equation for $c_s(\mathbf{R}, t)$ is

$$i\frac{\partial c_s(\mathbf{R},t)}{\partial t}=-\sum_e\mathbf{d}_{se}\mathbf{E}(\mathbf{R},t)c_e(\mathbf{R},t)\exp\{i\omega_{se}t\}. \quad (3.2)$$

In the solution of (3.2) we shall assume that the transitions 3 - 2 and 2 - 1 are due only to the resonant components of the fields, the transitions 0 - 2 are forbidden, and the higher levels E_n ($n \geq 4$) have little effect on the populations of the levels E_1, E_2, and E_3, When the resonant field acts on the excited levels, it is assumed that the field has time to transfer an electron from one level to the other during the lifetime of the excited state. The transfer frequency should therefore be larger than the width of each level:

$$|V_{21}|\gg\gamma_1,\gamma_2,\qquad |V_{32}|\gg\gamma_3,\gamma_2$$
$$(V_{21}=-{}^1/_2\mathbf{d}_{21}\mathbf{E}_{01},\quad V_{32}=-{}^1/_2\mathbf{d}_{32}\mathbf{E}_{02}). \quad (3.3)$$

Taking the foregoing into account, we can obtain from (3.2) a system of equations for $c_3(\mathbf{k}, \omega)$, $c_2(\mathbf{k}, \omega)$ and $c_1(\mathbf{k}, \omega)$:

$$\omega c_3(\mathbf{k},\omega)=V_{32}c_2(\mathbf{k}-\mathbf{k}_2,\omega-\varepsilon_2)+U_{30}(\mathbf{k},\omega),$$
$$\omega c_2(\mathbf{k},\omega)=V_{21}c_1(\mathbf{k}-\mathbf{k}_1,\omega-\varepsilon_1)+V_{23}c_3(\mathbf{k}+\mathbf{k}_2,\omega+\varepsilon_2), \quad (3.4)$$
$$\omega c_1(\mathbf{k},\omega)=V_{21}c_2(\mathbf{k}+\mathbf{k}_1,\omega+\varepsilon_1)+U_{10}(\mathbf{k},\omega),$$

where

$$U_{n0}(\mathbf{k},\omega)=-\mathbf{d}_{n0}\mathbf{E}(\mathbf{k},\omega)$$

under the assumption that the atom is excited from the ground state only by the particle field.

The finite energy widths of the excited levels can be taken into account by replacing ωc_s with $(\omega + i\gamma_s)c_s$ in the left-hand sides of (3.4). Neglecting the level width, we readily obtain

$$c_3(\mathbf{k},\omega)=\frac{1}{\omega}U_{30}(\mathbf{k},\omega)+$$
$$\frac{(\omega-\varepsilon_1-\varepsilon_2)|V_{32}|^2U_{30}(\mathbf{k},\omega)+\omega V_{32}V_{21}U_{10}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2,\omega-\varepsilon_1-\varepsilon_2)}{\omega[\omega(\omega-\varepsilon_2)(\omega-\varepsilon_2-\varepsilon_1-\omega|V_{21}|^2+(\omega-\varepsilon_1-\varepsilon_2)|V_{32}|^2]} \quad (3.5)$$

At frequencies close to ω_{30},

$$|\omega-\omega_{30}|\ll\omega,$$

the dipole moment induced in the atom differs from that induced in the atom in the absence of an external field by an amount

$$\mathbf{d}^{NL}(\mathbf{k},\omega)=\mathbf{d}_{03}c_3^{NL}(\mathbf{k},\omega-\omega_{30}) \quad (3.6)$$
$$=-\frac{\mathbf{d}_{03}V_{32}V_{21}(\mathbf{d}_{10}\mathbf{E}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2,\omega-\omega_1-\omega_2))}{(\omega-\omega_{30})(\omega-\omega_{30}-\varepsilon_2)(\omega-\omega_{30}-\varepsilon_1-\varepsilon_2)-|V_{21}|^2(\omega-\omega_{30})-|V_{32}|^2(\omega-\omega_{30}-\varepsilon_1-\varepsilon_2)}$$

A significant difference between the resonant case and the nonresonant case considered in the preceding section is the nonlinear dependence of the polarization on the amplitude of the external exciting field, which is more complicated than in (2.1). It is therefore convenient to calculate not the nonlinear polarizability χ, but directly the quantity $Q_{ij}(\omega)$ which enters in the expression (2.3) for the induction and in the final results (2.10) -(2.17). All the results of the preceding section, except (2.1) and the definition of $Q_{ij}(\omega, \alpha, \beta, \xi, \eta)$ in (2.4), remain in force.

Recognizing that the wave function is spherically symmetrical in the ground state, $l = 0$, $m = 0$, we assume that the excited state 2 also has $l = 0$ and $m = 0$.

If the direction of the z axis is chosen along the external field E_0, then the external field causes resonant transitions between state 2 and states 1 and 3, in which $l = 1$ and $m = 0$. It follows therefore that

$$Q_{ij}(\omega) = \frac{-1}{36} \frac{n_0 \zeta E_{01} E_{02} d_{03} d_{32} d_{21} d_{10}}{(\omega-\omega_{30})(\omega-\omega_{30}-\varepsilon_2)(\omega-\omega_{30}-\varepsilon_1-\varepsilon_2) - |V_{21}^0|^2(\omega-\omega_{30}) - |V_{32}|^2(\omega-\omega_{30}-\varepsilon_1-\varepsilon_2)} \quad (3.7)$$

where d_{if} is the matrix element of the quantity er in terms of the radial wave functions of the states i and f,

$$V_{21}^0 = -\frac{1}{2\sqrt{3}} d_{21} E_{01}, \quad V_{32}^0 = -\frac{1}{2\sqrt{3}} d_{32} E_{02}.$$

If the frequency of the exciting field coincide exactly with the transition frequencies, it is necessary to take into account the finite energy width of the excited levels of the atom. Allowance for the width yields in this case

$$Q_{ij}(\omega) = \frac{1}{36} e_i e_j n_0 \zeta E_{01} E_{02} \frac{d_{03} d_{32} d_{21} d_{10}}{(\omega - \omega_{30} - i\Gamma) G};$$
$$G = (\omega - \omega_{30})^2 - |V_{21}^0|^2 - |V_{32}^0|^2 \equiv (\omega - \omega_{30})^2 - \Omega^2, \quad (3.8)$$
$$\Gamma G = (\omega - \omega_{30})^2 (\gamma_1 + \gamma_2 + \gamma_3) - \gamma_3 |V_{21}^0|^2 - \gamma_1 |V_{32}^0|^2.$$

Consequently, when $\epsilon_1 = \epsilon_2 = 0$ the energy width must be taken into account in the narrow frequency region $\omega - \omega_{30} \sim \Gamma$, $\omega - \omega_{30} \pm \Omega \sim \Gamma$.

4. INTENSITY OF COHERENT EMISSION IN THE RESONANT CASE

Substituting the value (3.7) of Q_{ij} in (2.17), we can obtain the energy carried away by coherent emission in the direction n into a solid angle $d\Omega$ and in the frequency interval $d\omega$, assuming that the external field gives rise to small anisotropy, $\kappa \ll \epsilon_0$:

$$d\mathscr{E} = \frac{2\pi^3}{9} T \omega^4 d\omega d\Omega \sqrt{\varepsilon}\, n_0^2 \zeta^2 [\mathbf{ne}]^2 |d_{03} d_{10}|^2 |e\mathbf{E}^0(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2)|^2$$
$$\times \frac{|V_{21}^0|^2 |V_{32}^0|^2 \delta(\omega - \omega_1 - \omega_2 - \mathbf{kv} + \mathbf{k}_1\mathbf{v} + \mathbf{k}_2\mathbf{v})}{[(\omega-\omega_{30})(\omega-\omega_{30}-\varepsilon_2)(\omega-\omega_{30}-\varepsilon_1-\varepsilon_2) - |V_{21}^0|^2(\omega-\omega_{30}) - |V_{32}|^2(\omega-\omega_{30}-\varepsilon_1-\varepsilon_2)]^2} \quad (4.1)$$

($\mathbf{E}^0(\mathbf{k})$ is defined in (2.6)). In the case of exact resonance and exact tuning of the exciting field, it is necessary to use for $Q_{ij}(\omega)$ formula (8), which takes explicit account of the finite energy widths of the excited levels:

$$d\mathscr{E} = \frac{2\pi^3}{9} T \omega^4 d\omega d\Omega \sqrt{\varepsilon}\, n_0^2 \zeta^2 [\mathbf{ne}]^2 |d_{03} d_{31}|^2 |e\mathbf{E}^0(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2)|^2$$
$$\times \frac{|V_{21}^0|^2 |V_{32}^0|^2}{[(\omega - \omega_{30})^2 + \Gamma^2] G^2} \delta(\omega - \omega_1 - \omega_2 - \mathbf{kv} + \mathbf{k}_1\mathbf{v} + \mathbf{k}_2\mathbf{v}). \quad (4.2)$$

It is of interest to compare (4.1) and (4.2) with the intensity of Cerenkov radiation in a medium with $\epsilon - 1 \sim 1$. Considering emission angles that are not too small, $\vartheta \sim 1$, we readily see that the ratio of the energies emitted in the frequency interval $d\omega$ in the case of coherent emission in a field and in the case of Cerenkov radiation is, generally speaking,

$$\frac{d\mathscr{E}}{d\mathscr{E}_C} \sim (n_0 r_{at}^3)^2 \left(\frac{\omega_{at}}{\Delta\omega}\right)^2, \quad (4.3)$$

where r_{at} is of the order of the atomic dimension and $\Delta\omega$ is the largest of the quantities $\omega - \omega_{30}$, $\omega_{32} - \omega$, $\omega_{21} - \omega_1$, G, γ_0, γ_2, and γ_3.

It follows therefore in particular, that even in a gas ($n_0 \sim 10^{19}\ \mathrm{cm}^{-3}$) the intensity of the coherent emission in a field is of the same order as the intensity of Cerenkov radiation in a dense medium ($\epsilon - 1 \sim 1$) if the wavelength of the exciting field in the visible region differs from the wavelength of the radiation emitted by the atom by not more than 100 Å. A decrease in the deviation from the resonance of the exciting field or an increase of the density of the medium increase the intensity of the coherent emission in a field, so that the latter can exceed the intensity of the Cerenkov radiation. The published data[8-10] indicate that there are possibilities of having a set of frequencies of excited coherent radiation such as to ensure a coherent-emission frequency larger than the Cerenkov radiation intensity in a medium with $\epsilon - 1 \sim 1$. A detailed analysis of the variants of the optimal choice of the working medium and of the exciting-field sources will be carried out separately.

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[1] I. E. Tamm and I. M. Frank, Dokl. Akad. Nauk SSSR 14, 107 (1937).
[2] M. I. Ryazanov, ZhETF Pis. Red. 15, 437 (1972) [JETP Lett. 15, 310 (1972).
[3] S. A. Akhmanov and R. V. Khokhlov, Problemy nelineĭnoĭ optiki (Problems of Nonlinear Optics), M., 1964.
[4] N. Bloembergen, Nonlinear Optics, Benjamin, 1966.
[5] V. M. Faĭn, Kvantovaya radiofizika (Quantum Radiophysics), Soviet Radio, 1972.
[6] P. D. McWane and D. A. Sealer, Appl. Phys. Lett., 8, 278 (1966).
[7] P. D. Maker and R. W. Terhune, Phys. Rev., 137, A801 (1965).
[8] A. R. Striganov and N. S. Sventitskiĭ, Tablitsy spektral'nykh liniĭ (Tables of Spectral Lines, M., 1966.
[9] Z. Kiss and R. Pressley, Proc. IEEE, 54, 7 (1966).
[10] A. Bloom, Proc. IEEE, 54, 39 (1966).

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