Scattering of radiation by thermal electrons in a magnetic field

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(Submitted February 10, 1973)
Zh. Eksp. Teor. Fiz. **65**, 102–116 (July 1973)

Satellites of the fundamental frequency $\omega \pm n\omega_H$ appear following scattering of radiation by thermal electrons. Compton exchange of energy between the plasma and radiation at $(kT_e/\hbar) > \omega > \omega_H > \Delta\omega_D$ is due mainly to scattering into the satellites, but the energy exchange rate is not altered compared to the case of absence of a magnetic field. Classical and quantum derivations of the satellite intensities are presented. The scattering gives the main contribution to absorption of radiation with $\omega \sim \omega_H$ in the case of low density and weak plasma turbulence. In this case nonlinear scattering of resonance radiation into the high frequency harmonics $\omega' \sim n\omega_H$ may be great; this scattering simulates synchrotron radiation of a hot plasma with $kT_* = m_e v^2/2 = e^2E^2/2m_e(\omega - \omega_H)^2$ despite the fact that the plasma may be cold $T_e \ll T_*$. At a high brightness temperature of the resonance radiation $kT_b > m_e c^2$ and $|\omega - \omega_H| \ll \omega_H$, the electron moves in the field of the wave with a relativistic velocity $eE/m_e|\omega - \omega_H|c > 1$. The presence of a resonance in the scattering cross section at $\omega = \omega_H$ makes feasible the observation of "strong" wave effects when the wave is in fact a "weak" one in the usual sense of the word: $(eE/m_e\omega c) > 1$. At $\omega \ll \omega_H$ the transition of an ordinary wave to an extraordinary one and the converse transition during scattering occurs with a small cross section $\sim \sigma_T(\omega/\omega_H)^2$.

1. INTRODUCTION

The circular polarization of optical radiation observed in the case of white dwarfs[1] (and possibly for x-ray stars[2]) suggests the presence of strong magnetic fields $H \sim 10^6$–10^8 G in these objects. We note that still stronger fields (10^{12}–10^{13} G) appear to be present in neutron stars and, in particular, in pulsars. X-ray stars, which are compact objects, have a large optical depth due to Thomson scattering ($\tau_T = N_e\sigma_T R > 1$) and small bremsstrahlung depth for photons with $\hbar\omega \lesssim kT_e$.[3] As a result, the intensity in region $\hbar\omega \sim kT_e$ is much lower than the Planck intensity. Under these conditions, the Compton effect may be an important mechanism for energy transfer between plasma and radiation, and for the generation of the x-ray spectrum. Thus, the low-frequency bremsstrahlung photons increase their energy (owing to the Doppler effect) during scattering by hot electrons and diffuse into the Wien region of the spectrum.[4,5]

Published analyses of the interaction of radiation with electrons in a magnetic field refer to two extreme astrophysical situations, namely, 1) an optically thin medium, where it is sufficient to consider the radiation (its spectrum, polarization, and angular distribution), and 2) an optically thick medium, where local thermodynamic equilibrium is set up and radiative heat transfer has to be considered. In the latter case, the scattering process and its dependence on frequency, direction, and polarization play an important role.

Scattering by a stationary electron in a magnetic field has been considered by Gurevich and Pavlov,[6] Loskutov and Leventuev,[7] and, recently, by Kanuto et al.[8] in connection with astrophysical applications. In this paper we shall consider in detail the change in frequency upon scattering. This is important in the intermediate case when scattering has an important effect on the radiated spectrum, i.e., the spectrum is shifted, approaching the Wien form but not yet reaching the Planck equilibrium. The ideas developed by Kompaniets[4] on the kinetics of processes leading to equilibrium between plasma and radiation are extended to the case of magnetized electrons.

The frequency change and energy transfer between photons and electrons are connected with the recoil effect in the case of scattering by an electron at rest and the Doppler effect (when the thermal velocity of the electron is taken into account). At first sight, the situation in a magnetic field is quite different. The motion across the field occurs over circular orbits and quantum theory predicts the presence of discrete Landau levels with energies $n\hbar\omega_H = nheH/2\pi m_e c$. The longitudinal motion is not quantized but its energy is only half the total thermal energy. The change in the photon energy on scattering consists of the recoil and the Doppler effect associated with the longitudinal motion, and the change by an integer in the number of Larmor quanta due to transition of the electron from one Landau level to another. After scattering, the monochromatic radiation is transformed into a set of bands. However, detailed analysis shows that when the frequency ω of the radiation is greater than the Larmor frequency ω_H, all the physical conclusions are only slightly modified when the magnetic field is switched on. The total scattering cross section, the average change in energy, and the root mean square change in the photon energy on scattering, calculated from the band system, do not differ from those calculated from the Gaussian scattering function for free unmagnetized electrons. This result may be regarded as a natural consequence of the Bohr correspondence principle. Nevertheless, it is very instructive to examine how this principle becomes operative in the calculation.

The Landau levels form an equidistant system: $E_{n+1} - E_n = \hbar\omega_H$ independently of n. In the dipole approximation the absorption or emission of photons occurs with $\Delta n = \pm 1$ and, therefore, in the case of scattering, i.e. a process involving two photons, one would expect $\Delta n = 0, \pm 2$. Because of the equidistant property, all that remains is $\Delta n = 0$, and transitions with $\Delta n = \pm 2$ are forbidden in the dipole approximation. Harmonics ($\Delta n \neq 0$) appear in the next orders of the expansion in terms of the ratio of the orbit radius to the wavelength (and in principle when relativistic effects, which violate the equidistant property, are taken into account).

Since the bands are equidistant, it turns out that, as in the case of the harmonic oscillator, the classical analysis of the motion of the electron over an orbit in the field of the wave gives correct formulas for the scattering cross section and the intensity of the harmonics (satellites). Satellites of the carrier frequency, which appear in the problem of the scattering of waves by free thermal electrons, are directly connected with the presence of the magnetic field and the thermal electron velocities. The intensity of the satellites $\omega \pm n\omega_H$ decreases with increasing n in proportion to

$$\frac{1}{2^n}\frac{1}{n!}\left(\frac{v\omega}{c\omega_H}\right)^{2n}=\frac{1}{2^n n!}\left(\frac{kT_e}{m_e c^2}\right)^n\left(\frac{\omega}{\omega_H}\right)^{2n}=\frac{1}{2^n n!}\left(\frac{\Delta\omega_D}{\omega_H}\right)^{2n}.$$

The classical calculation of scattering by a harmonic oscillator of frequency ω_0 leads to a similar result: when $\omega > \omega_0$ the intensity of the satellites $\omega \pm n\omega_0$ is

$$\frac{\sigma I_\omega}{2^n n!}\left(\frac{\omega}{\omega_0}\right)^{2n}\left(\frac{kT}{mc^2}\right)^n,$$

where I_ω is the intensity of the incident radiation and σ is the scattering cross section.

As already noted, the scattering cross section approaches the Thomson value when the frequency of the radiation is high ($\omega \gg \omega_H$). At low frequencies ($\omega \ll \sigma_H$) the scattering cross section decreases and the electron becomes nearly free in the magnetic field, but when the electric field of the wave is perpendicular to the magnetic field the electron experiences a drift with the velocity $(c/H^2)[\mathbf{E}\times\mathbf{H}]$. When $\omega \ll \omega_H$ the Thomson cross sections for the ordinary and extraordinary waves are very different. It is shown below that the transition from one wave to the other during scattering is strongly suppressed and takes place with the cross section $\sigma \sim \sigma_T(\omega/\omega_H)^2$.

Finally, for magnetized plasma exposed to an external source in astrophysical or laboratory situations one is particularly interested in the resonance at $\omega \approx \omega_H$. A relatively weak wave (weak in the sense that the rotational velocity of the electron $\mathbf{v} = e\mathbf{E}/m_e\omega$ in the field $\mathbf{E}$ of the wave is small in comparison with the velocity of light) may produce circular motion of the electron at high—even relativistic—energy. The interaction between a monochromatic strong wave and electrons has been widely discussed in the literature,[9,12] and Zel'dovich and Illarionov[12] have considered the effect of a strong wave on an electron in a strong magnetic field. It is shown below that a weak wave, whose frequency is equal to the gyrofrequency ω_H, may behave as if it were strong. This is connected with the resonance in the rotational velocity of the electron in the field of the wave: $v/c = eE/m_e c|\omega - \omega_H|$. When the magnetized plasma is illuminated by radiation with the resonant frequency, the resonance take-up of energy by the electrons can be detected through the emission of harmonics. We note that the necessary spectral width of the band of external radiation must be greater than the resonance width, so that the Doppler effect and the relativistic effects do not take the electron out of resonance. Because of the presence of resonance in the Thomson cross section,[6-8] this effect is of interest even when the optical thickness of the plasma for Thomson scattering is small, and it is particularly important under laboratory conditions.

The intensity of the harmonics $\omega' = n\omega_H$ for

$$v/c = eE/m_e c|\omega - \omega_H| \ll 1$$

decreases in proportion to[1)]

$$\frac{\omega^2}{(\omega-\omega_H)^2}\frac{1}{n!}\left(n\frac{v}{c}\right)^{2n}\approx\frac{\omega^2}{(\omega-\omega_H)^2}\frac{1}{n!}\left[\frac{neE}{m_e c|\omega-\omega_H|}\right]^{2n}.$$

We recall that, in the absence of the magnetic field, the intensity of the harmonics $\omega' = n\omega$ during the scattering of waves of finite amplitude by free electrons is very small and decreases as

$$\frac{1}{n!}\left(\frac{nv}{c}\right)^{2n}=\frac{1}{n!}\left(\frac{neE}{m_e c\omega}\right)^{2n}.$$

The interaction between radiation and magnetized electrons and, in particular, the question of the scattering of radiation, are of major interest both for astrophysics and the theory of laboratory plasma.

2. THOMSON SCATTERING BY THERMAL ELECTRONS IN A MAGNETIC FIELD

The equation of motion for a nonrelativistic electron in an external field $\mathbf{H}$ and the electric field $\mathbf{E}e^{i\omega t - i\mathbf{k}\cdot\mathbf{r}}$ of an electromagnetic wave is

$$m_e\frac{d\mathbf{v}}{dt}=e\mathbf{E}e^{i\omega t-i\mathbf{kr}}+\frac{e}{c}[\mathbf{vH}] \qquad (1)^*$$

where we have neglected the reaction of the radiation and, in the special case when $kr \ll 1$,

$$v_x=\frac{ie\omega}{m_e(\omega_H^2-\omega^2)}e^{i\omega t}\left(E_x-i\frac{\omega_H}{\omega}E_y\right)+v_{0x}\exp(i\omega_H t),$$
$$v_y=\frac{ie\omega}{m_e(\omega_H^2-\omega^2)}e^{i\omega t}\left(\frac{\omega_H}{\omega}E_x-iE_y\right)+v_{0y}\exp(i\omega_H t), \qquad (2)$$
$$v_z=v_{0z}+(eE_z/i\omega m_e)e^{i\omega t},$$

where $\mathbf{v}_0$ is the thermal velocity of the electron. The magnetic field lies along the z axis and $\omega_H = eH/m_e c$ is the cyclotron frequency. We recall that, in magnetoactive plasma, two types of wave may propagate independently of one another, namely, extraordinary and ordinary waves, each of which has its own absorption coefficient, phase velocity, and polarization. In the general case of propagation at an arbitrary angle to the magnetic field these waves are elliptically polarized.

The Thomson cross section of a resting electron in a strong magnetic field was calculated in[6,8] with the aid of Eq. (2) for $v_0 = 0$. In this paper we shall consider the noncoherent scattering of an electron in a magnetic field and, in particular, the re-emission by the electron of the energy carried by the wave of frequency $\omega \approx \omega_H$ at frequencies which are multiples of the gyrofrequency.

Using the solution of Eq. (1), we shall calculate the intensity radiated by thermal plasma from the well-known formula:[9,13]

$$I_{\alpha\beta}(\omega,\Omega)=\frac{e^2\omega^2}{4\pi^2c^3}\left\langle\left|\int\exp'-i\omega\left(t-\mathbf{n}_\alpha\frac{\mathbf{r}}{c}\right)\right)[\mathbf{n}_\alpha[\mathbf{n}_\alpha\mathbf{v}_\beta]]dt\right|^2\right\rangle, \qquad (3)$$

where α and β assume the following values: 1—extraordinary wave, 2—ordinary wave; $\langle\ldots\rangle$ represents averaging over the Maxwellian distribution and $\mathbf{n}_\alpha$ is the direction of observation of the normal wave of type α.

Equation (3) describes four processes: 1—the well-known synchrotron radiation by thermal plasma (the other three correspond to the scattering of electromagnetic radiation by thermal plasma), 2—coherent scattering, including the Doppler effect (which in the absence of the magnetic field and for $\omega \gg \omega_H$ corresponds to Thomson scattering), 3—emission by the electron in the field of the wave into harmonics of the wave frequency, and 4—processes in which the photon frequency changes by multiples of the gyrofrequency.

A. Synchrotron Radiation

When $v_{0\perp} \gg eE/m_c|\omega - \omega_H|$ the formula given by Eq. (3) describes the well-known synchrotron radiation by a thermal electron at the resonances of the cyclotron frequency $\omega = n\omega_H$. The intensity of this radiation decreases with increasing n in accordance with the formula[12]

$$\frac{1}{2^n n!}\left(\frac{n^2 kT_e}{m_e c^2}\right)^n$$

when $nv/c < 1$.

B. Satellites of the Carrier Frequency[2)]

For plasma in a magnetic field, the square of the transverse (perpendicular to the magnetic field) component of the thermal electron momentum is quantized, i.e., it assumes discrete values which are multiples of the gyrofrequency:

$$p_\perp^2 / 2m_e = s\hbar\omega_H = sheH / 2\pi m_e c,$$

where s is the number of the Landau level. This leads to the following conservation laws:

$$\begin{gathered}\frac{p^2}{2m_e} + \hbar\omega = \frac{(p')^2}{2m_e} + \hbar\omega', \quad p_\parallel + \frac{\hbar\omega}{c}\cos\theta = p_\parallel' + \frac{\hbar\omega'}{c}\cos\theta' \\ p_\perp'^2/2m_e - p_\perp^2/2m_e = n\hbar\omega_H, \quad n = s - s' = 0, \pm1, \pm2, \ldots .\end{gathered} \tag{4}$$

In these expressions p, ω and p′, ω' are electron momenta and photon frequencies before and after scattering, and θ, θ' are the angles between the directions of motion of the photon and the magnetic field before and after scattering.

In general, the solution of Eq. (1) is difficult to obtain. We shall therefore consider a number of special cases. When $\omega \gg \omega_H$ the solution of Eq. (1) is

$$\mathbf{v} = \frac{e\mathbf{E}}{im_e\omega} e^{i\omega t - i\mathbf{kr}},$$

where $\mathbf{r}$ defines the trajectory of the thermal electron in the magnetic field in the absence of the wave. By substituting this solution in Eq. (3), we obtain the differential cross section[3)] for a photon with wave vector $\mathbf{k} \to \mathbf{k}'$:

$$\begin{gathered}\frac{d\sigma}{d\Omega' dk'} = \frac{d\sigma}{d\Omega'}\left(\frac{k'}{k}\right)^2 \left(\frac{m_e c^2}{2\pi kT_e q_\parallel^2}\right)^{1/2} \sum_{n=-\infty}^{\infty} X_n(q_\perp) \\ \times \exp\left[-\frac{m_e c^2}{2kT_e q_\parallel^2}\left(k' - k - n\frac{\omega_H}{c}\right)^2\right],\end{gathered} \tag{5}$$

$$X_n(q_\perp) = \frac{m_e}{kT_e}\int_0^\infty v_\perp dv_\perp \exp\left(-\frac{m_e v_\perp^2}{2kT_e}\right) J_n^2\left(\frac{q_\perp v_\perp}{\omega_H}\right). \tag{6}$$

In the above expansion J_n is the Bessel function of the first kind, $\mathbf{q} = \mathbf{k}' - \mathbf{k}$ is the momentum transferred to the electron on scattering,

$$d\sigma / d\Omega' = 3\sigma_T(1 + \cos^2\vartheta) / 16\pi,$$

and ϑ is the angle of scattering. The total scattering cross section

$$\int \frac{d\sigma}{d\Omega' dk'} d\Omega' dk'$$

is equal to the Thomson cross section, and the coherent scattering cross section (the term with n = 0) is somewhat less than σ_T. The integral in Eq. (6) can readily be evaluated in an explicit form:

$$X_n(q_\perp) = \exp\left(-\frac{q_\perp^2 kT_e}{m_e\omega_H^2}\right) I_n\left(\frac{kT_e q_\perp^2}{m_e\omega_H^2}\right), \tag{7}$$

where I_n is the Bessel function of the first kind of an imaginary argument.

Equation (5) can be obtained from the quantum-mechanical formula for the differential probability of scattering of a photon by an electron in a magnetic field:[6,8][4)]

$$\left(\frac{dW}{d\Omega' d\omega'}\right)_{fi} = \frac{r_e^2}{2}\left(\frac{\omega'}{\omega}\right)^2 |\mathbf{e}'^*\mathbf{e}\langle f|e^{-i\mathbf{qr}}|i\rangle - M_1 - M_2|^2, \tag{8}$$

where $r_e = e^2/m_e c^2$ is the classical radius of the electron. The first term inside the modulus is the matrix element for the transition of the system from the initial to the final state, valued for the operator $(e^2/2m_e c^2)\mathbf{A}\cdot\mathbf{A}$ in first-order perturbation theory. M_1 and M_2 are the matrix elements of the operator $(e/m_e c)\mathbf{P}\cdot\mathbf{A}$[5)] calculated in second-order perturbation theory. These elements are small when $\omega \gg \omega_H$. The vectors $\mathbf{e}$ and $\mathbf{e}'$ are the polarization unit vectors before and after scattering.

The total scattering probability can be obtained by summing Eq. (8) over all the final states and averaging over the initial states

$$dW = \sum_f dW_{fi}.$$

Since the wave functions for the electron in the magnetic field form a complete system, we have

$$\begin{gathered}\frac{dW}{d\Omega' d\omega'} = \frac{r_e^2}{2}\left(\frac{\omega'}{\omega}\right)^2 |\mathbf{e}'^*\mathbf{e}|^2 \int_{-\infty}^{+\infty} dt_1 \int_{-\infty}^{+\infty} dt_2 \exp[i(\omega' - \omega)(t_1 - t_2)] \\ \times\langle i|\exp[i\mathbf{q}\hat{\mathbf{r}}(t_1)]\exp[-i\mathbf{q}\hat{\mathbf{r}}(t_2)]|i\rangle, \\ \hat{\mathbf{r}}(t) = \exp\left(\frac{i}{\hbar}H_0 t\right)\mathbf{r}\exp\left(-\frac{i}{\hbar}H_0 t\right),\end{gathered} \tag{9}$$

where $\hat{\mathbf{r}}(t)$ is the coordinate operator in the Heisenberg representation, which acts on the electron wave functions in the magnetic field. The operator in parentheses is reduced to the product of commuting operators and, therefore, the evaluation of the diagonal matrix element $\langle i|\ldots|i\rangle$ reduces to the replacement of these operators by the classical values (functions of time) of the corresponding quantities

$$\langle i|\exp(i\mathbf{q}\hat{\mathbf{r}}(t_1))\exp(-i\mathbf{q}\hat{\mathbf{r}}(t_2))|i\rangle = \langle i|\exp(i\mathbf{q}[\mathbf{r}(t_1) - \mathbf{r}(t_2)])|i\rangle,$$

$$\mathbf{r}(t) = \mathbf{r}(0) + \frac{\mathbf{x}}{x}\frac{v_\perp}{\omega_0}\sin\omega_0 t - \frac{\mathbf{y}}{y}\frac{v_\perp}{\omega_0}\cos\omega_0 t + \frac{\mathbf{z}}{z}v_\parallel t,$$

where $\mathbf{r}(t)$ is the equation of the trajectory of the electron in the magnetic field and $\omega_0 = \omega_H(1 - v^2/c^2)^{1/2}$.

When we integrate with respect to t_1 and t_2 we use the following well-known expansion in terms of the Bessel functions:

$$e^{ix\sin\theta} = \sum_{n=-\infty}^{\infty} e^{in\theta} J_n(x).$$

Dividing Eq. (9) by the total interaction time $\mathbf{T} = 2\pi\delta(0)$, we obtain the expression for the scattering cross section of an electron for a photon in a magnetic field. This result must be then averaged over the initial states of the electrons, and the polarization states of the incident and scattered photons. When $\hbar\omega_H \ll kT_e$ the magnetic field does not affect the electron distribution function, and Eq. (9) may be averaged over the Maxwell distribution. The final result is Eq. (5).

In the case of a strong field, the argument of the Bessel function $q_\perp^2 kT_e/m_e\omega_H^2 < 1$. Expanding $I_n(z)$ into a series in terms of the small arguments, we have from Eqs. (5) and (7)

$$\begin{gathered}\frac{d\sigma}{d\Omega' dk'} = \left(\frac{d\sigma}{d\Omega'}\right)\left(\frac{k'}{k}\right)^2\left(\frac{m_e c^2}{2\pi kT_e q_\parallel^2}\right)^{1/2}\left\{\exp\left[-\frac{m_e c^2}{2kT_e q_\parallel^2}(k' - k)^2\right]\right. \\ + \sum_{n\neq0,\, n=-\infty}^{\infty} \frac{1}{2^{|n|}|n|!}\left(\frac{kT_e}{m_e c^2}\right)^{|n|}\left(\frac{cq_\perp}{\omega_H}\right)^{2|n|} \\ \left.\times\exp\left[-\frac{m_e c^2}{2kT_e q_\parallel^2}\left(k' - k - n\frac{\omega_H}{c}\right)^2\right]\right\}.\end{gathered} \tag{10}$$

The summation over negative n in this expression is carried out only up to n_m equal to the integer part of the ratio ck/ω_H. This eliminates the possibility of negative frequency.

The first term in Eq. (10) describes the Doppler broadening in the case of Thomson scattering in a magnetic field; the width of the profile is determined only by the parallel, nonquantized, component of the electron velocity. In the case of isotropic radiation, the line width is reduced by the factor of $\sqrt{3}$ in comparison with the usual Doppler profile. The higher terms in the expansion describe the change (both increase and reduction) in the frequency of the photon by an amount which is a multiple of ω_H ($\omega' = \omega \pm n\omega_H$). Radiation into high satellites $\omega' - \omega \gg \omega_H$ has a directional angular distribution which is symmetric about $\theta' = \pi/2$, since

$$\mathscr{P} \approx \sin^2\theta / (1+\cos^2\theta).$$

Because of the strong angular dependence, this radiation should be plane polarized and its degree of polarization should be

$$q_\perp^2 = k'^2\sin^2\theta' + k^2\sin^2\theta - 2kk'\sin\theta'\sin\theta\cos\varphi.$$

The fraction of energy re-emitted into the satellite of number n is lower by the factor $2^{-2|n|}(|n|!)^{-1}(\Delta\omega_D/\omega_H)^{2|n|}$ than the energy re-emitted into the fundamental frequency ω. We recall that this result was obtained on the assumption that $\Delta\omega_D \lesssim \omega_H$.

In the opposite case of a weak field, $\omega_H \ll \Delta\omega_D$, we can use the asymptotic behavior of the Bessel function for large values of the argument to show quite readily that Eq. (20) reduces to the well-known expression for the Doppler broadening in the case of Thomson scattering by Maxwellian electrons:

$$\frac{d\sigma}{d\Omega'\,dk'} = \frac{d\sigma}{d\Omega'}\left(\frac{m_ec^2}{2\pi kT_eq^2}\right)^{1/2}\exp\left[-\frac{m_ec^2}{2kT_eq^2}(k'-k)^2\right].$$

Equation (10) depends on the square of the transferred transverse momentum, and hence the probabilities of scattering into the satellites $\omega + n\omega_H$ and $\omega = n\omega_H$ are not equal to one another: the transition probability increases with increasing frequency, i.e., in the case of noncoherent scattering the radiation receives energy from the electrons. It will be shown below that noncoherent scattering described by Eq. (10) is equivalent to the root mean square Doppler increase in the photon frequency.

Equation (10) is valid only for satellites whose numbers satisfy the condition

$$\frac{c^2q_\perp^2}{\omega_H^2}\frac{kT_e}{m_ec^2} \sim n^2\frac{kT_e}{m_ec^2}\sin^2\theta' < 1. \tag{11}$$

For slightly relativistic plasma with $T_e \sim 10^9$ °K the condition given by Eq. (11) is violated near $\theta' \approx \pi/2$ even for the third resonance. The differential photon scattering cross section for weakly relativistic plasma is evaluated in the same way as the intensity of cyclotron radiation.[16] It is

$$d\sigma/d\Omega'dk' = \text{const}\cdot(k'/k)^2\exp\left[-R^{1/2}(\omega'-\omega;\, q_\perp,\, q_\parallel)\right];$$

$$R(\omega'-\omega;q_\perp,q_\parallel) = \left(\frac{m_ec^2}{kT_e}\right)^2 + 2\frac{m_ec^2}{kT_e}\frac{\omega'-\omega}{\omega_H}x_0 + \tag{12}$$

$$+\frac{1}{\omega_H^2}\{2q_\perp^2c^2(1-\operatorname{ch}x_0) - x_0^2[c^2q_\parallel^2-(\omega'-\omega)^2]\}. \tag{13}$$

The function x_0 is defined as the root of the transcendental equation

$$x_0 - \frac{c^2q_\perp^2}{(\omega'-\omega)^2-c^2q_\parallel^2}\operatorname{sh}x_0 = -\frac{m_ec^2}{kT_e}\frac{\omega_H}{\omega'-\omega}\left[1-\frac{c^2q_\parallel^2}{(\omega'-\omega)^2}\right]^{-1}. \tag{14}$$

The angular distribution for the higher satellites $\omega + |n|\omega_H \gg \omega$ is the same as the angular distribution of cyclotron radiation by weakly relativistic plasma:[16]

$$\frac{d\sigma}{d\Omega'\,dk'} = \text{const}\cdot\left(\frac{k'}{k}\right)^2\exp\left[-\frac{\omega'-\omega}{\omega_H}f\left(\frac{\omega'-\omega}{\omega}\frac{kT_e}{m_ec^2}\right)\left(\theta'-\frac{\pi}{2}\right)^2\right]. \tag{15}$$

The function f(x) is tabulated in,[16] and when $x \gg 1$

$$f(x) \approx (48x^2)^{-1/3}.$$

C. The Resonance Region $\omega \approx \omega_H$ and the Probability of the Various Processes

The coherent scattering cross section (without change in the photon frequency) is usually calculated by solving Eq. (1) and then substituting the result in Eq. (3). This method involves very laborious calculations.[8] We shall use a simpler approach based on the use of the optical theorem and the dispersion relation for magnetoactive plasma. The refractive index of magnetoactive plasma, with thermal motion neglected, is[17]

$$n_\alpha^2 = 1 - \frac{2v(1-v-i\delta)}{2(1-i\delta)(1-i\delta-v) - u\sin^2\theta + (-)^\alpha[u^2\sin^4\theta + 4u(1-i\delta-v)^2\cos^2\theta]^{1/2}},$$

$$u = \omega_H^2/\omega^2,\quad v = \omega_p^2/\omega^2 \equiv 4\pi e^2N_e/m_e\omega^2,\quad \delta = \Gamma/\omega, \tag{16}$$

where Γ which governs the absorption of waves in plasma.

In the special case of propagation in the direction of the magnetic field, the refractive index given by Eq. (16) assumes the simplest form

$$n_\alpha^2 = 1 - \omega_p^2/\omega(\omega \mp \omega_H - i\Gamma). \tag{17}$$

On the other hand, it is well known (see, for example,[18]) that the refractive index is readily expressed in terms of the forward scattering amplitude $a_\alpha \equiv a_\alpha$ for an individual electron:

$$n_\alpha^2 = 1 - 4\pi c^2N_ea_\alpha/\omega^2. \tag{18}$$

Using Eqs. (17) and (18), the optical theorem σ_a $4\pi c\omega^{-1}\,\mathrm{Im}\,a_\alpha$, and the well-known expression for the damping of a harmonic oscillator $\Gamma = \gamma \equiv 2/3(e^2\omega^2/m_ec^3)$, we obtain the following expression for the total scattering cross section of an electron in a magnetic field outside resonance $|\omega - \omega_H| \gg \Gamma$ for $\theta = 0$:

$$\sigma_1(\theta=0) = \sigma_T\frac{\omega^2}{(\omega-\omega_H)^2},\qquad \sigma_2(\theta=0) = \sigma_T\frac{\omega^2}{(\omega+\omega_H)^2}. \tag{19}$$

Similarly, when $\theta = \pi/2$ we have

$$\sigma_1\left(\theta=\frac{\pi}{2}\right) = \sigma_T\frac{\omega^2(\omega^2+\omega_H^2)}{(\omega^2-\omega_H^2)^2},\qquad \sigma_2\left(\theta=\frac{\pi}{2}\right) = \sigma_T. \tag{20}$$

Thus, the cross section for the extraordinary wave on an electron at rest has a resonance near the gyrofrequency, which is connected with the electron velocity resonance (2). It tends to the Thomson value for $\omega \gg \omega_H$ and falls as $(\omega/\omega_H)^2\sigma_T$ when $\omega \ll \omega_H$. The cross section for the ordinary wave does not have a resonance and its magnitude is close to the Thomson value σ_T with the exception of the narrow angular region $\theta < (\omega/\omega_H)^{1/2}$ for $\omega/\omega_H \ll 1$, where $\sigma \approx \sigma_T(\omega/\omega_H)^2$. In collisional plasma

$$\Gamma = \nu_e = {}^4/_3(2\pi/m_e)^{1/2}e^4N_eL/(kT_e)^{3/2}$$

where L is the Coulomb logarithm. In this case, we have the well-known formulas for bremsstrahlung absorption by the electron in the field of the ion in plasma with a magnetic field:[17]

$$k_{ff}^{(\alpha)}(\omega,H,\theta=0) = k_{ff}(\omega,H=0)\,\omega^2/(\omega\mp\omega_H)^2,$$

$$k_{ff}^{(1)}\left(\omega,H,\theta=\frac{\pi}{2}\right) = k_{ff}(\omega,H=0)\frac{\omega^2(\omega_H^2+\omega^2)}{(\omega^2-\omega_H^2)^2},$$

$$k_{ff}^{(2)}\left(\theta=\frac{\pi}{2}\right)=k_{ff}(\omega, H=0).$$

In plasma with thermal motion of the electrons, the refractive index for $\theta = 0$ is (neglecting collisions)[17]

$$n_\alpha^2=1-\frac{\omega_p^2}{\omega(\omega\mp\omega_H)}J\left(\frac{\omega\mp\omega_H}{\omega}\left(\frac{m_ec^2}{kT_e}\right)^{1/2}\right),$$

$$J(x)=x\exp\left(-\frac{x^2}{2}\right)\int_{i\infty}^{x}dt\exp\left(\frac{t^2}{2}\right).$$

Using Eq. (18) and the optical theorem, we obtain the following expression for the bremsstrahlung absorption coefficient for the extraordinary wave in the first resonance $\omega \approx \omega_H$ in the thermal plasma:[17]

$$k_e^{(1)}(\theta=0)=\left(\frac{\pi m_ec^2}{8kT_e}\right)^{1/2}\frac{\omega_p^2}{\omega^2}\exp\left[-\frac{m_ec^2}{2kT_e}\frac{(\omega-\omega_H)^2}{\omega^2}\right].$$

D. The Resonance Region $\omega \approx \omega_H$ and Re-emission Into the Harmonics

The reradiation of scattered energy into the harmonics of the carrier frequency $\omega \sim \omega_H$ may be due to two processes: the first is independent of the amplitude of the incident wave and is analogous to the emission of satellites $\omega' = \omega \pm n\omega_H$ which was considered above and is connected with the presence of thermal electron velocities, while the second is nonlinear in E and is connected with the rotational motion of the electron in the field of the wave. The cross section for the first process for $\omega \sim \omega_H$ or $\omega' \sim \omega_H$ can be calculated from the quantum-mechanical formula,[6-8] averaging the final result over the Boltzmann distribution. Calculations show that when $\omega \gtrsim \omega_H$ the transition cross section for the first satellites have the resonance form

$$\sigma(\omega\to\omega+\omega_H)\approx\sigma_T\frac{kT_e}{m_ec^2}\frac{\omega^2}{(\omega-\omega_H)^2+(\Delta\omega_D)^2},$$

$$\sigma(\omega\to\omega-\omega_H)\approx\frac{kT_e}{m_ec^2}\frac{\sigma_T\omega^2}{(\omega-2\omega_H)^2+\Delta\omega_D^2}.$$

Near resonance these cross sections are of the order of σ_T. It is interesting that the cross section for the transition from $2\omega_H$ to ω_H may exceed the cross section for bremsstrahlung absorption in the second resonance.

We must now consider scattering at the harmonics of the carrier frequency, which is nonlinear in E. When $\omega \approx \omega_H$ the rotational velocity of the electron in the field of the extraordinary wave has the resonance

$$\mathrm{v}=\frac{e\mathrm{E}}{m_e|\omega-\omega_H|}e^{i\omega t}$$

whatever the dependence on the magnitude of the thermal velocity (the resonance does not occur in the field of the ordinary wave). This velocity may exceed the thermal velocity of the electron and may even reach the velocity of light although the wave itself is weak in the ordinary sense: $eE/m_ec\omega \equiv b < 1$. In the field of the wave the electron will reradiate its energy at the harmonics of the fundamental frequency. This process is quite weak during the usual Thomson scattering when $\omega \gg \omega_H$ and $b \ll 1$. It can be readily shown from Eq. (3) that the intensity of the harmonics decreases as $(nb)^{2n}/n!$. However, at the resonance $\omega \sim \omega_H$, where the rotational velocity of the electron is high, this radiation may compete with the synchrotron radiation by the thermal electron and may even exceed it. Moreover, at low plasma densities, and in the absence of appreciable plasma turbulence, the situation may arise where the rate at which the thermal energy is supplied is much less than the radiation width, i.e., the time taken by the electron in the magnetic field to radiate its transverse momentum. In this case, the thermal velocity of the electrons may be neglected, the synchrotron absorption is unimportant, and the growth in the vibrational velocity (characteristic time $\sim 1/|\omega - \omega_H|$) is not prevented by collisions. The high rotational velocities of the electrons will prevent plasma recombinations and the radiation escaping from it at the high harmonics of the carrier frequency $\omega = n\omega_H$ will simulate the synchrotron emission by hot plasma with $kT_* \approx 3/2m_e v^2$, despite the low plasma temperature.

As already noted, the solution of the equation of motion (1) for the electron in the field of the wave in the presence of the magnetic field is difficult to obtain in the general case. It is, however, possible to use the special case which has been treated by Zel'dovich and Illarionov.[12]

Consider a strong, circularly polarized, electromagnetic wave propagating in the direction of the magnetic field, which is scattered by an electron whose motion along the field is compensated by a longitudinal electric field, i.e., $v_{\parallel} = 0$. In this case, there is no difficulty in allowing for the radiative friction. If Γ is the effective width of the resonance and is determined by damping processes, the velocity of the electron in the field of the ordinary wave and the magnetic field is ($v \ll c$):

$$v_x=\frac{eE}{m_e\Gamma}\cos\omega_H t,\qquad v_y=\frac{eE}{m_e\Gamma}\sin\omega_H t. \tag{21}$$

The radiation intensity emitted by an electron traveling with this velocity when $\omega \approx \omega_H$ is, according to Eq. (3),

$$I(\omega',\theta')=\frac{e^2\omega'^2}{2\pi c}\left\{\sum_{n=1}^{\infty}\left[\operatorname{ctg}^2\theta' J_n^2\left(\frac{\omega'}{\omega_H}\frac{eE}{m_ec\Gamma}\sin\theta'\right)\right.\right.$$
$$\left.\left.+\left(\frac{eE}{m_ec\Gamma}\right)^2 J_n'^2\left(\frac{\omega'}{\omega_H}\frac{eE}{m_ec\Gamma}\sin\theta'\right)\right]\right\}\delta(n\omega_H-\omega'). \tag{22}$$

When $v_{0\perp} \ll eE/m_e\Gamma < c$, Eq. (22) determines the process of reradiation of the wave energy by a nonrelativistic electron into the harmonics $n\omega_H$.

Expanding Eq. (22) into a series in powers of $eE/m_ec\Gamma$, integrating with respect to the angles and frequencies, and dividing the result by $cE^2/4\pi$, we obtain an expression for the scattering cross section in the form of a sum of harmonics:

$$\sigma_e=\sigma_T\frac{\omega_H^2}{\Gamma^2}\left[1+\sum_{n=2}^{\infty}\frac{(n+1)n^{2n+1}}{(2n+1)!}\left(\frac{eE}{m_e\Gamma c}\right)^{2n}\right]. \tag{23}$$

The first term in this sum corresponds to resonance coherent scattering, and the last term to reradiation into the $\omega' = n\omega_H$. If Γ is determined by the radiation width $\gamma = 2r_e\omega^2/3c$, the coefficient in front of the square brackets in Eq. (23) becomes $\sigma_e = 6\pi\lambda^2 = 6\pi c^2/\omega_H^2$.

Scattering provides a contribution of the order of γ/Γ to the total absorption of radiant energy with $\omega \approx \omega_H$. In tenuous plasma, and for a low degree of excitation of plasma turbulence, we have $\Gamma = \gamma$. In this case, scattering becomes the dominant process whose cross section exceeds the true absorption cross section by a factor of $\gamma/(\Gamma - \gamma)$. If, on the other hand, $eE/m_e\Gamma_c > 1$, then we have the case analogous to scattering in a strong wave which was discussed in[12]. Under these conditions $b_r = eE/m_ec\Gamma$. The resonance is accompanied both by the additional increase in the cross section by a factor of b_r^2 and by the synchrotron reradiation of the wave energy into the harmonics $\omega' = \omega_H b_r^2$. Because of the resonance in the scattering cross section there is a rapid increase in light pressure on the electron, which facilitates its acceleration. The harmonics are reradiated mainly at right-angles to the field. It is also important to remember that with increasing rotational and translational velocity (due to acceleration) the resonance will occur not at the frequency ω_H but at ω_H/b_r. However, for a sufficiently broad spectrum of the incident radiation, $\Delta\omega \sim \omega_H$, and high brightness temperature

$$\kappa T_b = 2\pi^2 I_\omega c^2 / \omega^2 > m_e c^2$$

the strong-wave condition can be satisfied even for relativistic electrons in this spectral range. In the above expression I_ω is the radiation intensity in erg/cm$^2\cdot$sec.

In conclusion, let us estimate the plasma parameters for which the damping γ connected with the reradiation of the wave energy exceeds the damping ν_e due to collisions in the plasma. In a magnetic field $H = 10^5$ gauss the gyrofrequency is $\omega_H = 2\times 10^{12}$ Hz and $\gamma = 2r_e\omega_H^2/3c = 40$ Hz. In plasma with temperature $T_e = 5\times 10^4$ °K and density $N_e = 10^6$ cm^{-3} the collision frequency is $\nu_e \sim 4$ Hz, i.e., $\gamma \gg \nu_e$ and the reradiation of energy into the harmonics is more effective than synchrotron absorption.

E. The Case $\omega \ll \sigma_H$ and Mutual Transformation of Normal Waves

The cross section for coherent scattering of the ordinary wave by an individual electron in a magnetic field is close to the Thomson cross section, and when $\omega \ll \omega_H$ we have $\sigma_2 \approx \sigma_T \sin^2\theta$ in a broad range of values of the angle θ between the direction of propagation and the magnetic field **H**. In the narrow angular range $\theta \lesssim (\omega/\omega_H)^{1/2}$ the cross section is $\sigma_2 \sim (\omega/\omega_H)^2\sigma_T$. The cross section for the extraordinary wave $\sigma_1 \sim (\omega/\omega_H)^2\sigma_T$ is small in the above range of frequencies ω and angles θ.[8] This is readily seen from the following discussion. When $(\omega/\omega_H) \ll \sin^2\theta/2\cos\theta$, the electric vector of the incident wave oscillates practically at right-angles to the plane of propagation of the wave **k** and the magnetic field **H** (the ellipticity of the polarization is $\sim\omega\cos\theta/\omega_H\sin^2\theta \ll 1$). When $\omega \ll \omega_H$ it follows from Eq. (1) that the vibrational velocity of the electron is equal to its drift velocity:

$$\mathbf{v} = cH^{-2}[\mathbf{EH}]e^{i\omega t}.$$

Substituting the expression for the drift velocity in Eq. (3), we obtain the scattering cross section for the extraordinary wave:

$$\sigma_1 \approx (\omega/\omega_H)^2\sigma_T.$$

The essential point is that the cross sections for the transformation of the ordinary wave into the extraordinary wave and vice versa are also smaller by a factor of $\sim(\omega/\omega_H)^2$ than the Thomson cross section. In the case of the differential cross sections for the transformation of the ordinary wave into the extraordinary wave and vice versa we can proceed by analogy with the treatment given by Kanuto et al.[8] to obtain the following formulas which are valid for any ratio of ω to ω_H:

$$\begin{aligned}\frac{1}{\sin\theta'}\frac{d\sigma_{\alpha\beta}(\theta\to\theta')}{d\theta'} &= 2\pi r_e^2\left(\frac{\omega^2}{\omega_H^2-\omega^2}\right)^2\frac{1}{1+K_\alpha^2(\theta)}\frac{1}{1+K_\beta^2(\theta')}\\ &\times\left\{\frac{(\omega_H^2-\omega^2)^2}{\omega^4}K_\alpha^2(\theta)\sin^2\theta K_\beta^2(\theta')\sin^2\theta' + \frac{1}{2}\left(1+\frac{\omega_H^2}{\omega^2}\right)\right.\\ &\times[1+K_\alpha^2(\theta)\cos^2\theta][1+K_\beta^2(\theta')\cos^2\theta'] - 2\frac{\omega_H}{\omega}[K_\alpha(\theta)\cos\theta\\ &\times(1+K_\beta^2(\theta')\cos^2\theta') + K_\beta(\theta')\cos\theta'(1+K_\alpha^2(\theta)\cos^2\theta)]\\ &\left.+2\left(1+\frac{\omega_H^2}{\omega^2}\right)K_\alpha(\theta)\cos\theta K_\beta(\theta')\cos\theta'\right\},\end{aligned} \qquad (24)$$

where the parameter characterizing the polarization ellipticity is

$$K_\alpha(\theta) = -2\cos\theta \Big/ \frac{\omega_H}{\omega}\sin^2\theta - (-1)^\alpha\left(\frac{\omega_H^2}{\omega^2}\sin^4\theta + 4\cos^2\theta\right)^{1/2}. \qquad (25)$$

The differential cross section given by Eq. (24) satisfies the condition

$$d\sigma_{\alpha\beta}(\theta\to\theta')/d\Omega' = d\sigma_{\beta\alpha}(\theta'\to\theta)/d\Omega. \qquad (26)$$

This condition ensures that an equilibrium is set up between the two types of normal wave well inside the medium. On the other hand, condition (24) can be obtained from thermodynamic considerations or from the reciprocity theorem. When $\omega \ll \omega_H$ the ellipticity parameter is

$$|K_1| = 1/|K_2| \approx (\omega\cos\theta/\omega_H\sin^2\theta) \ll 1.$$

We can then readily obtain the following approximate expressions for the transformation cross sections:

$$\sigma_{12}(\theta) \equiv \int\frac{d\sigma_{12}(\theta\to\theta')}{\sin\theta'\,d\theta'}\sin\theta'\,d\theta' \sim \sigma_{21}(\theta) \sim \left(\frac{\omega}{\omega_H}\right)^2\sigma_T. \qquad (27)$$

It follows that in a plasma with a strong magnetic field one ordinary wave will be effectively generated and propagated when $\omega \ll \omega_H$ and $\tau_T(\omega/\omega_H)^2 \ll 1$ even when $\tau_T \gg 1$. The radiation emitted by such a plasma has strong plane polarization given by

$$\mathcal{P} = |1-K_2^2(\theta)|/(1+K_2^2(\theta)).$$

The noncoherent scattering cross section for $\omega \ll \omega_H$ must be calculated from the quantum-mechanical equation given by Eq. (8). The transition to the satellite with n = 1 takes place with the cross section

$$\sigma(\omega\to\omega_H+\omega) \sim \frac{kT_e}{m_e c^2}\left(\frac{\omega}{\omega_H}\right)^2\sigma_T \ll \sigma_T.$$

The cross section for the next satellites falls rapidly as $(kT_e/m_e c^2)^n(\omega/\omega_H)^{2n}$. The cross section for the reverse process, i.e., from the satellite into the frequency range $\omega \ll \omega_H$ is of the same order of magnitude:

$$\sigma(\omega+\omega_H\to\omega) \sim (kT_e/m_e c^2)(\omega/\omega_H)^2\sigma_T \ll \sigma_T.$$

F. Quantum Recoil Effect

In accordance with the conservation laws, the change in the photon frequency on recoil in a magnetic field is

$$\omega' = \frac{\omega - n\omega_H}{1+(\hbar\omega/m_e c^2)\cos\theta\cos\theta'}, \quad n = 0, \pm1, \pm2, \ldots. \qquad (28)$$

Evaluating the matrix elements for Compton scattering by a resting electron in a magnetic field as given by Eq. (8) (see also[6]) we can show that, when $\omega \gg \omega_H \gg \Delta\omega_q = \hbar\omega^2/m_e c^2$, the cross section for scattering with photon frequency change $\omega - n\omega_H$ is smaller by the factor $(\Delta\omega_q/\omega_H)^n$ than the scattering cross section for n = 0.

The formation of the satellites is connected with transitions between Landau levels. The electron can freely receive the momentum component parallel to the field. The satellites and the profile of the scattered radiation near the carrier frequency are shifted toward lower frequencies by an average amount of $(1/3)^{1/2}\Delta\omega_q$. When the satellites are taken into account the rate of energy transfer from the scattered photons to the plasma is the same as in the absence of the field.

3. COMPTON ENERGY TRANSFER BETWEEN ELECTRONS AND RADIATION IN HIGH-TEMPERATURE PLASMA WITH A STRONG MAGNETIC FIELD

When the Doppler broadening by scattering is $\Delta\omega_D \gg \omega_H$, the rate of Compton energy transfer is the same as in plasma without the magnetic field. We shall show that the rate of Compton transfer remains the same when

$\omega \gg \omega_H > \Delta\omega_D$. Consider the mean square change in the photon frequency

$$\overline{\Delta\omega^2} = \frac{1}{\sigma_T}\int\frac{d\Omega}{4\pi}\int(\omega'-\omega)^2\frac{d\sigma}{d\Omega'\,d\omega'}d\omega'\,d\Omega'. \qquad (29)$$

Using Eq. (9), we can readily show that

$$\overline{\Delta\omega^2} = \frac{r_e^2}{4\pi}\int d\Omega'|\mathbf{e}'^*\mathbf{e}|^2\int_{-\infty}^{+\infty}d\varepsilon\,\varepsilon^2(1+\varepsilon)^2\int_{-\infty}^{+\infty}dt\,e^{-i\varepsilon t}$$
$$\times\left\langle i\left|\exp\left(\frac{i}{\hbar}H_0t\right)e^{-i\mathbf{qr}}\exp\left(-\frac{i}{\hbar}H_0t\right)e^{i\mathbf{qr}}\right|i\right\rangle. \qquad (30)$$

In this expression **e** and **e′** are the polarization unit vectors before and after scattering, and H_0 is the Hamiltonian for the electron in a magnetic field. Using the operator equation

$$e^{i\mathbf{qr}}H(\mathbf{p})e^{-i\mathbf{qr}} = H(\mathbf{p}-\mathbf{q})$$

and averaging Eq. (30) over the initial states of electrons with

$$F(p_z,n) = \frac{kT_e}{\hbar\omega_H}\frac{1}{(2\pi m_e kT_e)^{1/2}}\exp\left[-\frac{n\hbar\omega_H}{kT_e}-\frac{p_z^2}{2m_e kT_e}\right],$$

we obtain for $\hbar\omega \ll kT_e$

$$\overline{\Delta\omega^2} = \frac{r_e^2}{2\sigma_T}\int d\Omega'|\mathbf{e}'^*\mathbf{e}|^2\int_{-\infty}^{\pm\infty}d\varepsilon\,\varepsilon^2(1+\varepsilon)^2\exp\left[-\frac{m_e}{2kT_eq^2}\left(\varepsilon+\frac{q^2}{2m_e}\right)^2\right]$$
$$= \frac{2kT_e}{m_ec^2}\omega^2\frac{r_e^2}{2\sigma_T}\int d\Omega'(1+\cos^2\vartheta')(1-\cos\vartheta') = \frac{2kT_e}{m_ec^2}\omega^2. \qquad (31)$$

The same result is obtained through direct evaluation of Eq. (29) after substitution into it of the differential cross section given by Eq. (10).

The Kompaneets equation, therefore, which describes spontaneous Compton interaction between radiation and thermal electrons in plasma need not be modified in the presence of a strong magnetic field $\Delta\omega_D < \omega_H < \omega$

$$\frac{\partial n}{\partial t} = \frac{\sigma_T N_e kT_e}{m_e c}\frac{1}{\omega^2}\frac{\partial}{\partial\omega}\omega^4\frac{\partial n}{\partial\omega} \qquad (32)$$

In this expression $n = 2\pi^3c^2I_\omega/\hbar\omega^3$ is the photon occupation number in phase space.

We are indebted to Ya. B. Zel'dovich for stimulating remarks and to A. Z. Dolginov and A. F. Illarionov for useful discussions.

*[vH] ≡ v × H.

1) This formula is valid for $nv/c < 1$.

2) The existence of the satellites is not only a simple consequence of quantum-mechanical conservation laws but follows also from the classical formula given by Eq. (3).

3) Salpeter [14] has obtained the fluctuation spectrum for magnetoactive plasma. Sitenko [15] has shown that the scattering cross section is proportional to the fluctuation spectrum. Salpeter [14] has used this spectrum to investigate in detail the case of a weak field for $\omega_H \ll \Delta\omega_D$. We shall be largely interested in the opposite limiting case. We note also that in the dipole approximation there are no satellites; they appear as the higher multipoles in the expansion in powers of kr.

4) The differential probability given by Eq. (8) is the ratio of the light energy dI′ scattered into the solid angle $d\Omega'$ to the energy flux density in the incident radiation. When Eq. (8) is converted so that it gives the number of photons, it must be multiplied by the ratio ω/ω'.

5) **A**—vector potential, and **P**—generalized photon momentum.

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Translated by S. Chomet
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