Localization of electrons in a magnetic field

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The theory of magnetic freezeout in a compensated semiconductor is developed. For this purpose a quantum mechanical generalization is made of the authors' previously developed[6] theory of nonlinear screening. The dependence of the critical magnetic field (i.e., of the magnetic field strength at which the transition from metallic conductivity to activational conductivity occurs) on the degree of compensation is found. It turns out that in a compensated semiconductor the electrons are not localized on individual impurity centers but on large-scale fluctuations of the potential, which include many charged centers. The activation energy after the transition is therefore greater than the ionization energy of an isolated center. It is also shown that some of the electrons experience an analogous transition every time the Fermi level approaches the bottom of one of the Landau sub-bands. This explains the previously observed oscillations of the Hall coefficient when the magnetic field is varied. The amplitude of the oscillations is estimated.

1. INTRODUCTION

Let us consider a semiconductor doped with shallow hydrogen-like impurities (to be definite, we assume the impurities to be donors). We assume that the doping is strong, i.e., the Bohr radius of the electron on an isolated donor, a, is much larger than the average distance between donors ($(Na^3)^{1/3} \gg 1$). Here N is the donor concentration, $a = \hbar^2\kappa_0/me^2$, m is the effective mass, κ_0 is the dielectric constant of the lattice, and e is the electron charge. In the case of strong doping the donor levels merge with the conduction band, and the electrons form a weakly-nonideal Fermi gas of high density. The conductivity of such a semiconductor is metallic in nature, i.e., it is large and does not depend on the temperature.

The nature of the conductivity is substantially modified in a sufficiently strong magnetic field. The magnetic field, which compresses the wave functions of the electrons, facilitates their localization in the potential of the impurities. If the magnetic field is large enough, the states in the neighborhood of the Fermi level (which exists in view of the low temperature) turn out to be localized, and a transition takes place from metallic conductivity to activational conductivity (the MA-transition). This phenomenon is called magnetic freezeout of the carriers. It has been observed in InSb,[1,2] in InAs,[3] and in GaAs.[4]

It is customary to assume[5] that the freezout begins when the volume occupied by the wave function of the electron localized on an isolated donor becomes smaller than the average volume N^{-1} per single donor. This wave function has the shape of a cigar with a radius of order $\lambda = (\hbar c/eH)^{1/2}$ and a length of order $a \ln^{-1}(a/\lambda)$ ($\lambda \ll a$). Therefore, apart from logarithmic factors, the indicated condition is of the form $N\lambda^2 a = 1$ or

$$H_1 = c\hbar Na/e, \tag{1}$$

where H_1 is the critical transition field.

We shall show below that condition (1) corresponds to a transition only in the case of a weakly compensated semiconductor. In experiments with III—V crystals one often has to deal with the case of strong compensation. In this case, in addition to the donors there are also acceptors in the semiconductor, with the acceptor concentration N_A close to N ($(N - N_A)/N \ll 1$). At low temperatures the acceptors capture N_A electrons, and only $n = N - N_A$ electrons per unit volume are left in the conduction band. The small number of electrons turns out to be in the random field created by the large number of negatively charged acceptors and positively charged donors. As is well known,[6] compensation leads to localization of the electrons in the potential wells and to a transition from metallic conductivity to activational conductivity even in the absence of any magnetic field. Therefore, it is quite clear that the critical magnetic field H_c associated with the MA-transition must, in general, depend on the degree of compensation. As we shall demonstrate, compensation even modifies the qualitative picture of magnetic freezeout. In the presence of compensation the electrons are no longer localized on individual impurities, but rather in potential wells, created by fluctuations of the density, which include a large number of charged impurities.

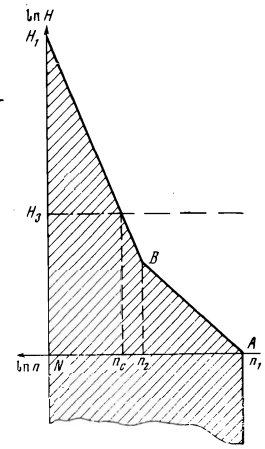

FIG. 1. Phase diagram of the MA-transition. The region located to the right of the ordinate axis, in which $n < N$, has physical significance. The region having metallic conductivity is cross-hatched. The dot-dash line describes an MA-transition under the conditions of resonance localization. Here n stands for the electron concentration in the "resonance" subband, and the field H_3 is determined by the resonance condition.

One of the results of the present work is the phase diagram which is shown schematically in Fig. 1, where the magnetic field and the electron concentration n are plotted along the axes. At points corresponding to the cross-hatched region the conductivity is metallic in nature, and in the remaining region—it is activational. The boundary between the two phases corresponds to the curve $H_c(n)$. This curve has three sections. The two, lower sections can be understood very easily. As we showed earlier,[6] when no magnetic field is present the

MA-transition appears when the rms fluctuation of the potential,

$$\gamma(r_0)=\frac{e^2}{\varkappa_0 r_0}(Nr_0^3)^{1/4}$$

becomes of the order of the Fermi energy $\hbar^2 n^{2/3}/m$ (r_0 denotes the electron screening radius). This equality corresponds to the electron concentration $n_1 = N^{2/3}a$. Since the quantizing magnetic field can only facilitate the breakdown of metallic conductivity, for concentrations $n < n_1$ the conductivity can only be activational. The vertical segment of the curve with $n = n_1$ (below point A) is associated with this.

Now let us go on to the description of the segment AB shown on the phase diagram. If n substantially exceeds n_1, the magnetic field H_c inducing the MA-transition must be such that all electrons are found in the lowest Landau band. In this case the Fermi energy, measured from the bottom of this band, decreases with magnetic field according to the formula

$$\mu = \hbar^2 n^2 \lambda^4 / m, \tag{2}$$

and the screening radius, obtained in the Thomas-Fermi approximation, is given by

$$r_0=\frac{\varkappa_0}{e}\left(\frac{d\mu}{dn}\right)^{1/2}=a^{1/2}\lambda^2 n^{1/4}. \tag{3}$$

(Here we do not write down numerical factors since we only need these formulas for estimates.)

As is shown in[6], one can regard the potential relief as classical at H = 0. First let us assume that it is also classical in the case under consideration. With the aid of Eq. (3), let us calculate the average amplitude of the potential relief

$$\gamma(r_0) = e^2\varkappa_0^{-1}N^{1/3}n^{1/12}a^{1/4}\lambda. \tag{4}$$

The Fermi energy μ decreases with λ faster than $\gamma(r_0)$. In accordance with[6] the MA-transition appears when $\gamma(r_0) \approx \mu$. Equating expressions (4) and (2), we find the following result for the critical magnetic field H_c:

$$H_c=\frac{\hbar c}{e}\frac{a^{1/4}n^{7/6}}{N^{1/3}}. \tag{5}$$

Now let us verify the assumption concerning the classical nature of the potential. It is obvious that in order to do this, one should compare the electron wavelength $\hbar(m\mu)^{-1/2}$ with the length r_0. It is easy to verify that the classical nature of the potential is preserved for $H = H_c$ provided that

$$n < n_2, \quad n_2 = N/(Na^3)^{1/3}. \tag{6}$$

Therefore, the dependence (5) is valid on the phase curve only between points A and B.

Now let us discuss the magnitude of the activation energy in the nonmetallic phase in the region $n < n_2$. Since one can regard the potential as classical in this region, the theory of nonlinear screening developed in[6] is applicable. According to this theory, the activation energy ϵ (provided that the correlation radius in the impurity distribution is larger than R_c) is determined by potential fluctuations of the order of $R_c = N^{1/3}/n^{2/3}$, and the order of magnitude of the activation energy is given by

$$\varepsilon = e^2 N^{2/3}/\varkappa_0 n^{1/3} \tag{7}$$

for $n \ll n_c$, and as $n \to n_c$ the activation energy tends to zero according to the law indicated in[6]. Here n_c denotes the concentration at the MA-transition point. We note that the electronic states themselves may significantly depend on the magnetic field. Since the field reduces the size of the wave function, the classical approach to large-scale fluctuations is preserved; however, the binding energy of the small-scale fluctuations increases logarithmically with the field and may become comparable with expression (7) in a very strong magnetic field (far from the transition).

Now let us go on to the region $n > n_2$. In this case upon increasing the magnetic field the electron wavelength becomes comparable with the screening radius until localization occurs, i.e., still in the free electron region. In this connection the electron gas also remains ideal with regard to the electron-electron interaction (the condition $(m\mu)^{1/2}a/\hbar \gg 1$ is necessary for this). However, in this region the screening is not described by the Thomas-Fermi equation and it becomes substantially anisotropic. (We shall call this quantum screening.) The linear theory of quantum screening in a magnetic field is basically worked out (see, for example, the review article by Horing[7]); however, it is, of course, only applicable in the free-electron regime, that is, before the MA-transition. The method of describing the MA-transition which is developed in[6] is essentially quasiclassical and is not applicable to this case. The quantum theory of nonlinear screening and localization of electrons is developed in the present article. We have shown that for $n > n_2$ the critical field H_c is given by the equation

$$H_c=\frac{\hbar c}{e}\frac{an^2}{N}. \tag{8}$$

The relation (8) is plotted on Fig. 1 to the left of the point B. When n = N the field $H_c = H_1$. In the nonmetallic phase the order of magnitude of the activation energy is given by

$$\varepsilon=\frac{\hbar^2}{ma^2}\left(\frac{N}{n}\right)^2 \tag{9}$$

for $n \ll n_c$, and tends to zero as $n \to n_c$. We note that the activation energy in a compensated semiconductor is larger than the binding energy of an isolated impurity.

In Sec. 3 we obtain results (8) and (9) from qualitative considerations. In order to do this, however, certain results of the linear theory of screening are required, which we did not find in the literature, and therefore we present these results in Sec. 2. Finally, the equations describing the electronic states both above and below the transition point are derived in Sec. 4.

The theory developed in Secs. 3 and 4 explains the oscillations of the Hall coefficient in a strong magnetic field, which have been observed in a large number of experiments (see, for example,[8,9]). Let us consider an uncompensated semiconductor in the metallic region. As the magnetic field is varied, the Landau levels move with respect to the Fermi level. If the Fermi level is close to the bottom of one of the Landau sub-bands (see Fig. 2), the electrons in this sub-band have small longitudinal kinetic energies and are localized inside the impurity potential wells. Thus, with a variation of the magnetic field near the Fermi surface the localized states periodically appear and disappear.[1)] The localized electrons do not contribute to the static conductivity. Therefore, spikes of the Hall coefficient should be observed every time the Landau level intersects the Fermi energy. We shall call this phenomenon resonance local-

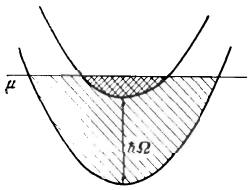

FIG. 2. Energy diagram illustrating the passage of the Fermi level through the bottom of one of the Landau sub-bands. The occupied states are cross-hatched. The densely shaded region corresponds to the resonance sub-band.

ization of the electrons, and we shall investigate it in Sec. 5. We note that it has previously been indicated[8] that localization of the electrons might explain the oscillations of the Hall effect. However, they had in mind localization on individual impurity centers, which cannot occur in the case under consideration.

2. CERTAIN RESULTS OF THE QUANTUM THEORY OF SCREENING

As is well known,[7] when the spatial dispersion is taken into account the dielectric constant of electrons in a magnetic field takes the form

$$\varkappa(\mathbf{q}) = \varkappa_0(1 + k_s^2(\mathbf{q})/q^2). \tag{10}$$

We are interested in the case $q_x, q_y \ll \lambda^{-1}$ (the magnetic field is directed along the z-axis). Then k_s^2 only depends on q_z and is of the form

$$k_s^2(q_z) = \frac{2}{\varkappa_0\lambda^2 q_z a}\sum_{N,s}\ln\left|\frac{p_{N,s}+q_z/2}{p_{N,s}-q_z/2}\right|,$$
$$p_{N,s} = \hbar^{-1}[2m(\zeta - \hbar\Omega(N+{}^1/_2) - s\mu_B H)]^{1/2}. \tag{11}$$

Here ζ denotes the Fermi energy, Ω is the Larmor frequency, N is the Landau quantum number, $s = \pm 1$, and μ_B is the effective magnetic moment. The expression inside the square root sign must be positive. As $q_z \to 0$ the quantity $k_s^2(q_z)$ coincides with the reciprocal of the screening radius in the Thomas-Fermi approximation:

$$k_s^2(0) = \frac{2}{\pi\lambda^2 a}\sum_{N,s} p_{N,s}^{-1} = r_0^{-2}. \tag{12}$$

Let us assume that the electrons exist only in the lowest Landau band. Then the sums in (11) and (12) consist of only the single term with $p_{N,s} = p_F$, where p_F is the wave vector of the longitudinal motion at the Fermi surface. Let us consider the case of quantum screening, when $p_F \ll r_0^{-1}$.

The variation of the potential along the magnetic field has been treated in the majority of articles on quantum screening. However, we shall be interested in screening in a direction perpendicular to the field. We confine our attention to two examples.

1. The screening of a point charge. The potential of a charge e, located at the origin of coordinates, is given by the formula

$$\varphi(\mathbf{r}) = \frac{4\pi e}{(2\pi)^3}\int\frac{d^3q}{q^2}\frac{e^{-i\mathbf{qr}}}{\varkappa(q_z)}$$
$$= \frac{e}{\pi\varkappa_0}\int dq_z e^{-iq_z z}K_0(\rho[q_z^2 + k_s^2(q_z)]^{1/2}), \tag{13}$$
$$\rho = (x^2+y^2)^{1/2},$$

where K_0 is Macdonald's function. At $z = 0$ the potential decreases exponentially for $\rho > r_\perp$, where $r_\perp = (1/2)(r_0 p_F^{-1})^{1/2}$. In this case the integral (13) is evaluated by the method of steepest descents and

$$\varphi(\rho) = \frac{e}{2\varkappa_0\rho}\exp\left(-\frac{\rho}{r_\perp}\right). \tag{14}$$

2. The screening of a charged filament. Let us consider a filament coincident with the z axis and having a charge density sin kz. Then

$$\varphi(\rho, z) = \frac{2\sin kz}{\varkappa_0}K_0(\rho[k^2 + k_s^2(k)]^{1/2}). \tag{15}$$

If $k \ll p_F$ the potential falls off like $\exp\{-\rho/r_0\}$. If $p_F \ll k \ll r_\perp^{-1}$ the potential falls off like $\exp\{-\rho k_s(k)\}$. Finally, screening is not at all important for $k \gg r_\perp^{-1}$, and the potential contains $e^{-\rho k}$.

3. THEORY OF THE MA-TRANSITION. QUALITATIVE CONSIDERATIONS

Now we can investigate which fluctuations of the impurity concentration lead to localization of the electrons. Let us assume that the electrons are free, but the magnetic field grows and the quantity $\mu = \hbar^2 p_F^2/2m$ decreases in accordance with Eq. (2). Localization begins when a potential relief arises having a characteristic size not smaller than the electron wavelength and an amplitude comparable with the Fermi energy. We see that since near the transition the wavelength along the magnetic field is larger than the transverse size of the wave function and the screening radius across the field, then fluctuations having the shape of cylinders extended along the z-axis are important for localization.

Let us consider a cylinder of length l_z and radius $l_\perp$. The excess number of impurities in the cylinder is $(N l_z l_\perp^2)^{1/2}$. If $l_z \gg l_\perp$, then the potential of the cylinder is of the order of the charge density per unit length:

$$V = \frac{e^2}{l_z\varkappa_0}(N l_z l_\perp^2)^{1/2}. \tag{16}$$

In order for an electron to be localized inside a single such well, the following condition is sufficient:

$$\frac{e^2}{l_z\varkappa_0}(N l_z l_\perp^2)^{1/2} = \frac{\hbar^2}{m l_z^2}. \tag{17}$$

The fluctuations with larger values of l_z have smaller amplitudes since, according to Eq. (16), V decreases with increasing l_z. As has already been stated, localization occurs when the amplitude V of the fluctuations becomes comparable with the Fermi energy. It follows from Eq. (17) that at the transition point $l_z = p_F^{-1}$. According to Eq. (16), V increases with increasing values of $l_\perp$. However, the size $l_\perp$ of the fluctuations in the transverse direction is limited by the screening radius. One can easily specify this screening radius by recalling the second example of the previous section. If the charge density of the filament is homogeneous over a length p_F^{-1}, then the potential is screened over a distance r_0. Therefore, to determine the critical field of the MA-transition, it is sufficient to substitute $l_z = p_F^{-1}$ and $l_\perp = r_0$ in Eq. (17) and use formulas (2) and (3). As a result we obtain expression (8). One can show that this condition is equivalent to stating that the electron damping $\hbar\tau^{-1}$ due to impurity scattering is comparable[2)] to the Fermi energy μ.

We note that the shift of the energy obtained from perturbation theory in the metallic phase turns out to be considerably larger at the transition point than the Fermi energy. This shift is determined by the spherical fluctuations which, according to the results of the previous section, are screened over distances $r_\perp$. The potential created by them is of the order of

$$\gamma(r_\perp) = \frac{e^2}{\varkappa_0 r_\perp}(N r_\perp^3)^{1/2}.$$

Since $r_\perp \ll p_F$ near the transition, the effect of these fluctuations averages out, and only a weak modulation of the wave function having a period $r_\perp$ and relative magnitude $\gamma(r_\perp)mr_\perp^2/\hbar^2 \ll 1$ is created. As a result of this modulation, an energy shift of the order of

$$\gamma^2(r_\perp)\frac{mr_\perp^2}{\hbar^2}=\frac{e^4m}{\varkappa_0^2\hbar^2}Nr_\perp^3$$

also appears, and exceeds the Fermi energy at the transition point.

Now let us go on to a description of the nonmetallic phase. Our problem is to determine the characteristic dimensions l_z and $l_\perp$ of the fluctuations which localize the electrons, and to determine the characteristic energy of localization, which is also the activation energy. It is obvious that the arguments leading to condition (17) are applicable as before. However, we can no longer use the linear theory of screening for the determination of $l_\perp$. Let us apply the method developed in [6]. In this case, however, the fluctuation is characterized by two quantities which can be chosen to be l_z and the volume $l_\perp^2 l_z$. It is necessary to find typical fluctuations (not having small probabilities) which create a maximal lowering of the level, taking into consideration that the fluctuations are screened by the electrons. For a fixed volume, the optimum value of l_z is determined by Eq. (17). The larger the volume the larger is V; however, fluctuations with a large volume have a small excess charge density and are neutralized by the electrons. Let us divide up all space into identical cells with dimensions $l_\perp$ and l_z. Roughly speaking, half of a cell has positive and half of a cell has negative excess impurity charge, with the density given by

$$\Delta N = e(Nl_\perp^2 l_z)^{1/2}/l_\perp^2 l_z.$$

If this density is large in comparison with the average electron charge density en, then such fluctuations can be neutralized by the electrons. Therefore, the maximum volume of a charged cell is determined from the condition

$$(Nl_\perp^2 l_z)^{1/2} = nl_\perp^2 l_z. \tag{18}$$

Simultaneously solving Eqs. (17) and (18) we find

$$l_\perp = N/a^{1/2}n^{3/2}, \quad l_z = an/N. \tag{19}$$

The ratio $l_\perp/l_z = (n_2/n)^{5/2} \ll 1$. The values of r_0 and $l_\perp$ are equal at the transition point when $H = H_c$. Substituting expressions (19) into Eq. (16), we obtain an expression for the localization depth, and hence formula (9) for the activation energy.

4. THE STRUCTURE OF THE ELECTRONIC STATES

To obtain a quantitative description of the nonmetallic phase, it is necessary to write down an expression for the electronic density $n(\mathbf{r})$. Here we use the fact that the potential in the transverse direction changes very little over a wavelength in this direction ($l_\perp \gg \lambda$). First let us assume that the potential only depends on z. Then the transverse motion is described by the Landau functions, and we obtain the following expression for $n(\mathbf{r})$

$$n(\mathbf{r})=\frac{1}{2\pi\lambda^2}\sum_\alpha |F_\alpha(z)|^2, \quad \int |F_\alpha(z)|^2\,dz = 1, \tag{20}$$

where F_α and ϵ_α are the wave function and energy eigenvalues of the longitudinal motion (as before, we assume that only the lowest Landau level is occupied), and the summation goes over the region $\epsilon_\alpha < \mu$. Now let us consider a smooth variation of the potential in the transverse direction. For this purpose it is sufficient to assume that F_α and ϵ_α depend on x and y through the Schrödinger equation:

$$-\frac{\hbar^2}{2m}\frac{d^2F_\alpha}{dz^2}+V(x,y,z)F_\alpha=\varepsilon_\alpha(x,y)F_\alpha, \tag{21}$$

where x and y enter as parameters. The potential V satisfies the self-consistent equation

$$\Delta V = 4\pi\frac{e^2}{\varkappa_0}[\xi(\mathbf{r})+n-n(\mathbf{r})], \tag{22}$$

where $\xi(\mathbf{r})$ denotes the excess concentration of charged impurities, which is determined by the correlation relationship

$$\langle\xi(\mathbf{r})\xi(\mathbf{r}')\rangle = 2N\delta(\mathbf{r}-\mathbf{r}'). \tag{23}$$

when the donor and acceptor concentrations are nearly equal.

Let us introduce the following dimensionless variables and unknown functions:

$$\frac{x}{l_\perp}=r_1, \quad \frac{y}{l_\perp}=r_2, \quad \frac{z}{l_z}=r_3, \quad \frac{\xi(\mathbf{r})}{n}=f(\mathbf{r}),$$
$$\frac{V}{\varepsilon}=\chi \quad \frac{\varepsilon_\alpha}{\varepsilon}=\eta_\alpha, \quad \frac{\mu}{\varepsilon}=\eta, \quad \Phi_\alpha(r_3)=F_\alpha l_z^{1/2}, \tag{24}$$

where ϵ, $l_\perp$, and l_z are determined by formulas (9) and (19). Neglecting terms of order $l_\perp/l_z \ll 1$, we can represent Eqs. (20)—(23) in the form

$$\frac{\partial^2\chi}{\partial r_1^2}+\frac{\partial^2\chi}{\partial r_2^2}=4\pi\Big[f(\mathbf{r})-\delta\sum_{\substack{\alpha\\ \eta_\alpha<\eta}}|\Phi_\alpha|^2+1\Big], \tag{25}$$

$$-\frac{d^2\Phi_\alpha}{dr_3^2}+\chi\Phi_\alpha=\eta_\alpha\Phi_\alpha, \tag{26}$$

$$\langle f(\mathbf{r})f(\mathbf{r}')\rangle = 2\delta(r_1-r_1')\delta(r_2-r_2')\delta(r_3-r_3'), \tag{27}$$

$$\int|\Phi_\alpha(r_3)|^2\,dr_3 = 1. \tag{28}$$

This system depends on the single parameter $\delta = (\lambda^2 l_z n)^{-1}$, which is of the order of unity at the MA-transition point. One can show that the potential in Eq. (26) can be regarded as a small perturbation for $\delta \ll 1$, and the functions Φ_α are then close to plane waves. Substituting the correction to Φ_α into (25), we obtain the results of the linear screening theory with dispersion along the magnetic field (in formula (20) we have already neglected dispersion in the transverse direction).

Now let us analyze the derived system of equations for the case $\delta \gg 1$. Let us show that isolated functions Φ_α, localized along r_3 in a unit length, satisfy this system of equations. Let us assume that this is so. Then, in the regions where $\eta_\alpha < \eta$, the electron concentration is according to (25) of order $\delta \gg 1$ (in units of n). Therefore, these regions should occupy a small fraction of the total volume, i.e., the electrons must be highly unevenly distributed. Let us estimate the transverse dimension of these regions. We assume that the energy $\eta_0(r_1, r_2)$ corresponding to the ground state of Eq. (26) is smaller than η inside a cylinder having a unit length along r_3 and a base area of the order of ρ^2. This deepening of the level is created by a positive fluctuation of the impurity charge inside the cylinder. According to Eq. (27) the typical value of f in such a fluctuation is of the order of ρ^{-1}. According to Eq. (25) the condition $\rho < \delta^{-1}$ is necessary in order that the electrons not overcompensate the charge fluctuations. Hence the transverse dimensions of the regions filled with electrons is of

order $\delta^{-1} \ll 1$ (or of order $\lambda^2 a^{1/2} n^{1/2} \ll l_\perp$ in dimensional units). The maximum distance $\eta - \eta_0$ is of the order of $\delta^{-1} \ll 1$. Thus, wherever the electron density is different from zero, the energy of the ground state of Eq. (26) is "clamped" to the Fermi level. As will be evident from the properties of the potential which are derived below, the closest excited state (if one exists) is above the ground state by an energy of the order of unity. Therefore, the probability that the excited state would turn out to be below the Fermi level is exponentially small (for the case $\eta - \eta_0 \gg \delta^{-1}$ one can investigate the level distribution by using the method of optimal fluctuations[10] with the aid of Eqs. (25)—(28)).

Now let us introduce the electron density $\Phi^2(\mathbf{r})$, averaged in the r_1, r_2 plane over an area that includes many electron droplets but is small in comparison with unity. In this connection we shall not take the small-scale fluctuations into account, and we assume that the equation $\eta_0 = \eta$ is valid in the region where $\Phi^2(r) > 0$. Then, instead of Eqs. (25) and (26) we obtain the following result in the zero-order approximation in δ^{-1}:

$$\frac{\partial^2 \chi}{\partial r_1^2} + \frac{\partial^2 \chi}{\partial r_2^2} = 4\pi [f(\mathbf{r}) - \Phi^2(\mathbf{r}) + 1], \tag{29}$$

$$\frac{d^2 \Phi}{dr_3^2} + \chi \Phi = \eta \Phi. \tag{30}$$

These equations can be derived directly from the condition for minimizing the total energy where, in contrast to the case considered in[6], it is necessary to take the kinetic energy of the longitudinal motion into consideration. The obtained system of equations (29), (30), (27) with the neutrality condition (the integral over all space of the right hand side of Eq. (29) vanishes) does not contain any parameters. In this way it is proved that η is of the order of unity and that the typical magnitude of the potential, which varies over unit distances, is of the order of unity.

In this approximation the transverse dimension of the regions where $\Phi^2 > 0$ is determined by the minimum dimension taken into account in the construction of the random function f. Taking account of the higher harmonics leads to a subdivision of the regions into many small ones and to a reduction in the fraction of the total volume occupied by them. These regions do not overlap each other and therefore in the vicinity of the Fermi level the wave functions are actually localized. At high energies, of course, the states are not localized. The activation energy is determined by the distance from the Fermi level to the boundary of the localization region. Since the system of equations which we have derived does not contain any parameters, the activation energy must be of the order of ϵ (see (9)).

5. RESONANCE LOCALIZATION OF THE ELECTRONS

Let us consider an uncompensated semiconductor (see Fig. 2) whose Fermi level occupies the position $\mu \ll \hbar\Omega$ above the bottom of one of the Landau sub-bands (not the very lowest sub-band). One can easily show that formulas (2) and (3) are valid for μ and r_0, where n is the electron concentration in the indicated sub-band. The radius of the screening due to the electrons of the lower sub-bands is much larger than r_0, and at first we shall not take this screening into consideration. Localization of the electrons existing in the upper sub-band and having a small longitudinal kinetic energy occurs upon reducing the value of μ. These electrons are located in the field of a large number of charged impurities; therefore, the transition is analogous to the transition in a compensated semiconductor treated above. During the transition the magnetic field varies very little. It is determined by the condition for coincidence of the Fermi energy with the bottom of the appropriate Landau sub-band. For not too high sub-bands, this field is of the order of $H_3 = \hbar c N^{2/3}/e$. In return the concentration changes abruptly upon a change in the mutual position of the bottom of the sub-band and the Fermi level.

Since the electrons existing in the lower Landau sub-bands do not play any role, the problem concerning the MA-transition reduces to the one treated in the preceding sections. One can classify the transition with the aid of the phase diagram (Fig. 1). Let us draw the straight line $H = H_3$ on it and find the concentration corresponding to the transition. It is easy to see that the transition occurs in the region of quantum screening ($n > n_2$) at a concentration

$$n_c = N / (Na^3)^{1/4}, \tag{31}$$

which is obtained if we substitute $H_c = H_3$ in (8). The localizing fluctuations are characterized by the lengths $l_\perp$ and l_z determined by expressions (19). The activation energy is determined by formula (9). As the value of n is reduced, the value of $l_\perp$ increases and the value of l_z decreases, and these lengths become equal when $n = n_2$. With further decrease of n, one can regard the potential classically, the activation energy is expressed by formula (7), and the dimension of the fluctuations is of the order of R_c. Finally, when $n = N/(Na^3)^{1/4}$ the length R_c is comparable with the screening radius due to the electrons of the lower Landau sub-bands. Beginning with this concentration, the potential relief does not depend on n, and n decreases exponentially with increasing height of the Landau sub-band above the Fermi level. The electrons then fill only the Gaussian tail of the Landau sub-band under consideration.

We note that the Hall-coefficient maxima corresponding to resonance localization, must have a large value since, according to Eq. (31), the ratio of the number of localized electrons to their total number is of the order of $(Na^3)^{-1/6}$, i.e., it is not very small. The relative magnitude of the maxima observed in experiments[8,9] is of the order of 10 to 15%.

[1)] It should be kept in mind that in this case the states are not strictly localized since they are in the continuum corresponding to the lowest Landau sub-band. However the interaction between electrons belonging to different sub-bands is small as a consequence of the large difference in their momenta. Therefore, to a high degree of accuracy one can regard the states as localized.

[2)] Since $\mu \ll \hbar\Omega$ in the region of interest to us, the condition $\Omega\tau \gg 1$ is certainly satisfied right up to the MA-transition.

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