Suppression of the inelastic channels of a nuclear reaction in a crystal under the conditions of hyperfine splitting

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A dynamical theory of the diffraction of γ quanta resonantly interacting with the nuclei in a crystal under conditions of hyperfine splitting is developed. A wide class of cases when complete suppression of the inelastic channels occurs is discovered. The obtained results allow us to find the optimum conditions for the observation of the suppression effect practically from the knowledge only of the hyperfine structure of the ordinary resonant-absorption spectrum. As an example, the case of quadrupole splitting is considered in detail.

1. INTRODUCTION

In previous investigations of the problem of the resonant interaction of γ quanta[1] and neutrons[2] with the nuclei in regular crystals (see also[3]), the present authors predicted the possibility (owing to the collective nature of the interaction between the particles and the nuclei in the crystals) of the suppression of the inelastic nuclear-reaction channel (the γ-quantum $\rightarrow$ electron conversion in the case of γ quanta and the $n \rightarrow \gamma$ reaction in the case of neutrons). Strong suppression can occur even in those cases when for the individual nucleus the inelastic reaction is the main result of the interaction of the particles with the nucleus. The suppression of the nuclear reaction occurs under diffraction conditions when the particles impinge on the crystal at an angle close to the Bragg angle. The crystal then becomes transparent to the resonant nuclear particles, whereas under ordinary conditions a thin layer of the substance practically completely absorbs the radiation.

The suppression effect (SE) has been observed experimentally both in the case of γ quanta[4,5] and in the case of neutrons[6].

In the analysis in[1] of the resonant interaction of γ quanta with nuclei it was assumed for simplicity that hyperfine splitting was absent. Such cases are encountered quite often, and the first experiments on the detection of the SE were performed on samples in which the nuclei were not subjected to hyperfine interaction. On the other hand, it is clear that the number of cases with hyperfine splitting is enormous, and their analysis is undoubtedly of great interest. Thus, the experiments[5] by Sklyarevskiĭ, Smirnov, et al. were performed under conditions of hyperfine splitting.

A number of concrete variants with hyperfine splitting (among them the cases considered in[5]) easily reduce to the case of the unsplit line, when the results of the paper[1] can be used directly (see[3]). The general situation, however, turns out to be much more complex. In the first place, there arises here an enormous number of diverse variants differing in the multipole order of the nuclear transition, in the nature of the hyperfine splitting, in the structure of the unit cell, etc. Furthermore, the problem also becomes quite complicated from the mathematical point of view. Nevertheless, as will be shown below, it is possible to find a fairly wide class of cases when complete (a hundred percent) suppression of the inelastic channels occurs. The obtained results will be presented in a form which will allow us practically from the knowledge only of the hyperfine structure of the ordinary resonant-absorption spectrum to choose the optimum conditions for observing the SE.

2. GENERAL FORMULAS AND THE FORMULATION OF THE PROBLEM

Below we shall use extensively the results of[1], references to the formulas of which will be cited as (0.0),I. Let us restate that we shall assume that the γ quanta (with the wave vector $\mathbf{k}_0$) impinge on the crystal at an angle close to the Bragg angle and that only one diffracted wave (with a wave vector $\mathbf{k}_1 = \mathbf{k}_0 + \mathbf{K}$) arises as a result of the diffraction. The motion of the γ quanta through the crystal is determined by the system of dynamical equations (3.1),I.

On account of the weakness of the interaction between the γ quanta and the crystal, the electromagnetic field in the crystal remains practically transverse, i.e., $\mathbf{E}(\mathbf{k}_\alpha)\cdot\mathbf{k}_\alpha = 0$. Thus, the system (3.1),I is already a system of not six, but only four equations. Let us choose arbitrarily two transverse polarizations: $\mathbf{e}_0^{(1,2)}$ in the incident and $\mathbf{e}_1^{(1,2)}$ in the diffracted waves. We can represent the fields $\mathbf{E}(\mathbf{k}_0)$ and $\mathbf{E}(\mathbf{k}_1)$ in the form

$$\mathbf{E}(\mathbf{k}_\alpha) = \mathbf{e}_\alpha^{(1)} E_\alpha^{(1)} + \mathbf{e}_\alpha^{(2)} E_\alpha^{(2)}. \tag{1}$$

From (3.1),I we obtain for the scalar amplitudes $E_\alpha^{(s)}$ the following system of equations:

$$\begin{aligned}(k_0^2/\varkappa^2 - 1)E_0^{(s)} &= \sum_{s'=1,2}(g_{00}^{ss'}E_0^{(s')} + g_{01}^{ss'}E_1^{(s')}),\\ (k_1^2/\varkappa^2 - 1)E_1^{(s)} &= \sum_{s'=1,2}(g_{10}^{ss'}E_0^{(s')} + g_{11}^{ss'}E_1^{(s')}),\end{aligned} \tag{2}$$

where

$$g_{\alpha\beta}^{ss'} = \sum_{i,l}(\mathbf{e}_\alpha^{(s)})_i g_{\alpha\beta}^{il}(\mathbf{e}_\beta^{(s')})_l^*; \tag{3}$$

$\varkappa = |\boldsymbol{\varkappa}|$, $\varkappa$ is the wave vector of the γ quanta in vacuum. The coefficients $g_{\alpha\beta}^{il}$ are determined by the formulas (3.2),I and (2.11),I.

Let a γ-ray beam be incident on a crystal in the form of a plate. Owing to the weak refraction at the boundary, the vector $\mathbf{k}_0$ will differ slightly from the vector $\boldsymbol{\varkappa}$:

$$\mathbf{k}_0 = \boldsymbol{\varkappa} + \varkappa\delta\mathbf{n}, \quad |\delta| \ll 1, \tag{4}$$

where $\mathbf{n}$ is the inward normal to the surface of the crystal.

Let us, as usual, introduce the quantity

$$\varepsilon_0 = \gamma_0\delta, \quad \gamma_{0,1} = \cos \sphericalangle (\mathbf{k}_{0,1}, \mathbf{n}),$$

and rewrite the system of equations (2) in the matrix form:

$$(\hat{g} - \hat{\alpha} - 2\varepsilon_0\hat{\beta}^{-1})\underline{E} = 0; \tag{5}$$

here $\hat{g}$, $\hat{\alpha}$, and $\hat{\beta}$ are four-dimensional matrices of the form:

$$\hat{g} = \begin{pmatrix} \hat{g}_{00} & \hat{g}_{01} \\ \hat{g}_{10} & \hat{g}_{11} \end{pmatrix}, \quad \hat{g}_{\alpha\beta} = \begin{pmatrix} g_{\alpha\beta}^{11} & g_{\alpha\beta}^{12} \\ g_{\alpha\beta}^{21} & g_{\alpha\beta}^{22} \end{pmatrix},$$

$$\hat{\alpha} = \begin{pmatrix} \hat{0} & \hat{0} \\ \hat{0} & \alpha\hat{1} \end{pmatrix}, \quad \hat{\beta} = \begin{pmatrix} \hat{1} & \hat{0} \\ \hat{0} & \beta\hat{1} \end{pmatrix},$$

$$\hat{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \hat{0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

while E is a four-dimensional column composed of the quantities $E_\alpha^{(s)}$:

$$\underline{E} = \begin{pmatrix} E_0^{(1)} \\ E_0^{(2)} \\ E_1^{(1)} \\ E_1^{(2)} \end{pmatrix}.$$

The parameter α determines the deviation from the exact fulfilment of the Bragg condition: $\alpha = \mathbf{K}\cdot(\mathbf{K} + 2\kappa)/\kappa^2$, while $\beta = \gamma_0/\gamma_1$.

The conditions for the existence of a nontrivial solution to the system (5) lead to the following equation for ϵ_0:

$$\det(\hat{g} - \hat{\alpha} - 2\varepsilon_0\hat{\beta}^{-1}) = 0. \tag{6}$$

The wave field inside the crystal will be given by an expression of the form

$$\mathbf{E}(\mathbf{r}) = e^{i\varkappa\mathbf{r}} \sum_{m=1}^{4} \exp\{i\varkappa\varepsilon_0^{(m)} t/\gamma_0\} \Big\{ \sum_{s=1,2} [\mathbf{e}_0^{(s)} E_0^{(s)}(m) + e^{i\mathbf{K}\mathbf{r}} \mathbf{e}_1^{(s)} E_1^{(s)}(m)] \Big\}, \quad t = \mathbf{nr}. \tag{7}$$

Here $\epsilon_0^{(m)}$ are the roots of Eq. (6) and E(m) are the solutions corresponding to these roots. Since our basic system (5) is homogeneous, to determine E(m) we must also invoke boundary conditions.

In the present paper we shall be interested in only diffraction in the Laue geometry. The boundary conditions (when the small difference between E and D is neglected) then has the form

$$\sum_{m=1}^{4} E_0^{(s)}(m) = \mathscr{E}_0^{(s)}, \quad \sum_{m=1}^{4} E_1^{(s)}(m) = 0, \tag{8}$$

where $\mathscr{E}_0^{(s)}$ is the amplitude of the electric field with the polarization s incident on the crystal.

From (6) and (8) it is not difficult to obtain

$$\underline{E}(m) = \Big\{ \prod_{m_1 \neq m} (\varepsilon_0^{(m_1)} - \varepsilon_0^{(m)}) \Big\}^{-1} \Big\{ \prod_{m_1 \neq m} (\varepsilon_0^{(m_1)} - \hat{f}) \Big\} \underline{\mathscr{E}},$$

$$\hat{f} = {}^1/_2\hat{\beta}(\hat{g} - \hat{\alpha}), \quad \underline{\mathscr{E}} = \begin{pmatrix} \mathscr{E}_0^{(1)} \\ \mathscr{E}_0^{(2)} \\ 0 \\ 0 \end{pmatrix}. \tag{9}$$

The formulas (7), (6), (9), (3), (3.2),I, and (2.11),I completely solve the problem of the motion of the γ quanta through the crystal for an arbitrary character of the hyperfine splitting. The general case is however quite complex for analysis, since it is not possible to find in explicit form the roots of Eq. (6). The analysis is made substantially easier if by an appropriate choice of the polarizations we are able to reduce the tensor $g_{\alpha\beta}^{ss'}$ to the diagonal form with respect to the polarization indices:

$$g_{\beta\alpha}^{ss'} = g_{\beta\alpha}^{(s)} \delta^{ss'},$$

as obtains, for example, in the case of the unsplit line[1]. The system of equations (2) then breaks up into two pairs of independent equations, and the direct analysis (see[1,5]) shows that if the parameter

$$\Delta^{(s)} = g_{00}^{(s)} g_{11}^{(s)} - g_{01}^{(s)} g_{10}^{(s)} = 0,$$

then one of the roots $\epsilon_0^{(m)}$ vanishes when the Bragg condition is exactly fulfilled. In this case, as can be seen from (7), part of the γ-ray beam will travel through the crystal completely unabsorbed. In the absence of hyperfine splitting, for the E1 and M1 transitions, the parameter $\Delta^{(s)}$ is equal to zero for one of the polarizations. In the case of the E2 transitions, however, neither of the parameters $\Delta^{(s)}$ vanishes identically, but we can, by an appropriate choice of the scattering geometry, sharply reduce $\Delta^{(s)}$, and thereby sharply decrease the effective coefficient of absorption of the γ quanta.

Situations in which

$$\operatorname{Im} \varepsilon_0^{(m)} = 0$$

for one of the roots will be called cases of realization of the total suppression effect, and the determination of these situations will be the principal problem of the following section.

3. THE PRINCIPAL RESULTS. THE GENERAL CASE

Let us consider the general case, when hyperfine splitting is present, and let us address ourselves to the expression for the coefficients $g_{\alpha\beta}^{jl}$, (3.2),I and (2.11),I, an expression which we find convenient to rewrite in the following form:

$$g_{\alpha\beta}^{jl} = -g_0 \sum_p A_\alpha^j(p) R(p) A_\beta^{l*}(p). \tag{10}$$

Here p is the set of indices j, ζ_0, and ζ characterizing the position of the nucleus in the unit cell and the quantum numbers of the nuclear sublevels in the ground and excited states respectively;

$$g_0 = \frac{4\pi\eta}{\varkappa^3 V} \frac{2I+1}{2(2I_0+1)} \frac{\Gamma_1}{\Gamma}, \tag{11}$$

$$\mathbf{A}_\alpha(p) = \frac{2f_j(\mathbf{k}_\alpha)}{c[\Gamma_1(2I+1)]^{1/2}} \hat{\mathbf{j}}(\mathbf{k}_\alpha)_{\zeta_0\zeta} \exp\{i\mathbf{k}_\alpha \mathbf{R}_j\}, \tag{12}$$

$$f_j(k_\alpha) = \exp\Big\{-\frac{Z_j(\mathbf{k}_\alpha)}{2}\Big\}, \quad R(p) = \frac{\Gamma/2}{\omega - \omega_{\zeta_0\zeta} + i\Gamma/2}. \tag{13}$$

The remaining notation is standard (see[1]).

As was shown in[1], to the total suppression of the inelastic channels corresponds that coherent superposition of the incident and diffracted waves in which the amplitude of excited-nucleus formation strictly vanishes. In the presence of hyperfine splitting, we should require the vanishing of the corresponding amplitudes for all the hyperfine transitions which turn out to be important for the spectral region in question. This, as is easily verified, is equivalent to the following conditions:

$$A(p)\sim\sum_{\alpha,s}(\mathbf{A}_\alpha(p)\mathbf{e}_\alpha^{(s)*})E_\alpha^{(s)}(m)=0 \qquad (14)$$

for all the p over which the summation is carried out in (10).

The superposition of the fields that satisfies the conditions (14) is, as can directly be verified, a solution to Eq. (3) for $\alpha = 0$, and to this solution corresponds the root

$$\varepsilon_0^{(m)}=0, \qquad (15)$$

i.e., this superposition will travel through the crystal without absorption.

Let us now assume that the energy of the γ quanta falls in a region of the spectrum where not more than three lines of the hyperfine structure are grouped and the rest are located sufficiently far away, so that their contribution to (10) can be neglected. In this case (14) is only a system of not more than three homogeneous equations for the four quantities $E_\alpha^{(s)}$. It is clear that a solution always exists, and, consequently, the total suppression effect can be realized.

If in the spectral region in question all the three lines turn out to be important, then only one linearly independent solution of Eqs. (14) exists. A hundred percent SE can then be realized for γ quanta of a definite polarization. If, however, only two lines lie in the corresponding spectral region, then the system of equations (14) has two linearly independent solutions. This means that in this case the SE can be realized for γ quanta of both polarizations.

First of all it should be noted that the obtained result is very general and encompasses a huge number of diverse cases differing from each other in the multipole order of the nuclear transition, in the nature of the hyperfine splitting, in the geometry of the directions of the magnetic and electric hyperfine fields relative to the scattering plane and the crystallographic axes, in the structure of the unit cell, etc. On the other hand, it is clear that in the cases when the hyperfine splitting is large, we can practically always choose a section of the spectrum where not more than three lines group together and thereby a priori ensure the conditions for strong suppression of the inelastic $\gamma \to e$ process. (In computing the number of lines, we should take into account the multiplicity of their degeneracy.)

Notice that the simultaneous presence of two anomalously absorbable waves leads, in the case of two close lines, to peculiar interference phenomena. Indeed, for small deviations from the exact fulfilment of the Bragg condition, i.e., for $|\alpha| \ll g_0$, the roots $\epsilon_0^{(m)}$ which strictly vanish for $\alpha = 0$ ($m = 1, 2$) will be equal to

$$\varepsilon_0^{(1,2)}=-d(1,2)\alpha-\tfrac{1}{2}c(1,2)\alpha^2. \qquad (16)$$

Here the d's and c's are some real and complex constants, respectively. Since in the general case $\mathrm{Re}\,\epsilon_0^{(1)} \neq \mathrm{Re}\,\epsilon_0^{(2)}$, the two waves "accumulate" different phase factors as they approach the exit surface of the crystal. This leads at once to an oscillatory dependence of the intensities of the transmitted and diffracted waves on the crystal thickness t for fixed α, or on α for fixed t. An analogous phenomenon is well known in x-ray physics under the name of the pendellosung effect[5]. The pendellosung effect is however observed only in weakly absorbing crystals. Our phenomenon is characterized by the fact that the oscillations take place under strong-absorption conditions.

Furthermore, in the case of γ quanta interference phenomena of the type indicated above can, in a number of situations, also manifest themselves in the integrated characteristics, predetermining quite peculiar energy dependences.

In the case when the energy of the γ quanta falls in a region of the spectrum where only one line is located, the system (14) has three linearly independent solutions and the dispersion equation (6) has accordingly three trivial roots. However, not all the trivial roots are connected with the SE. Indeed, under conditions of hyperfine splitting, cases are possible when γ quanta of definite polarization in the incident or diffracted beams do not at all interact with the nuclei. Thus, in the case of one line γ quanta with polarizations $\mathbf{e}_\alpha^{(s)}$ respectively perpendicular to the vectors $\mathbf{A}_\alpha(p_1)$, i.e., $\mathbf{e}_\alpha^{(s)}\cdot\mathbf{A}_\alpha^*(p_1)=0$, do not interact with the nuclei. Thus, two trivial roots are connected with this trivial circumstance. The existence of a third root is due wholly to the suppression effect. It should be noted that in the cases of two and three lines one of the trivial roots may also be connected not with the SE, but with the presence of γ quanta which do not interact with the nuclei. However, such situations in the cases of two and three lines are encountered quite seldom and are not a general rule, as in the case of one line.

The dependence of the SE on the structure of the unit cell turns out to be quite distinctive. Let us suppose that the section of the spectrum of interest to us contains three (or two) lines, each line corresponding to transitions in nuclei occupying different nonequivalent positions in the unit cell and subjected to different hyperfine interactions. In this case, according to the general result, the total SE is always realized and the structural factors $S_j = \exp(i\mathbf{K}\cdot\mathbf{K}_j)$ ($\mathbf{R}_j$ is the vector determining the position of the j-th nucleus in the unit cell) can have an arbitrary value. We recall that in the case of x rays the deviation of the structural factors S_j from unity sharply decreases the Borrmann effect. Physically, this is connected with the fact that in the case of x rays to the sharp reduction in the photoelectric absorption corresponds that coherent superposition of the waves in which electric-field nodes are formed at the locations of the atoms. But if the factors $S_j \neq 1$, then it is not possible for purely geometrical reasons for the electric field to vanish simultaneously at all the locations of the atoms in the unit cell. In the case, however, of resonant γ quanta the realization of the SE requires the formation of a coherent superposition in which the corresponding amplitude of the excited nucleus vanishes for each nucleus in the unit cell. But if the nuclei are subjected to different hyperfine interactions, then the excited-state production amplitudes are also different. Under these conditions the geometrical factors turn out to be unimportant. If, on the other hand, the same hyperfine structure obtains in the various nuclei, then the realization of the total SE requires the equality to unity of the structural factors S_j.

4. THE E1 AND M1 TRANSITIONS

Among the nuclear transitions of different multipole order, the E1 and M1 transitions turn out to occupy a special position, since in their case the number of

cases (besides the above-considered general cases) when the total SE is realized sharply increases. Let us to begin with consider the E1 transitions. For these transitions the current operator $\hat{\mathbf{j}}(\mathbf{k})$ in (12) does not depend of the direction of the vector $\mathbf{k}$. Let us suppose now that the reflection has been chosen in such a way that

$$S_j = 1, \quad f_j(\mathbf{k}_0) = f_j(\mathbf{k}_1). \tag{17}$$

The vectors $\mathbf{A}_\alpha(\mathbf{p})$ then do not depend on the index α.

Let us now turn to the system of equations (14). It is not difficult to see that, irrespective of the number of equations in this system, it always has the solution

$$E_1^{(\sigma)} = -E_0^{(\sigma)}, \tag{18}$$

where, as the polarization vectors $\mathbf{e}_{0,1}^{(\sigma)}$, we have chosen the unit vector perpendicular to the plane $(\mathbf{k}_0, \mathbf{k}_1)$. Thus, under the assumptions made above about the choice of the reflection plane and about the isotropy of the Mössbauer factors, in the case of the E1 transition, no matter how complex the spectrum of the hyperfine structure is (including the case of the unresolved spectrum), the total suppression effect is always realized.

If the unit cell contains only one resonant nucleus, or if the Mössbauer factors are the same for all the nuclei, then the total SE is also realizable when $f(\mathbf{k}_0) \neq f(\mathbf{k}_1)$. The solution of the system (14) will then be

$$f(\mathbf{k}_1) E_1^{(\sigma)} = -f(\mathbf{k}_0) E_0^{(\sigma)}. \tag{19}$$

A similar situation obtains for the M1 transitions. The current operator in this case has the form

$$\hat{\mathbf{j}}(\mathbf{k}) = ic[\mathbf{k}\hat{\boldsymbol{\mu}}], \tag{20}$$

where $\hat{\boldsymbol{\mu}}$ is the nuclear magnetic moment operator. It is not difficult to verify directly that in this case, when (17) is fulfilled, the system of equations (14) always has the solution

$$E_1^{(\pi)} = -E_0^{(\pi)}, \tag{21}$$

where, as the polarization vectors $\mathbf{e}_{0,1}^{(\pi)}$, we have chosen the vectors

$$\mathbf{e}_\alpha^{(\pi)} = [\mathbf{k}_\alpha \mathbf{e}^{(\sigma)}]/k_\alpha. \tag{22}$$

If, on the other hand, the Mössbauer factors do not depend on the position of the nucleus in the unit cell, then the solution is determined by the formula (19) in which $E_\alpha^{(\sigma)}$ must be replaced by $E_\alpha^{(\pi)}$. The result concerning the M1 transitions is the most important result, since to this type of multipole order pertains a large group of Mössbauer transitions, including the 14.4-keV transition in Fe^{57}, as well as the 24.6-keV transition in Sn^{119}.

5. CONCRETE EXAMPLES

We shall demonstrate below the general statements of the preceding sections by means of a number of concrete examples. Let us restrict ourselves to the consideration of only M1 transitions, taking our cue mainly from the nuclei Fe^{57} and Sn^{119}, Let us for the same reason set $I_0 = 1/2$ and $I = 3/2$.

Let us now suppose that there exists in the nucleus an axially symmetric electric-field gradient. The spectrum of the hyperfine structure will in this case consist of two lines: the two-fold degenerate line corresponding to the $\pm 3/2 \to \pm 1/2$ transitions and the four-fold degenerate line of the $\pm 1/2 \to \pm 1/2$ transitions.

Using now the expression for the current operator (20), as well as the formulas (10)—(13), we find*

$$g_{\alpha\beta}^{il} = -g_0 f(\mathbf{k}_\alpha) f(\mathbf{k}_\beta) \{a[(\mathbf{k}_\alpha \mathbf{k}_\beta)\delta^{il} - \mathbf{k}_\beta^i \mathbf{k}_\alpha^l]/\varkappa^2 + b[\mathbf{k}_\alpha \mathbf{q}]^i [\mathbf{k}_\beta \mathbf{q}]^l/\varkappa^2\},$$
$$a = 3/4 R(+) + 1/4 R(-), \quad b = 3/4 [R(+) - R(-)], \tag{23}$$
$$R(\mp) = (x \pm 1/2\Delta + i)^{-1}, \quad x = 2\omega/\Gamma.$$

Here $\mathbf{q}$ is the unit vector directed along the electric-field gradient and Δ is the magnitude of the quadrupole splitting in units of $\Gamma/2$. The $x = -\Delta/2$ line corresponds to the $\pm 1/2 \to \pm 1/2$ transitions and the $x = \Delta/2$ line, to the $\pm 3/2 \to \pm 1/2$ transitions.

In writing down (23), we assumed that the unit cell contains only one atom. In order to make the analysis most transparent, let us direct the vector $\mathbf{q}$ towards the scattering plane $(\mathbf{k}_0, \mathbf{k}_1)$. In this case, as can be easily obtained directly from (23), the system of dynamical equations (2) splits up into two pairs of equations separately for the σ- and π-polarized waves. In this case

$$g_{\alpha\beta}^{(\pi)} = -g_0 f(\mathbf{k}_\alpha) f(\mathbf{k}_\beta) a, \tag{24}$$

$$g_{\alpha\beta}^{(\sigma)} = -g_0 f(\mathbf{k}_\alpha) f(\mathbf{k}_\beta) (aP^{(\pi)} - b q_\alpha^{(\pi)} q_\beta^{(\pi)}), \tag{25}$$

where

$$P^{(\pi)} = \mathbf{e}_0^{(\pi)} \mathbf{e}_1^{(\pi)}, \quad q_\alpha^{(\pi)} = \mathbf{q}\mathbf{e}_\alpha^{(\pi)}.$$

The roots of the dispersion equation (6) will be determined by the formula (3.10),I.

It is easy to show (see[3]) that the minimum value of $\operatorname{Im} \epsilon_0$ is realized for

$$\alpha = \alpha_1 = -\operatorname{Im}[(g_{00}^{(s)} - \beta g_{11}^{(s)}) \tilde{g}_{01}^{(s)*}]/\beta \operatorname{Im} \tilde{g}_{01}^{(s)}, \tag{26}$$

and

$$\operatorname{Im} \varepsilon_0(\alpha_1) = 1/4 \operatorname{Im}(g_{00}^{(s)} + \beta g_{11}^{(s)}) - 1/4 \{[\operatorname{Im}(g_{00}^{(s)} + \beta g_{11}^{(s)})]^2 - 4\beta \tilde{\Delta}^{(s)}\}^{1/2},$$
$$\tilde{\Delta}^{(s)} = \operatorname{Im} g_{00}^{(s)} \operatorname{Im} g_{11}^{(s)} - (\operatorname{Im} \tilde{g}_{01}^{(s)})^2, \quad \tilde{g}_{01}^{(s)} = (g_{01}^{(s)} g_{10}^{(s)})^{1/2}, \tag{27}$$

It is already easy to see from formulas (25)—(27) that for the π-polarized quanta $\tilde{\Delta}^{(\pi)} = 0$ and $\operatorname{Im} \epsilon_0(\alpha = 0) = 0$, i.e., a hundred percent SE is realized. For the σ-polarized γ quanta

$$\tilde{\Delta}^{(\sigma)} = g_0^2 f^2(\mathbf{k}_0) f^2(\mathbf{k}_1) (1 - P^{(\pi)2}) \operatorname{Im} a \operatorname{Im} (a - b). \tag{28}$$

It follows immediately from (23) that in the vicinity of the $\pm 3/2 \to \pm 1/2$ transition a = b and, consequently, $\tilde{\Delta}^{(\sigma)} = 0$.

Thus, in this spectral region a hundred percent SE is also realized for σ-polarized γ quanta. In the vicinity of the $\pm 1/2 \to \pm 1/2$ transition, where the condition $\tilde{\Delta}^{(\sigma)} = 0$ is not fulfilled, γ quanta of this polarization are strongly absorbed. All this is in complete agreement with the results of preceding sections. The realization of the hundred percent SE for π-polarized quanta is the result of the multipole order of the nuclear transition, while the characteristics of the absorption of σ-type γ quanta are determined by the multiplicity of the degeneracy of the corresponding spectral lines.

For $\alpha \neq \alpha_1$, but $|\alpha| \ll g_0$, we easily find from (3.10),I

$$\operatorname{Im} \varepsilon_{0s} = \operatorname{Im} \varepsilon_{0s}(\alpha_1) + (\alpha - \alpha_1)^2 / 2\alpha_0^2, \tag{29}$$

$$\alpha_0^2 = \frac{|g_{01}^{(s)}|^2}{\beta^3 \operatorname{Im} \bar{g}_{01}^{(s)}} \left\{ \frac{[\operatorname{Im}(g_{00}^{(s)} + \beta g_{11}^{(s)})]^2 - 4\beta \bar{\Lambda}^{(s)}}{(\operatorname{Im} \bar{g}_{01}^{(s)})^2} \right\}^{3/2}. \quad (30)$$

Furthermore, in the case of a thick crystal, when only the weakly absorbable superpositions corresponding to the roots (29) reach the exist surface of the crystal, it is not difficult to obtain for the intensities of the transmitted (J_T) and diffracted (J_R) beams the following formulas

$$J_{T,R} \approx J_{T,R}(\alpha_1) \exp\left\{ -\frac{(\alpha - \alpha_1)^2}{\alpha_0^2} \frac{\varkappa l}{\gamma_0} \right\}, \quad (31)$$

$$J_T = \left| \frac{\beta g_{11}^{(s)} - \alpha_1}{g_{00}^{(s)} + \beta g_{11}^{(s)} - \alpha_1} \right|^2 \exp\left\{ -\frac{\varkappa l}{\gamma_0} \cdot 2 \operatorname{Im} \varepsilon_0(\alpha_1) \right\},$$

$$J_R = |g_{01}^{(s)} / (\beta g_{11}^{(s)} - \alpha)|^2 J_T.$$

If the beam incident on the crystal is uncollimated, then the intensities of the transmitted and diffracted quanta will be proportional to the integrals

$$J_{T,R} = \int_{-\infty}^{\infty} d\alpha \, J_{T,R}(\alpha). \quad (32)$$

From (31) we easily find that

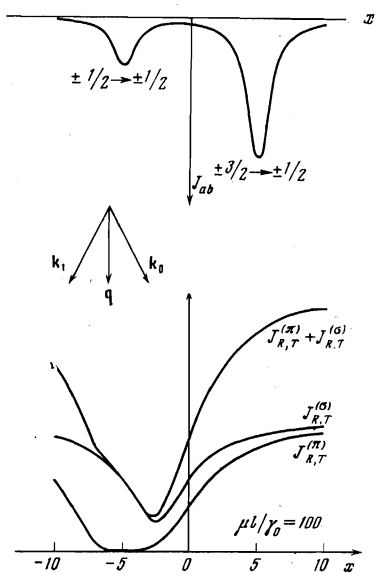
$$J_{T,R} = (\pi \gamma_0 / \varkappa l)^{1/2} \alpha_0 J_{T,R}(\alpha_1). \quad (33)$$

The above-noted distinguishing features of the SE are, in the presence of quadrupole splitting, distinctly manifested in the integrated characteristics as well (and, consequently, they can relatively easily be experimentally observed). Thus, for the π-polarization, $\alpha_1 = 0$ and $\operatorname{Im} \epsilon_{0\pi}(0) = 0$, and from the formulas (31)–(33), (24), and (23) we have

$$J_{T,R} = i_{T,R} \left\{ \operatorname{Im}\left(-\frac{1}{a} \right) \right\}^{-1/2} = i_{T,R} \left(1 + \frac{3}{16} \frac{\Delta^2}{(x + \Delta/4)^2 + 1} \right)^{-1/2}, \quad (34)$$

where $i_{T,R}$ are constants not depending on x.

The corresponding curve of the dependence $J_{T,R}^{(\pi)}(x)$ is shown in the figure. The upper part of the figure shows the usual absorption spectrum of a thin crystal corresponding to the direction of the axis of the gradient of the crystalline field in the $k_0 - k_1$ plane (see the sketch in the figure). It can be seen from the curve $J_{T,R}^{(\pi)}(x)$, as well as from the formula (34), that the intensities of the π-polarized quanta transmitted through the crystal are approximately the same in the vicinities of the $\pm 3/2 \to \pm 1/2$ ($x = \Delta/2$) and $\pm 1/2 \to \pm 1/2$ ($x = -\Delta/2$) transitions, in spite of the fact that the strengths of the interaction between the γ quanta and the nuclei differ strongly in the vicinities of the indicated transitions. The deep dip in the curve $J_{T,R}^{(\pi)}(x)$ is determined by the interference of the amplitudes of the nuclear scattering from the two lines. The corresponding amplitudes are absolutely quenched when $x = \Delta/4$. For σ-polarized quanta the situation changes abruptly. As can be seen from the corresponding curve $J_{T,R}^{(\sigma)}$, strong absorption occurs near the $\pm 1/2 \to \pm 1/2$ transition, while in the vicinity of the $\pm 3/2 \to \pm 1/2$ transition we have $J_{T,R}^{(\sigma)} \approx J_{T,R}^{(\pi)}$.



The dependence of the integrated characteristics on the energy of the incident quanta that is qualitatively described above remains for an arbitrary direction of the axis of the electric-field gradient. Only the directions of the vector q drop out from the general case, when q is perpendicular to either k_0 or k_1 (or to both vectors at once). In fact, let us direct q along the vector $e^{(\sigma)}$. The π-polarized γ quanta do not then interact at all with the nuclei near the $\pm 3/2 \to \pm 1/2$ transition. The interaction of these quanta with the nuclei in the vicinity of the $\pm 1/2 \to \pm 1/2$ transition guarantees them a hundred percent SE. For the σ-polarized quanta a hundred percent SE is not realized in any region of the spectrum. They will be strongly absorbed in the vicinities of both the $\pm 3/2 \to \pm 1/2$ and $\pm 1/2 \to \pm 1/2$ transitions.

As to the magnetic hyperfine splitting, we shall not here dwell on the cases when the magnetic unit cell contains two groups of Mössbauer nuclei acted on by hyperfine magnetic fields that are close in value but different in direction. The hematite Fe_2O_3 can serve as an example (see[7]). In the case of strong splitting when the spectral lines are sufficiently far apart, in order for total SE to be realized, it is sufficient that the structural factors S_j be equal only within each group. The relation between the structural factors of different groups can be arbitrary. A hundred percent SE is then realized at once for both polarizations. This result very much enlarges the number of reflections in which total SE is realized. It is worth noting that even a small difference among the structural factors within one group leads to an appreciable restoration of the nuclear absorption, a fact which has been experimentally observed in[7].

6. ROLE OF THE INTERACTION WITH THE ATOMIC ELECTRONS

In the resonant γ-ray diffraction process there participate not only the nuclei, but the atomic electrons as well. Since diffraction scattering is a purely elastic coherent process, interference occurs between the resonant nuclear and electron Rayleigh scatterings. This interference is sharply manifested in the energy dependence of the intensity of the transmitted and diffracted quanta and has already been quite well investigated from both the theoretical and experimental points of view (see[8-11] and the references cited therein). It is natural for an enormous number of diverse manifestations of this interference to arise under hyperfine-splitting conditions. However, in the present case there emerges another aspect of the role of the electrons.

In the preceding section we found a large class of

cases when the total SE is realized under purely-nuclear-scattering conditions. The question arises: In which of these cases is the total SE preserved in the presence of electron scattering, and to what extent is nuclear absorption restored in the remaining situations.

The interaction with the electrons introduces into the coefficients of the dynamical system of equations (2) the additive terms

$$\chi_{\alpha\beta}^{ii} = -\delta^{ii}\frac{4\pi r_0}{\varkappa^2 V}\sum_a F_a(\mathbf{k}_\alpha - \mathbf{k}_\beta) f_a(\mathbf{k}_\alpha - \mathbf{k}_\beta)\exp\{i(\mathbf{k}_\alpha - \mathbf{k}_\beta)\rho_a\}, \quad (35)$$

where ρ_α is the location of the a-th atom in the unit cell and $F_\alpha(\mathbf{k})$ is the atomic structural factor in which we have included small imaginary corrections determined by the photoelectric-absorption process (see, for example,[12,13]). In the absence of nuclear scattering the system of dynamical equations (2) with the coefficients (35) has as solutions superpositions of either the π-polarized waves only, or the σ-polarized waves only. It follows immediately from this that the switching on of electron scattering will not lead to the restoration of nuclear absorption only in those cases in which the superimposed waves, which are responsible for the realization of the total SE, are only π-type, or only σ-type waves. The cases considered in the preceding section for the E1 and M1 transitions are just cases in point.

The above-noted conditions are, however, necessary but not sufficient. In order to find the situations of interest to us, we must verify that the superpositions (19), (30), or (18) are, for some value of α, a solution to the diffraction problem (2) when both nuclear and electron scattering are taken into account. Direct analysis leads to the requirement that the coefficients χ_{01}^{il} and χ_{10}^{il} be real. The imaginary corrections in the atomic amplitudes $F_\alpha(\mathbf{k})$ can be neglected, since allowance for them leads to the restoration of absorption of the order of just the ordinary photoelectric absorption.

Thus, we arrive at the following general statement. In order for the total SE to be preserved in the cases of the E1 or M1 transitions in the presence of electron scattering, it is sufficient to choose the reflection so as to satisfy, besides the condition (17), also the condition

$$\mathrm{Im}\left\{\sum_a e^{i\mathbf{K}\rho_a} f_a(\mathbf{K})\,\mathrm{Re}\,F_a(\mathbf{K})\right\} = 0. \quad (36)$$

If the unit cell contains only one atom, then the condition (17) is automatically fulfilled. But even in complex lattices it is easy to satisfy this condition. In the remaining cases electron scattering leads to a partial restoration of nuclear absorption. If the electron-scattering amplitude is small compared to the nuclear-scattering amplitude, then this restoration will also be small—of the order of $\mu_n^{res}|\chi_{01}/g_0|^2$.

Finally, let us briefly discuss the role of electronic absorption. Under diffraction conditions there occurs not only suppression of nuclear reactions, but also a reduction in the photoelectric absorption (the Borrmann effect), the maximum decrease in the photoabsorption occurring in the superposition of σ-waves. As was shown above, the superposition of σ-waves provides a strong suppression of the inelastic channel in the case of E1 transitions. It follows immediately from this that we need not, in the E1 transitions, particularly worry about the thickness of the crystal, since there occurs in this case a sharp decrease in both the nuclear and the electronic absorptions.

For the M1 transitions the situation changes abruptly. Here the nuclear-absorption suppression occurs in the superposition of π-waves, while the electronic-absorption suppression occurs in the superposition of σ-waves. It is clear that in the present case it is not possible to work with crystals that are too thick. The thickness l of the crystal should be of the order of, or less than the photoelectric absorption length l_{ph}. It should be noted here however that in a number of cases it is disadvantageous to take crystals which are too thin, since more expressive curves of the energy dependence $J_{T,R}$ are obtained for thicknesses $l_{ph} \sim l$.

*$[\mathbf{k}_\alpha \mathbf{q}] \equiv \mathbf{k}_\alpha \times \mathbf{q}$.

[1] A. M. Afanas'ev and Yu. Kagan, Zh. Eksp. Teor. Fiz. 48, 327 (1965) [Sov. Phys.-JETP 21, 215 (1965)].
[2] Yu. Kagan and A. M. Afanas'ev, Zh. Eksp. Teor. Fiz. 49, 1504 (1965) [Sov. Phys.-JETP 22, 1032 (1966)].
[3] Yu. Kagan and A. M. Afanas'ev, Mössbauer spectroscopy and its applications (Proc. Panel Vienna, 1971), Vienna, 1972, p. 143.
[4] V. K. Voĭtovetskiĭ, I. L. Korsunskiĭ, and Yu. F. Pazhin, ZhETF Pis. Red. 8, 611 (1968) [JETP Lett. 8, 376 (1968)]; V. K. Voĭtovetskiĭ, I. L. Korsunskiĭ, A. I. Novikov, and Yu. F. Pazhin, ZhETF Pis. Red. 11, 149 (1970) [JETP Lett. 11, 91 (1970)].
[5] V. V. Sklyarevskiĭ, G. V. Smirnov, A. N. Artem'ev, R. M. Mirzababaev, B. Shestak, and S. Kadechkova, ZhETF Pis. Red. 11, 531 (1970) [JETP Lett. 11, 363 (1970)]; G. V. Smirnov, V. V. Sklyarevskiĭ, and A. N. Artem'ev, ZhETF Pis. Red. 11, 579 (1970) [JETP Lett. 11, 400 (1970)].
[6] S. Sh. Shil'shteĭn, V. A. Somenkov, and V. I. Marukhin, ZhETF Pis. Red. 13, 301 (1971) [JETP Lett. 13, 214 (1971)].
[7] A. N. Artem'ev, I. P. Perstnev, V. V. Sklyarevskiĭ, G. V. Smirnov, and E. P. Stepanov, Zh. Eksp. Teor. Fiz. 64, 255 (1973).
[8] P. J. Black and P. B. Moon, Nature (Lond.) 188, 461 (1960); P. J. Black, D. E. Evans, and D. A. O'Connor, Proc. Roy. Soc. A270, 168 (1962); P. J. Black, G. Longworth, and D. A. O'Connor, Proc. Phys. Soc. 83, 925, 937, 941 (1964).
[9] V. K. Voĭtovetskiĭ, I. L. Korsunskiĭ, I. A. Novikov, and Yu. F. Pazhin, Zh. Eksp. Teor. Fiz. 54, 1361 (1968) [Sov. Phys.-JETP 27, 729 (1968)].
[10] Yu. Kagan, A. M. Afanas'ev, and I. P. Perstnev, Zh. Eksp. Teor. Fiz. 54, 1530 (1968) [Sov. Phys.-JETP 27, 819 (1968)].
[11] V. V. Sklyarevskiĭ, G. V. Smirnov, A. N. Artem'ev, R. M. Mirzababaev, and E. P. Stepanov, Zh. Eksp. Teor. Fiz. 64, 934 (1973).
[12] M. V. Laue, Rontgenstrahlinterferenzen, Akademische Verlagsg., Leipzig, 1960.
[13] A. M. Afanas'ev and Yu. Kagan, Acta Cryst. A24, 163 (1968).

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