Phonon transition radiation

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The emission of acoustic waves upon passage of a charged particle through the interface between two media with different acousto-electrical properties is considered. The intensity of the phonon radiation is calculated and studied in detail for the case in which the particle crosses the boundary between a dielectric and a piezodielectric. The intensity of the "forward" and "backward" radiation is investigated as a function of the emission angle and the charged particle velocity. The radiation is produced at any velocity v; upon increase in v ($v \gg v_s$) the phonon intensity decreases as v^{-4}. Phonon transition radiation can be observed most effectively at low temperatures and when dense particle clusters cross the boundary.

1. PHYSICS OF THE PHENOMENON. FUNDAMENTAL EQUATIONS

If there is an effective mechanism of interaction of the electric field in a crystal with the lattice, then a charged particle moving in such a medium is accompanied by local elastic deformations of the lattice. The passage of the particle through the interface between two crystalline media with different elastic and electrical properties leads to the emission of a broad spectrum of sound waves. The source of these waves is the motion of the interface under the action of the electric field which accompanies the particle. By analogy with the similar effect in electrodynamics,[1,2] we shall call such a generation of acoustical waves the phonon transition radiation. Evidently the greatest interest attaches to the study of this phenomenon in piezoelectric crystals, in which the interaction of the charged particle with the lattice is most effective.

The character of the phonon radiation depends on which physical quantities undergo discontinuities at the interface. In the simplest case, the boundary arises from a difference in the elastic properties of the neighboring materials. This means that the crystalline media on the two sides of the boundary correspond either to different sound velocity ellipsoids or to identical ellipsoids that are differently oriented relative to the boundary. However, even in the case in which the elastic properties are the same, a boundary exists if the media are characterized by different values of the piezoelectric interaction. For this, they should differ in their piezoelectric characteristics or in their dielectric permittivities. In particular, the phonon transition radiation can arise in the p-n junction in a piezosemiconductor. In the general case, the crystals bordering on each other can have different elastic and piezoelectric properties. One must keep in mind the interesting fact that the intensity of the phonon transition radiation generally depends weakly on the nature of the discontinuity at the boundary. It is determined primarily by the characteristic value of the piezoelectric interaction in the crystal. The character of the boundary conditions affects mainly the directivity pattern of the radiation. The transition radiation of acoustic waves, naturally, develops in crystals with a different mechanism of electron-phonon interaction.

We note that similar phonon radiation ought to be observed upon passage of charged particles through the interface over the surface of two crystals with different electro-acoustic properties. It is important to emphasize that, in contrast with the case of Cerenkov radiation of phonons,[3,4] the transition phonon radiation is possible at any velocity of the charged particle, including values less than the phase velocity of sound.

Let us consider two abutting media, which differ in their piezoelectric properties: in the first, the piezointeraction is lacking, while the second possesses the piezoelectric effect. Analysis of this special case permits us to explain the features of the phenomenon under consideration. Let a charged particle pass through the boundary of a medium $z = 0$ with constant velocity along the z direction. The energy losses of the particle per unit path length will be assumed to be small in comparison with its kinetic energy.

We shall suppose the velocity of the particles to be much less than the velocity of light, so that the electric field can be assumed to be potential in character. Poisson's equation, which describes this electric field in the piezo-active medium, takes the form

$$\varepsilon_0 \operatorname{div} \mathbf{E} = 4\pi\beta_{i,jk}\frac{\partial u_{jk}}{\partial x_i} + 4\pi e\delta(\mathbf{r} - \mathbf{v}t), \tag{1.1}$$

where ϵ_0 is the dielectricconstant, $\beta_{i,jk}$ the piezotensor, u_{jk} the deformation tensor, and u the lattice displacement vector. This vector satisfies the equation of elasticity theory with allowance for the piezoeffect:

$$\rho_0\frac{\partial^2 u_i}{\partial t^2} - c_{ijkl}\frac{\partial u_{kl}}{\partial x_j} + \mu_{ijkl}\frac{\partial^2 u_{kl}}{\partial t\,\partial x_j} = \beta_{l,ij}\frac{\partial E_l}{\partial x_j}. \tag{1.2}$$

Here ρ_0 is the density of the lattice, c_{ijkl} the elasticity tensor, and μ_{ijkl} the viscosity tensor. The electric field produced by the moving particles in the piezomedium produces a deformation of the lattice, the displacement vector of which $\mathbf{u}(\mathbf{r}, t)$, is a particular solution of the inhomogeneous equations (1.1) and (1.2).

We represent u in the form of the Fourier integral

$$\mathbf{u}(\mathbf{r}, t) = \int \sum_\alpha \mathbf{b}^\alpha u_\alpha(\mathbf{q}) e^{i(\mathbf{q}\mathbf{r} - \omega t)}\, d\mathbf{q}, \quad \omega = \mathbf{q}\mathbf{v} = q_z v; \tag{1.3}$$

Here b^α is the vector of the α-th polarization. Then the Fourier component of the stimulated acoustic oscillation takes the form

$$u_\alpha(\mathbf{q}) = \frac{e\beta_\alpha}{2\pi^2\varepsilon_0\rho_0(-\omega^2 + q^2 v_\alpha^2 + iqv_\alpha\Gamma_\alpha)}, \tag{1.4}$$

where

$$v_\alpha=(c_\alpha/\rho_0)^{1/2}(1+\eta_\alpha^2),\quad \eta_\alpha^2=4\pi\beta_\alpha^2/c_\alpha,$$
$$\beta_\alpha=\beta_{i,jk}n_in_kb_j^\alpha,\quad c_\alpha=c_{ijkl}n_jn_kb_i^\alpha b_l^\alpha,\quad \mathbf{n}=\mathbf{q}/q,$$

v_α is the velocity of sound with account of the piezoeffect, and Γ_α the viscous damping.

In a non-piezoactive medium, the particle itself will not excite oscillations of the medium. The only natural oscillations possible in such a medium satisfy the homogeneous equation of elasticity theory. The lattice displacements, which arise in the second medium due to interaction with the electric field of the particle, produce motion of the interface, which leads to excitation of characteristic acoustic oscillations in both media when account is taken of the usual boundary conditions, namely equality of the displacement vectors and of the normal stresses. These oscillations can be represented in the form

$$\mathbf{u}_{1,2}(\mathbf{r},t)=\int\mathbf{u}_{1,2}(\mathbf{q})\exp\{i[\varkappa\rho+\lambda_\alpha(1,2)z-\omega t]\}\,d\mathbf{q},\tag{1.5}$$

where ρ and $\varkappa$ are the components of the vectors $\mathbf{r}$ and $\mathbf{q}$ in the (x, y) plane.

The amplitudes

$$\mathbf{u}_{1,2}(\mathbf{q})=\sum_\alpha \mathbf{b}^\alpha(1,2)u_\alpha(1,2)$$

are determined from the boundary conditions, in which both the stimulated solution (1.4) and the electric field $\mathbf{E}(\mathbf{q})$ of the particle are taken into account:

$$\mathbf{u}_1(\mathbf{q})=\mathbf{u}_2(\mathbf{q})+\mathbf{u}(\mathbf{q}),\quad \sigma_{iz}(1)=\sigma_{iz}(2)+\sigma_{iz};\tag{1.6}$$

here $\sigma_{iz}=ic_{izlj}q_iu_j-\beta_{l,iz}E_l$ is the elastic stress and $\mathbf{E}=e\mathbf{q}/2\pi^2i\epsilon_0q^2$.

In order that (1.5) be the solution of the homogeneous equations of elasticity theory, the equality

$$\lambda_\alpha^2(1,2)=[\omega/v_\alpha(1,2)]^2[1+i\Gamma_\alpha(1,2)/\omega]-\varkappa^2\tag{1.7}$$

must be satisfied.

The solutions (1.5) must not diverge as $z\to\pm\infty$; we therefore assume $\mathrm{Im}\,\lambda_\alpha(1)<0$ and $\mathrm{Im}\,\lambda_\alpha(2)>0$. Inasmuch as the sound waves can propagate from the boundary in the direction $-z$ in the first medium and in the direction $+z$ in the second, one must assume $\mathrm{Re}\,\lambda_\alpha(1)<0$ and $\mathrm{Re}\,\lambda_\alpha(2)>0$. To simplify the derivation, we consider media with identical isotropic elasticities and dielectric properties, in which case the first medium is a dielectric and the second is a piezoelectric with a non-zero piezoconstant $\beta_{z,zz}\equiv\beta$. The latter is similar in its piezoproperties to a hexagonal piezoelectric, for which the symmetry axis C is directed along z. In such a medium, only two acoustic modes will be piezoactive, with polarization vectors lying on the (z, m) plane: a longitudinal mode

$$\mathbf{b}^\parallel=\mathbf{n}=\frac{\varkappa}{q}+\frac{q_z}{q}\frac{\mathbf{z}}{z},\tag{1.8a}$$

and a transverse one

$$\mathbf{b}^\perp=\frac{\mathbf{n}n_z-\mathbf{z}/z}{(1-n_z^2)^{1/2}}=\frac{q_z}{q}\frac{\varkappa}{\varkappa}-\frac{\varkappa}{q}\frac{\mathbf{z}}{z}.\tag{1.8b}$$

Since the elastic properties of the media are the same, waves will propagate in the first medium from the oscillating boundary, with the same polarization vectors (1.8). The elasticity tensor of the isotropic medium can be written in the usual form:[5]

$$c_{ijkl}=\rho_0\{{}^1/_2(v_\parallel^2-v_\perp^2)(\delta_{il}\delta_{jk}+\delta_{ik}\delta_{jl})+v_\perp^2\delta_{ij}\delta_{kl}\},\tag{1.9}$$

where $v_\parallel$ and $v_\perp$ are the velocities of longitudinal and transverse waves, and δ_{ij} is the Kronecker delta.

Inasmuch as the polarization vectors lie in the $(z, \varkappa)$ plane, we can rewrite the set of equations (1.6) in the form

$$\begin{gathered}\sum_\alpha[b_\varkappa^\alpha(1)u_\alpha(1)-b_\varkappa^\alpha(2)u_\alpha(2)-b_\varkappa^\alpha u_\alpha]=0,\\ \sum_\alpha[b_z^\alpha(1)u_\alpha(1)-b_z^\alpha(2)u_\alpha(2)-b_z^\alpha u_\alpha]=0;\end{gathered}\tag{1.10}$$

$$\begin{gathered}\sigma_{\varkappa z}(1)=\sigma_{\varkappa z}(2)+\sigma_{\varkappa z},\quad \sigma_{zz}(1)=\sigma_{zz}(2)+\sigma_{zz},\\ \sigma_{\varkappa z}=i\rho_0\frac{v_\parallel^2-v_\perp^2}{2}\sum_\alpha(q_zb_\varkappa^\alpha+\varkappa b_z^\alpha)u_\alpha,\\ \sigma_{zz}=i\rho_0\sum_\alpha(b_z^\alpha q_zv_\parallel^2+b_\varkappa^\alpha\varkappa v_\perp^2)u_\alpha-\frac{\beta e}{2\pi^2\varepsilon_0q}.\end{gathered}\tag{1.11}$$

The values of $\sigma_{iz}(1, 2)$ are expressed in a fashion similar to σ_{iz}, only the last term in $\sigma_{zz}(1, 2)$ is absent.

The system (1.10), (1.11) is a system of algebraic equations relative to the unknown amplitudes $u_\alpha(1)$ and $u_\alpha(2)$. It is easy to find their solution with accuracy to within the constant of electromechanical coupling $\eta_\alpha^2\ll 1$, but they have a rather cumbersome form. Substituting them in (15), we can obtain expressions for the components $\mathbf{u}_{1,2}(\mathbf{r}, t)$ in explicit form.

We shall be interested in the fraction of energy lost by the particle to excitation of oscillations of the lattice within the time of its flight in the medium. For this purpose, it is necessary to find the Poynting vector $\mathbf{S}(\mathbf{r}, t)$, which can be represented in the form[5]

$$S_i(\mathbf{r},t)=-\sigma_{ik}(\mathbf{r},t)\partial u_k(\mathbf{r},t)/\partial t.\tag{1.12}$$

Since the properties of the medium are axially symmetric relative to the $\mathbf{z}$ direction, we restrict ourselves to the calculation of the vector $\mathbf{S}$ at points in the (x, z) plane. Then the component S_y is equal to zero, inasmuch as the y component of the displacement vector is not piezoactive in the given case and consequently, no displacement $u_y(\mathbf{r}, t)$ will be produced by the flight of the charged particle. Thus there exist only two components of the Poynting vector, S_x and S_z, which are easily expressed in terms of the displacement vector.

For further calculations, it is convenient to represent the vector $\mathbf{S}$ in the form of two projections: on the direction of observation $\mathbf{R}$ and on the direction of the vector $\mathbf{P}$ perpendicular to $\mathbf{R}$:

$$S_R=S_z\cos\theta+S_x\sin\theta,\quad S_P=S_x\cos\theta-S_z\sin\theta,\tag{1.13}$$

here θ is the angle between the vectors $\mathbf{R}$ and $\mathbf{z}$, where $\mathbf{z}$ is the distance from the origin of the coordinates (the point of intersection of the particles with the boundary of the two media) to the point of observation. The amount of transition radiation emitted in the direction $\mathbf{R}$ in the solid angle $d\Omega=\sin\theta\,d\theta d\varphi$ during the entire time of flight of the particle is

$$\frac{dW_R^{\mathrm{tr}}(\mathbf{R})}{d\Omega}=R^2\int_{-\infty}^{\infty}dt\,S_R(\mathbf{R},t).\tag{1.14}$$

In the direction transverse to the angle of viewing, the flux of the Poynting vector through a unit area ds during the time of flight of the particle in the medium is equal to

$$dW_P^{\mathrm{tr}}(\mathbf{R})=\int_{-\infty}^{\infty}dt\,S_P(\mathbf{R},t)\,ds.\tag{1.15}$$

2. INTENSITY OF THE "BACKWARD" RADIATION

We now determine the amount of energy of the sound oscillations emitted by the particle in the first medium—the dielectric— counter to the direction of its motion, by calculating the components of the lattice displacement vector $u_1(r, t)$ and substituting them in Eqs. (1.12)—(1.15).

After substituting in (1.5) the expressions for the polarization vectors $b^{\alpha}(1)$ and the amplitudes $u_{\alpha}(1)$, we find separately the components $u_{1p}(r, t)$ and $u_{1z}(r, t)$ by carrying out integration, similar to that of Garibyan,[2] over the angle Φ between the direction ρ and κ along κ (by the saddle-point method)

$$u_{1\rho}(\mathbf{r}, t) = V_{\parallel}(1)\sin\theta + V_{\perp}(1)\cos\theta, \qquad u_{1z}(\mathbf{r}, t) = -V_{\parallel}(1)\cos\theta + V_{\perp}\sin\theta. \quad (2.1)$$

Here

$$V_{\alpha}(1) = \frac{ie\beta g_1^{\alpha}(\theta)}{2\pi\rho_0\varepsilon_0 v_{\alpha}^2 R}\int\frac{d\omega}{\omega}\exp\left\{i\omega\left(\frac{R}{v_{\alpha}} - t\right)\right\}, \quad (2.2)$$

$$g_1^{\parallel}(\theta) = \frac{-\cos\theta}{(1+\gamma_{\parallel}^2\sin^2\theta)(\xi^2\sin^2\theta+\cos^2\theta)}\left[1+\frac{1-\gamma_{\parallel}\xi^2\sin^2\theta/\cos\theta}{(1+\gamma_{\parallel}\cos\theta)(1+\gamma_{\parallel}^2\sin^2\theta)} - (\xi\sin\theta)^2\frac{(\xi^{-2}-1)(2+\gamma_{\parallel}^2\sin^2\theta)\gamma_{\parallel} - \gamma_{\parallel}^2\sin^2\theta - \gamma_{\perp}^2\cos^2\theta}{1+\gamma_{\parallel}^2\sin^2\theta}\right] \quad (2.3)$$

$$g_1^{\perp}(\theta) = \frac{\sin\theta(2+\gamma_{\perp}\cos\theta)}{(1+\gamma_{\perp}\cos\theta)(1+\gamma_{\perp}^2\sin^2\theta)} \quad (2.4)$$

and the notation

$$\gamma_{\alpha} = v/v_{\alpha}, \quad \xi = v_{\perp}/v_{\parallel}.$$

is introduced.

It is seen from (2.1)—(2.4) that the lattice vibrations in transition radiation of phonons in the first medium ("backward" radiation) represent two types of waves at large distances from the point where the particle crosses the interface. In the first, the lattice displacement $V_{\parallel}(1)$ takes place in the plane which passes through the trajectory of the particle and the vector $\mathbf{R}$, but in a direction perpendicular to the beam. The amplitude of the lattice displacements, as was to have been expected, is proportional to the interaction constant (to the piezomodulus β). For high velocities of the particle, $v \gg v_{\alpha}$, we have $g_1^{\alpha}(\theta) \propto (v_{\alpha}/v)^2$, i.e., the amplitude of the displacements tends to zero as v^{-2}. The fact is that the interface, by virtue of inertia, does not succeed in reacting to the change in the electric field of the rapidly moving particle. The dependence of the displacement amplitude on the angle θ characterizes the function $g_1^{\alpha}(\theta)$: the amplitude $g_1^{\parallel}(\theta)$ (see (2.3)) changes phase with increase in (θ), vanishing at some angle $\tilde{\theta}$ that depends on the ratio $v/v_{\parallel}$, while the amplitude $g_1^{\perp}(\theta)$ (see (2.4) increases as θ approaches $\pi/2$.

Carrying out the elementary differentiation of Eqs. (2.1) with respect to the time and the coordinates, and substituting the results in (1.12), (1.13), we find the components of the Poynting vector:

$$S_R = -\frac{e^2\beta^2}{4\pi^2\rho_0\varepsilon_0^2R^2}\int d\omega\, d\omega'\left\{\frac{(g_1^{\parallel})^2}{v_{\parallel}^3}e_{\parallel}(\omega+\omega') + \frac{1}{2}\left(\frac{1}{\xi^2}-1\right)\frac{(g_1^{\perp})^2}{v_{\perp}^3}e_{\perp}(\omega+\omega')\right\}, \quad (2.5)$$

$$S_P = \frac{e^2\beta^2}{4\pi^2\rho_0\varepsilon_0^2R^2}\int d\omega\, d\omega'\frac{g_1^{\parallel}g_1^{\perp}}{v_{\parallel}^2 v_{\perp}}\left[1+\frac{1}{2\xi}(1-\xi^2)\right]e_{\parallel}(\omega)e_{\perp}(\omega'),$$

where

$$e_{\alpha}(\omega) \equiv \exp\{i(R/v_{\alpha} - t)\omega\}.$$

We are interested in the quantity of energy of the sound oscillations, excited by the particles during the time of flight in the medium. Using Eqs. (1.14) and (1.15) for the lattice-vibration energy radiated in the direction of the line of sight $\mathbf{R}$ (in the solid angle $d\Omega$) and in the direction perpendicular to it $\mathbf{P}$ (through the unit area ds), we get

$$\frac{dW_R^{\mathrm{tr}}}{d\Omega} = \sum_{\alpha}\frac{dW^{\mathrm{tr}}(\alpha)}{d\Omega}, \quad (2.6)$$

$$\frac{dW^{\mathrm{tr}}(\parallel)}{d\Omega} = \frac{\eta_{\parallel}^2e^2}{4\pi^2\varepsilon_0 v_{\parallel}}(g_1^{\parallel})^2\int_0^{\omega_{\lim}} d\omega, \quad (2.7)$$

$$\frac{dW^{\mathrm{tr}}(\perp)}{d\Omega} = \frac{\eta_{\perp}^2e^2}{4\pi^2\varepsilon_0 v_{\perp}}\left(\frac{1}{\xi^2}-1\right)(g_1^{\perp})^2\int_0^{\omega_{\lim}} d\omega, \quad (2.8)$$

$$\frac{dW_P^{\mathrm{tr}}}{ds} = \frac{\eta_{\parallel}^2e^2}{4\pi^2R^2\varepsilon_0 v_{\perp}}\left[1+\frac{1}{2\xi}(1-\xi^2)\right]g_1^{\parallel}g_1^{\perp}\int_0^{\omega_{\lim}} d\omega\cos\left\{\left(\frac{1}{v_{\perp}}-\frac{1}{v_{\parallel}}\right)\omega R\right\}. \quad (2.9)$$

Here $\eta_{\alpha}^2 = 4\pi\beta^2/\rho_0\epsilon_0 v_{\alpha}^2$; expressions for $g_1^{\alpha}(\theta)$ are given in (2.3) and (2.4). The energy flux in the angle $d\Omega$ consists of the fluxes of longitudinal (2.7) and transverse (2.8) spherical sound waves. The energy flux in the $\mathbf{P}$ direction appears as a result of the interference of stresses arising from the longitudinal and transverse waves. This flux oscillates in space with period $2\pi v_{\parallel}v_{\perp}/\omega(v_{\parallel} - v_{\perp})$ and changes its direction depending on the phase difference, at the given point, between the oscillations of the stresses of one polarization and the oscillations of the lattice displacement of another polarization.

In the integration over κ, it was assumed that $(\omega/v_{\alpha})R\sin^2\theta \gg 1$. This condition limits the applicability of the resultant formulas, excluding the description of the region bounded by the surface $R \sim \lambda/\sin^2\theta$ (λ is the acoustic wavelength)—the region of "formation" of the wave field of the transition radiation. It follows from this that the distance R to the point of observation should be greater than the characteristic length of the excited acoustical waves: $R \gg \lambda$. This latter leads to the result that the flux (2.9) vanishes after integration over all frequencies.

3. INTENSTIY OF THE "FORWARD" RADIATION

We now consider the phonon radiation in the second medium, which travels in the direction of motion of the particle ("forward" radiation). As in the preceding case, we find the components of the displacement vector $u_2(r, t)$ at large distances R ($\rho = R\sin\theta$, $z = R\cos\theta$).

As in Sec. 2, we calculate the integral over κ by the saddle-point method, where the points $\kappa_0 = \pm\omega\sin\theta/v_{\alpha}$ and the saddle lines in this case are the same. But it is necessary here to take into account the pole of the integrand $u_{\alpha}(2)$: $\tilde{\kappa} = \omega(\gamma_{\alpha}^2 - 1)^{1/2}$. If the pole $\tilde{\kappa}$ lies in the interval $(0, \kappa_0)$, i.e., when the condition

$$0 \leqslant \gamma_{\alpha}^2 - 1 \leqslant \gamma_{\alpha}^2\sin^2\theta \quad (3.1)$$

is satisfied, then the residue at this pole (the Cerenkov radiation) must be added to the integral along the deformed contour of integration. Upon satisfaction of the inequality (3.1), we obtain

$$u_{2\rho} = V_{\parallel}(2)\sin\theta - V_{\perp}(2)\cos\theta + u_{\rho}^{\mathrm{cer}}, \qquad u_{2z} = V_{\parallel}(2)\cos\theta + V_{\perp}(2)\sin\theta + u_z^{\mathrm{cer}}, \quad (3.2)$$

where $V_\alpha(2)$ are given by Eq. (2.2) with the corresponding replacement of g_1^α by g_2^α:

$$g_2^{\parallel}(\theta)=\frac{\cos\theta}{(1+\gamma_\parallel^2\sin^2\theta)(\xi^2\sin^2\theta+\cos^2\theta)}\left[1-\frac{1+\xi^2\gamma_\parallel\sin^2\theta/\cos\theta}{(1-\gamma_\parallel\cos\theta)(1+\gamma_\parallel^2\sin^2\theta)}-(\xi\sin\theta)^2\frac{(1/\xi^2-1)(2+\gamma_\parallel^2\sin^2\theta)\gamma_\parallel+\gamma_\parallel^2\sin^2\theta+\gamma_\perp^2\cos^2\theta}{1+\gamma_\parallel^2\sin^2\theta}\right],$$

and the expression for $g_2^\perp(\theta)$ is obtained from (2.4) by reversing the sign of $\cos\theta$;

$$u_\rho^{\text{cer}}=-\frac{\sqrt{-i}\,e\beta}{2\pi^2v^2\rho_0\varepsilon_0}\left(\frac{\pi}{2Rv\sin\theta}\right)^{1/2}\int\frac{d\omega}{\sqrt{\omega}}e^{-i\omega t}\times\sum_\alpha\left\{\frac{(\gamma_\alpha-1)^{3/4}}{\gamma_\alpha^2}\exp\left(iR\frac{\omega}{v}[(\gamma_\alpha^2-1)^{1/2}\sin\theta+\cos\theta]\right)\right\}\tag{3.3}$$

finally, u_z^{Cer} is obtained from (3.3) by replacing the factor $\sqrt{-i}$ by $\sqrt{i}$ and the factor $(\gamma_\alpha-)^{3/4}$ in the curly brackets by $(\gamma_\alpha-1)^{1/4}$.

The first components in Eqs. (3.2) describe the transition radiation and the second, the Cerenkov radiation of phonons, considered in[3,4].

Using (1.12)—(1.15), we find the amount of transition radiation emitted in the solid angle $d\Omega$ during the time of flight of the particle:

$$\frac{dW_R^{\text{tr}}}{d\Omega}=\frac{\eta_\parallel^2e^2}{4\pi^2\varepsilon_0}\int_0^{\omega_{\text{lim}}}d\omega\left[\frac{(g_2^\parallel)^2}{v_\parallel}+\frac{(g_2^\perp)^2}{2v_\perp}\frac{1}{\xi^2}\left(\frac{1}{\xi^2}-1\right)\right].\tag{3.4}$$

When the saddle point $\kappa_0=(\omega/v_\alpha)\sin\theta$ is close to the pole $\tilde{\kappa}$ or coincides with it (for $\theta=\theta_0^\alpha\pm\Delta\theta$, where $\theta_0^\alpha=\arccos(v_\alpha/v)$, $\Delta\theta\sim(v_\alpha/\omega R\sin^2\theta)^{1/2}\ll1$), the integral over κ along the saddle line is divided into the integral in the sense of its principal value, which corresponds to the transition radiation, and half the residue at the pole, corresponding to Cerenkov radiation. Estimate of the rather cumbersome expression for the intensity of the transition radiation near the angle θ_0^α shows that it is of the same order as it is far from the Cerenkov cone (cf. (3.4)).

4. BASIC CHARACTERISTICS OF THE PHONON TRANSITION RADIATION

The intensity of the transition radiation, as is seen from Eqs. (2.7), (2.8), (3.4) has the same value over the entire spectral interval (we have neglected any change of the sound velocity with frequency because of its weak dependence). Thus, it is important for us first to establish the factors which limit the spectrum of sound frequencies of the transition radiation: 1) the sound wavelength λ should be greater than the interatomic distance $a\sim10^{-8}$ cm, i.e., $\omega_{\text{lim}}<v_\alpha/a=10^{13}\ \text{cm}^{-1}$; 2) the wavelength of the sound vibrations, in the classical treatment of their excitation by the particle, should be greater than the deBroglie wavelength of this particle: $\lambda>\lambda\!\!\!^{-}=\hbar/mv$, where $\hbar$ is Planck's constant. For $m=10^{-27}$ g and $v\sim10^5$–10^7 cm/sec we have $\omega_{\text{lim}}<v_\alpha/\lambda\!\!\!^{-}=10^{10}$–$10^{12}\ \text{sec}^{-1}$; 3) the wavelength λ should be greater than the width of the transition layer; for $d=10^{-3}$–10^{-5} cm we have $\omega_{\text{lim}}<v_\alpha/d=10^8$–$10^{10}\ \text{sec}^{-1}$.

It is seen that the last two factors are fundamental. Starting out from this, we estimate the limiting frequency at $\omega_{\text{lim}}\sim10^{10}\ \text{sec}^{-1}$. We compare the characteristic values of the considered phonon transition radiation with the level of the thermal vibrations of the lattice. During the time of flight of the particle τ, there is incident on an area $ds=R^2d\Omega$ a thermal energy at frequencies $\omega\ll T/\hbar$, with a spectral density

$$\frac{d\mathscr{E}_T(\alpha)}{d\omega}=\frac{T}{4\pi^2}\left(\frac{\omega R}{v_\alpha}\right)^2\tau\,d\Omega,\tag{4.1}$$

where T is the temperature of the lattice in energy units. Inasmuch as (4.1) is proportional to ω^2, and the spectral density of the energy of the transition radiation (see (3.4)) does not depend on the frequency, since the dispersions of the elastic constants and piezoconstants are small, it is evident that the latter will be greater than the former in the low frequency range.

If we now consider the passage of a cluster of charged particles with density n and with dimensions Δ smaller than the characteristic wavelength of the radiated sound ($\Delta\ll\lambda$), then this cluster can be regarded, from the viewpoint of excitation of sound oscillations, as a single particle with charge eN ($N=n\Delta^3$, $N\ll n\lambda^3$). Then the radiation from such a cluster will be N^2 times greater than the radiation from a single particle, inasmuch as the energy of the transition radiation is proportional to the square of the charge of the particle. The spectral energy density of the phonon radiation in the passage of clusters of particles can be represented in the form

$$\frac{d\mathscr{E}^{\text{tr}}(\alpha)}{d\omega}=\frac{\eta_\alpha^2(eN)^2G^\alpha(\theta)}{4\pi^2\varepsilon_0v_\alpha}d\Omega,\tag{4.2}$$

$$G^\parallel(\theta)=[g^\parallel(\theta)]^2,\quad G^\perp(\theta)={}^1\!/_2(1/\xi^2-1)[g^\perp(\theta)]^2.$$

The threshold frequency ω_{th}, above which the thermal vibrations will predominate over the transition radiation, is determined from the equality $d\mathscr{E}_T/d\omega=d\mathscr{E}^{\text{tr}}/d\omega$ and is equal to

$$\omega_{th}=\frac{\eta_\alpha eN}{R}\left[\frac{v_\alpha G^\alpha(\theta)}{\varepsilon_0\tau T}\right]^{1/2}.\tag{4.3}$$

For parameters of crystals of the CdS type, $\eta_\alpha\sim10^{-1}$, $v_\alpha\sim10^5$ cm/sec, $\epsilon_0=10$ and for $T=4\times10^{-14}$ erg (300°K), path length $L\sim1$ cm, particle velocity $v\sim v_\alpha$, $\tau\approx L/v\sim10^{-5}$ sec, the threshold frequency is proportional to the number of particles in the cluster, i.e., $\omega_{\text{th}}\approx N\sqrt{G^\alpha(\theta)}\ [\text{sec}^{-1}]$. Thus, for the given parameters, a significant excess of the transition phonon radiation over the thermal at high sound frequencies can be obtained by using dense clusters of particles. For example, for frequencies $\omega\sim10^7\ \text{sec}^{-1}$ ($\lambda\sim10^{-2}$ cm) the necessary particle densities are $n\gg N/\lambda^3=10^{13}\ \text{cm}^{-3}$.

In the derivation of formulas of phonon transition radiation, it was assumed that the energy spent by the particles

$$\mathscr{E}^{\text{tr}}=\sum_\alpha\int d\Omega\,d\omega\left(\frac{d\mathscr{E}^{\text{tr}}(\alpha)}{d\omega}\right)=\frac{\eta^2(eN)^2\omega_{\text{lim}}}{8\pi\varepsilon_0}\sum_\alpha\int_0^{\pi/2}\frac{G^\alpha(\theta)}{v}d\theta$$

should be much less than their kinetic energy $\mathscr{E}=Nmv^2/2$. We write down the values of these energies for specific values of the parameters $N=10^6$, $\omega_{\text{lim}}\sim10^8\ \text{sec}^{-1}$ and $T\sim300$°K. For $v/v_\alpha=10$, we have

$$\mathscr{E}\sim10^{-9}\ \text{erg},\quad\mathscr{E}^{\text{tr}}\sim10^{-11}\ \text{erg},\quad\omega_{th}^{max}\sim2\cdot10^7\ \text{sec}^{-1}.$$

But even for $v/v_\alpha=4$, we have

$$\mathscr{E}\sim10^{-10}\ \text{erg},\quad\mathscr{E}^{\text{tr}}\sim10^{-10}\ \text{erg},\quad\omega_{th}^{max}\sim10^8\ \text{sec}^{-1},$$

i.e., the formulas for the transition radiation at the given parameters lie at the limit of applicability.

With decrease in temperature, the thermal acoustic background decreases and the excess of the transition radiation takes place for even higher frequencies of lattice vibration. For example, for $T = 4^\circ K$ and $v/v_\alpha \sim 10$ we have $\omega_h^{max} \sim 10^8\ sec^{-1}$ (for the angle θ^{max}, where $G^\alpha(\theta)$ is maximal).

Thus, the phonon transition radiation is effective at low temperatures for fluxes of slow charged particles. Evidently, it is very simple to observe such radiation when the particle passes over the surface of the medium or in a narrow channel between two crystals. Losses from scattering of the energy of the particle inside the medium are eliminated in this way.

It should be noted that when the limit $v \to 0$ is taken the relative amplitude of the oscillations tends toward a constant, and the upper value of the limiting frequency $\omega_{lim} \sim v/a$ ($\omega \sim q_z v$, a is the interatomic distance) tends to zero, so that the total energy of the transition radiation also tends to zero ($\mathscr{E}^{tr} \sim G^\alpha(\theta)\omega_{lim} \sim \sim v \cdot const$). However, it would be incorrect to take this limit, since it was assumed, within the framework of the considered approximation, that the energy ($\mathscr{E} \sim v^2$) of the particle (or cluster of particles) is much greater than the total radiation energy.

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