Thermal conductivity and the temperature dependences of the energy gaps of niobium samples containing large amounts of impurity atoms

É. E. Anders, B. Ya. Sukharevskiĭ, and I. V. Volchok
Physico-technical Institute of Low Temperatures, Ukrainian Academy of Sciences
(Submitted November 22, 1972)
Zh. Eksp. Teor. Fiz. **64**, 1688-1698 (May 1973)

A method is proposed for determining the temperature dependence of the energy gap on the basis of data obtained by measuring the thermal conductivity of superconductors with a high impurity-atom content. Results are presented of an investigation of the temperature dependence of the thermal conductivity of niobium (99.6%) and a solid niobium solution containing 1.7 at.% zirconium in the superconducting state, and also of the thermal conductivity of the samples in the 10–20°K temperature range. The dependence of the specific heat of the solid solution with 1.7 at.% zirconium on temperature between 2.5 and 18°K is presented. From the results obtained the temperature dependence of the energy gap of the samples is determined. Electron and lattice components of the thermal conductivity of the samples in the superconducting state are found. The contribution of phonon scattering by crystal structure defects to the lattice thermal resistance in the superconducting state is determined.

We consider in this paper the results of investigations of the formal conductivity and the electric conductivity of niobium samples 99.6% pure and niobium with 1.7 at% zirconium. For the latter sample, we investigated also the temperature dependence of the specific heat. The measurements of the thermal conductivity and of the electric conductivity were accurate to ~ 2% and ~ 0.5%, respectively. The error in the measurement of the specific heat in the temperature range 4–18°K was 1–18%[1,2]. The investigated samples were kindly supplied to us by V. V. Baron and V. A. Frolov of the Baĭkov Metallurgy Institute of the USSR Academy of Sciences. Some characteristic of the superconducting state of solid solutions of the niobium-zirconium system were considered by us earlier[3,4].

The investigated compositions were prepared by the method of multiple remelting of mixtures of initial components in a vacuum electric-arc oven on a copper water-cooled hearth in a medium of pure helium at a pressure 400–500 mm Hg. After mechanical working, the samples were etched to clean their surface, and then subjected to homogenizing annealing in a vacuum oven at a temperature of 1500°C for eight hours, after which the sample with 1.7 at% zirconium was quenched to fix the β-solid solution of zirconium in niobium, and the niobium sample was slowly cooled with the oven in a vacuum of ~ 10^{-6} Torr.

The temperature dependences of the thermal conductivity of the investigated samples are shown by curves 1 of Figs. 1 and 2. The results of the measurements of the electric conductivity are shown in Fig. 3. The change of the electric resistance in the temperature interval 10–18°K is much less than the residual resistance, and this gives grounds for assuming that the electrons are predominantly scattered by impurities. This is confirmed also by an analysis of the Lorentz numbers[3], which were calculated from the values of the residual resistances obtained by graphic extrapolation, or from an analytic relation of the form

$$\rho = \rho_0 + AT^2 + BT^5.$$

The obtained values of ρ_0 enable us to determine the electronic component of the thermal conductivity in the normal state: $\kappa_{en} = L_0 T/\rho_0$ (L_0 is the Sommerfeld number). For the investigated samples the values of κ_{en} coincide in practice with the general thermal conductivity in the normal state. Figure 4 shows the temperature dependence of the specific heat of the niobium sample with 1.7 at.% zirconium.

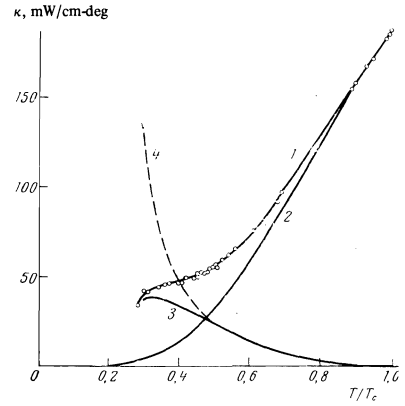

FIG. 1. Temperature dependence of the thermal conductivity of niobium sample: 1—total thermal conductivity κ_s; 2—electronic thermal conductivity κ_{es}; 3—phonon thermal conductivity $\kappa_{gs} = \kappa_s - \kappa_{es}$; 4—phonon thermal conductivity $\kappa'_{gs} = Bt^2\Omega(b)$.

The obtained experimental data can be used to determine the temperature dependence of the superconducting gap. There are several direct methods of observing the energy gap in the spectrum of electronic excitations of superconductors. The simplest and most reliable method of measuring the tunnel current between two layers of superconducting materials, separated by a thin oxide film, cannot be used for the investigated objects because of great technological difficulties encountered when an attempt is made to produce the "sandwich." This explains also why it is impossible to determine the energy gap from the absorption of infrared radiation in thin films of the investigated materials.

From the temperature dependence of the electronic thermal conductivity it is possible to obtain information on the value of the gap parameter at rather low tem-

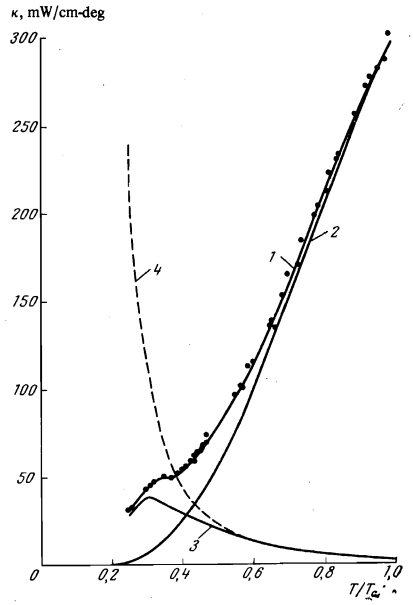

FIG. 2. Temperature dependence of the thermal conductivity of a solid solution of neobium with 1.7 at.% zirconium: 1–total thermal conductivity κ_s; 2–electronic thermal conductivity κ_{es}; 3–phonon thermal conductivity $\kappa_{gs} = \kappa_s - \kappa_{es}$; 4–phonon thermal conductivity $\kappa'_{gs} = Bt^2\Omega(b)$.

peratures (Δ_0) and its behavior near the transition temperature[5,6]:

$$\Delta(T)/T = \beta(1 - T/T_c)^{1/2}. \quad (1)$$

The factor β is connected with the discontinuity of the electronic thermal conductivity by the relation

$$\left.\frac{C_{es}}{C_{en}}\right|_{T_c} = 1 + \frac{3}{2\pi^2}\beta^2.$$

Data on the thermal conductivity, as will be shown later on, make it possible to determine with sufficient accuracy the values of the energy gap at intermediate temperatures, and in some cases practically in the entire temperature interval of the superconducting state.

In the calculations we used an expression that follows from the formulas given by Bardeen et al.[7] and by Geĭlikman[8] and describe the electronic thermal conductivity of superconductors in the case of scattering of electronic excitations by point defects:

$$\frac{\kappa_{es}}{\kappa_{en}} = \Phi(b) = \left\{\frac{b^2}{e^b+1} + 2\sum_{s=1}^{\infty}\frac{(-1)^{s+1}}{s^2}e^{-sb} + 2b\ln(1+e^{-b})\right\} \Big/ 2\sum_{s=1}^{\infty}\frac{(-1)^{s+1}}{s^2}, \quad (2)$$

where κ_{es} is the electronic thermal conductivity in superconducting state, κ_{en} is the electronic thermal conductivity in the normal state and is determined by extrapolation, into the region of low temperatures, of the linear dependence of the normal thermal conductivity measured at

$$T > T_c, \quad b = \Delta T/T_c t, \quad t = T/T_c.$$

The lattice thermal conductivity, which is controlled by the scattering of the phonons from the electronic excitations, was calculated with a formula proposed by Geĭlikman and Kresin[9]

$$\kappa_{gs} = Bt^2\Omega(b) = Bt^2\left[\int_0^{2b}\frac{x^4e^x\,dx}{(e^x-1)^2[2x - 2\ln(e^{b+x}+1)(e^b+1)^{-1}]} + \int_{2b}^{\infty}\frac{x^4e^x\,dx}{(e^x-1)^2[x+2b-2\ln(e^{b+x}+1)(e^{x-b}+1)^{-1}]}\right]. \quad (3)$$

Using formulas (2) and (3), we can write down an expression for the total thermal conductivity in the following form:

$$\kappa_s = \alpha t\Phi(b) + Bt^2\Omega(b), \quad (4)$$

where α and B are the coefficients characterizing the respective contribution of the electronic and lattice thermal conductivities to the total conductivity. At $T \geq T_2$ we have b = 0, $\Phi(0) = 1$, and $\Omega(0) = 7.209$. In this case, expression (4) describes the thermal conductivity of the sample in the normal state.

In (4), $\alpha = L_0T_c/\rho_0$; to determine the coefficient B it is necessary to know the value of the gap b_0 at a certain temperature t_0:

$$B = \frac{\kappa_s - \alpha t_0\Phi(b_0)}{t_0^2\Omega(b_0)}. \quad (5)$$

An analysis carried out with a computer shows that Eq. (4) has a unique root in the interval b = 0–10. The value of the root at each temperature makes it possible to determine the energy gap. The calculations can be carried out with a computer or manually. In the latter case, we used the functions $\Phi(b)$ and $\Omega(b)$, which were tabulated by us in detail for values b = 0–10 in steps of 0.01–0.03 using the VM-222 calculator.

Let us describe the procedure of the manual calculation. Choosing a certain (initial) curve passing through b_0, relation (2) is used to calculate κ_{es}, and then, using expression (3), we obtain again the values of b from the values of $\kappa_{gs} = \kappa_s - \kappa_{es}$. By comparing these values with the initial ones we can estimate the corrections that must be introduced in the initial curve to obtain a self-consistent condition. In the temperature region where κ_{gs} are small in comparison with the error with which the total thermal conductivity is determined, the criterion of the correct choice of the temperature

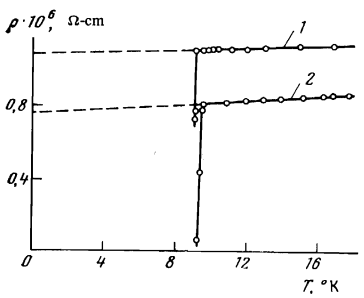

FIG. 3. Temperature dependence of the resistivity of the investigated samples: 1–investigated niobium sample, 2–niobium with 1.7 at.% zirconium.

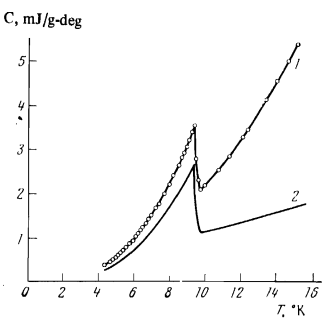

FIG. 4. Temperature dependence of the specific heat of a solid solution of niobium with 1.7 at.% zirconium: 1–total specific heat, 2–electronic specific heat.

TABLE I. Estimate of the gap errors due to the error in the thermal-conductivity measurement ($t_0 = 1$)

t	b	K	δb				
			$\frac{\alpha}{B}=2$	$\frac{\alpha}{B}=10$	$\frac{\alpha}{B}=50$	$\frac{\alpha}{B}=100$	$\frac{\alpha}{B}=1000$
0.9	1.03	0.10	−0.014	−0.07	*	*	*
		0.05	−0.007	−0.04	−0.18	−0,30	*
		0.01	−0.002	−0.008	−0.05	−0.24	0.15
0.8	1.56	0.10	−0.02	−0.1	*	*	*
		0.05	−0.01	−0.05	−0.2	*	*
		0.01	−0.002	−0.01	−0.06	−0.15	*
0.28	6.25	0.10	−0.03	−0.13	*	*	*
		0.05	−0.02	−0.07	−0.29	*	*
		0.01	−0.003	−0.02	−0 07	−0.16	*

* $|\delta b| > 0.3$.

TABLE II. Estimate of the gap errors due to the error in the measurement of the thermal conductivity ($t_0 = 0.6$)

t	b	K	δb		
			$\frac{\alpha}{B}=50$	$\frac{\alpha}{B}=100$	$\frac{\alpha}{B}=1000$
0.9	1.03	0.05	+0.20	*	*
		0.01	+0.04	+0.17	−0.1
0.8	2.80	0.05	+0.06	+0.25	−0.21
		0.01	+0.01	+0.05	−0.04
0.28	6.25	0.05	−0.02	+0.03	−0.25
		0.01	−0.003	+0.01	−0,06

* $|\delta b| > 0.3$.

dependence of the gap may be a comparison of the lattice thermal conductivity, calculated from formula (3), with $\kappa_{gs} = \kappa_s - \kappa_{es}$. The discrepancy must not exceed the error in the determination of the total thermal conductivity.

Let us see first what requirements must be imposed on the experimental accuracy to obtain sufficiently reliable values of the energy gap from measurements of the thermal conductivity. Let us assume, to simplify the subsequent arguments, that the relative error in the determination of the thermal conductivity is $\delta\kappa/\kappa_s = K > 0$ and does not depend on the temperature. In our case, this is indeed the case, since the errors are systematic and are due mainly to parasitic heat losses[1].

Using (4), we obtain

$$(1+K)\alpha t\Phi(b) + (1+K)Bt^2\Omega(b) = \alpha t\Phi(\bar{b}) + \bar{B}t^2\Omega(\bar{b}), \qquad (6)$$

where $\bar{B}$ and $\bar{b}$ are quantities corresponding to the measured value of the thermal conductivity. The coefficient α is determined from the values of the electric conductivity, which is measured with rather high accuracy. We therefore assume that the true value of this coefficient is used in (6).

Assuming that at a certain temperature t_0 the value of the gap is known and is equal to b_0, and putting $\bar{b} = b + \delta b$, we obtain, confinining ourselves to the first two terms of the expansions of $\Phi(b)$ and $\Omega(b)$ in powers of δb,

$$\delta b \approx \frac{K[\Phi(b) - \Phi(b_0)t\Omega(b)/t_0\Omega(b_0)]}{\Phi'(b) + B(1+K+K\Phi(b_0)\alpha/t_0\Omega(b_0)B)t\Omega'(b)/\alpha}; \qquad (7)$$

$$\Phi'(b) = \partial\Phi/\partial b, \quad \Omega'(b) = \partial\Omega/\partial b.$$

Formula (7) contains the parameter α/B, which depends on the degree contamination of the sample: $\alpha/B\Omega(0)$ is the ratio of the electronic and lattice thermal conductivities at $t = 1$. The relative contribution of the lattice thermal conductivity increases with increasing impurity-atom content. Therefore, to obtain numerical estimates of the error in the determination of the gap, we use values of α/B in the interval 2–1000. For the case $t_0 = 1$ ($b_0 = 0$), the results of the estimate are shown in Table I.

The values of b in Table 1 are set in correspondence with the values of t obtained from the temperature dependence of the BCS-model gap[10]. For any other model, naturally, the values of b in the table will correspond to other values of the temperature.

It follows from Table I that for samples with high impurity-atom content, and consequently with an appreciable contribution of the phonon thermal conductivity in the normal state ($\alpha/B \sim 10$), the errors in the determination of the gap do not exceed 4% even if the thermal-conductivity measurement is made with an error ~ 5%. This accuracy can be regarded as acceptable. However, at values $a/B \sim 100$, the gap is determined very roughly even with relatively accurate measurements of the thermal conductivity. This is due to the large error in the determination of the coefficient B. Indeed, at $\alpha/B \sim 100$, the lattice thermal conductivity at the transition temperature amounts to about 1% of the total thermal conductivity and consequently, when the thermal conductivity is measured with accuracy $K = 0.01$, the values of B are obtained with an error ~ 100%.

It might seem for samples with a ratio $\alpha/B \sim 1000$ one can neglect the phonon thermal conductivity when determining the energy gap near T_c. It is easy to see, however, that the error in the determination of b amount in this case to

$$\delta b \approx \frac{K\Phi(b) + (1+K)Bt\Omega(b)/\alpha}{\Phi'(b)}, \qquad (8)$$

from which it follows that even at $t = 0.9$ and $T = 0.01$ the gap is determined with an error exceeding 20%. The error increases rapidly with decreasing temperature. The phonon thermal conductivity can be neglected in the determination of the gap only in the case of very pure samples with a ratio $a/B \sim 10^4$.

The result of the determination of the gap can be greatly improved for relatively pure samples by choosing as t_0 a sufficiently low temperature, at which the contribution of the lattice thermal conductivity becomes noticeable, but it can be assumed as before that the only mechanism that limits the lattice thermal conductivity is the scattering of phonons by electrons. The errors in the determination of the gap, as functions of the error in the measurement of the thermal conductivity, are shown in Table II for the case $t_0 = 0.6$.

As seen from the table, the determination of the temperature dependence of the gap of a relatively pure sample, especially at temperatures close to T_c, calls for highly accurate measurements of the thermal conductivity ($K \sim 0.01$). This accuracy can be attained at the present state of the art.

The only limitation on the proposed method of determining the temperature dependence of the gap is the need for knowing its value at the temperature t_0, since this makes it possible to obtain the coefficient B of (4). As will be shown later on, we can use for this purpose also the values of the gap at 0°K or the temperature dependence of the gap near T_c, which can be obtained, for example, with the aid of (1).

By way of example we have determined the temperature dependence of the gap from the result of measurement of the thermal conductivity of a niobium sample.

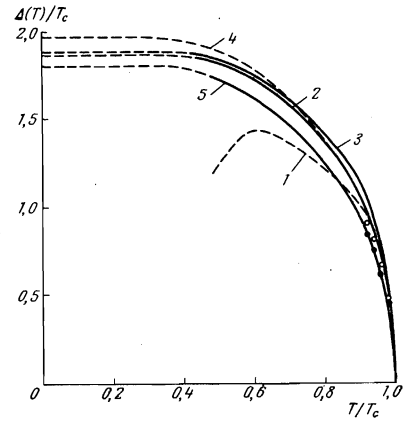

FIG. 5. Temperature dependence of the gap of niobium and of a solution of niobium with 1.7 at.% zirconium. 1–Niobium without allowance for the phonon thermal conductivity; 2–investigated niobium sample; 3–niobium in accordance with the data of [13]; 4–BCS gap model with 0 = 1.97. Calculation of the gap from the discontinuity of the specific heat: O–niobium in accord with the data of [11]; ●–niobium with 1.7 at.% zirconium (see Fig. 4), 5–niobium with 1.7 at.% zirconium.

In this case $\rho_0 = 1.08 \times 10^{-6}$ Ω-cm, and the Lorentz number is practically equal to the Sommerfeld value. This makes it possible to use for κ_{en} an extrapolation of the proton thermal conductivity of the normal state, $\kappa_{en} = 190t$ mW/cm-deg. Curve 1 in Fig. 5 shows the values of the gap determined without allowance for the phonon thermal conductivity. It is reasonable to assume that the maximum in this curve and the low values of the gap at $t \approx 0.6$ are due to the growth of the phonon contribution with decreasing temperature.

In the region $0.5 \leq t \leq 1$, Eq. (4) is satisfied, in the case of the investigated sample, by gap values represented by curve 2. The coefficient B was obtained by substituting in (5) the values $t_0 = 2.92$ and $t_0 = 0.6$ ($\alpha/B = 870$). The correctness of the choice of these values is confirmed by the agreement of the $\Delta(t)$ relation represented by curve 2 with the values of the gap calculated from the discontinuity in the specific heat, using the data of[11].

At low temperatures, curve 2 approaches the value of Δ_0 obtained from ultrasound absorption[12]. This value agrees with numerous determinations of Δ_0 from measurements of the specific heat[13].

At $t < 0.5$, the gap values calculated from κ_{gs} cannot be reconciled with the monotonic extrapolation of curve 2 into the low-temperature region (dashed). It must be assumed that phonon scattering by the crystal-lattice defects has a noticeable effect in this temperature region.

Figure 5 shows also the temperature dependence of the superconducting gap (curve 3) of the niobium sample ($\rho_0 = 5.2 \cdot 10^{-7}$ Ω-cm), calculated by us from thermal-conductivity measures performed by Wasim and Lebouni[13]. In spite of the difference in the purity and in the number of defects in the samples (the latter is illustrated by the difference in the temperatures up to which the only mechanism controlling the lattice thermal conductivity is the scattering of phonons by electrons), the temperature dependences of the gaps are very close. The discrepancy between curves 2 and 3 can be explained by assuming a systematic error on the order of 2–2.5% in the thermal conductivity measurement made by Wasim and Leboumi[13]. The authors of this paper themselves, assuming that the contribution of the phonon thermal conductivity can be neglected at $t \gtrsim 0.7$, interpreted their experimental data by using for the gap a temperature dependence calculated by the BCS model[10] with $\Delta_0 = 1.97$ (curve 4). Comparison with curves 2 and 3 shows that at $t < 0.5$ the values of the gap, and consequently the resolution of κ_s into electronic and lattice components with the aid of these values, are highly inaccurate.

We consider now a case when there are no additional data that make it possible to verify the correctness of the choice of the initial gap. Assuming at the temperature t_0 the erroneous gap value $\bar{b}_0 = b_0 + \delta b_0$, we incur by the same token the following error in the determination of the coefficient B in formula (4):

$$1 - \frac{\bar{B}}{B} \approx \delta b_0 \frac{\Omega'(b_0) + \alpha\Phi'(b_0)/Bt_0}{\Omega(b_0) - \delta b_0 \Phi'(b_0)\alpha/Bt_0}. \quad (9)$$

The use of the coefficient B in (4) in place of the correct value B leads to the following error in the determination of the gap:

$$\delta b \approx \frac{(1 - \bar{B}/B)\Omega(b)}{\Omega'(b) + \Phi'(b)B\alpha/\bar{B}Bt}. \quad (10)$$

At a ratio $\alpha/B \sim 10$, the coefficient B can be determined with sufficient reliability from the thermal conductivity and electric conductivity at temperature t = 1. The problem of the correct choice of the coefficient B arises only at the parameter values $\alpha/B \gtrsim 100$.

Figure 6 shows the results of calculations of the energy gap in accordance with formula (10) for the case when the true temperature dependence of the gap is given by the BCS model[7,11]. The calculations were performed with the parameter values $\alpha/B = 100$ and $\alpha/B = 1000$. The value $(B - \bar{B})/B$ corresponds to a gap error $\delta b_0 = \pm 0.13$ at $t_0 \approx 0.6$. For a given experimental $\kappa_s(t)$ curve it is possible to set in correspondence, with the aid of Eq. (4), each value of B with a separate $\Delta(t)$ curve. From the shapes of the curves in Fig. 6, however, it follows that the requirements that $\Delta(t)$ be monotonic and that the square-root dependence (1) be preserved near t = 1 are satisfied by the solutions of (4) only if the coefficient B is correctly chosen. The error in the choice of this coefficient comes most strongly into play at values $\alpha/B \sim 1000$. In this case, even a relatively small error in the coefficient B leads to a strong and easily observed violation of the monotonicity of the function $\Delta(t)$. In the case of samples whose thermal conductivity in the normal state is determined primarily by the electronic component, this makes it possible to obtain a sufficiently correct temperature dependence of the energy gap by solving Eq. (4) without using additional data, which can then be used to monitor the correctness of the choice of the coefficient B.

We have made use of this possibility to determine the temperature dependence of the gap from measured values of the thermal conductivity of the solid solution of niobium with 1.7 at.% zirconium. It is possible to obtain for Eq. (4) solutions that lead to a monotonic temperature dependence of the gap if B is equal to 0.424 mW/cm-deg. This dependence is represented by curve 5 of Fig. 5. Near t = 1, this curve agrees satisfactorily with the gap values calculated from formula (1) using data on the electronic specific heat of this sample. The values of the gap were determined from the thermal-conductivity data only in the interval $0.55 \leq t \leq 1$, inasmuch at lower temperatures the scattering of the phonons by the crystal-structure lattice becomes appreciable. At $t < 0.55$, the value of the gap is obtained by extrapolation (dashed

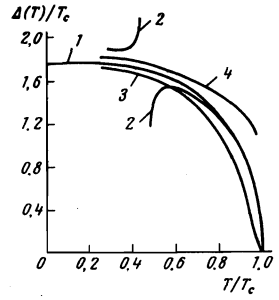

FIG. 6

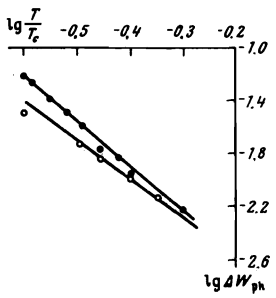

FIG. 7

FIG. 6. Temperature dependence of the gap, calculated from thermal-conductivity data at different values of $\bar{B}/B$: 1–$\delta b_0 = 0$, $\bar{B}/B = 1$; $\delta b_0 = 0.13$, $\bar{B}/B = 1.14$; 4–$\delta b_0 = 0.13$, $\bar{B}/B = 0.90$.

FIG. 7. Temperature dependence of additional thermal resistance due to scattering of phonons by crystal-structure defects: ○–niobium with 1.7 at.% zirconium, ●–niobium.

continuations of curves 2, 3, and 5 in Fig. 5). This extrapolation is justified to a considerable degree by the character of the curves in that temperature region where they correspond to Eq. (4).

The obtained temperature dependence of the gap make it possible to calculate the electron and phonon components of the thermal conductivity of the investigated samples (curves 2 and 3 in Figs. 1 and 2). Curves 4 in Figs. 1 and 2 correspond to the values of the phonon thermal conductivity calculated from formula (3), limited only by the scattering of the phonons from the electronic excitations. Curves 3 and 4 coincide in that temperature region in which this mechanism of phonon scattering is predominant. The presence of maxima on the plots of the total thermal conductivity indicates that below a certain temperature the contribution of the scattering of the phonons by the crystal-structure defect becomes significant. Therefore the discrepancy between curves 3 and 4 becomes reasonable.

Assuming that the processes of phonon scattering by electrons and by structure defects are independent, we obtained the thermal resistivity due to the last mechanism as the difference between the reciprocals of the ordinates of curves 3 and 4. Figure 7 shows the dependence of the logarithm of this thermoresistance (ΔW_{ph}) on the logarithm of the reduced temperature. This dependence is described by the relation

$$\Delta W_{ph} = (AT_c^n l^n)^{-1}, \qquad (11)$$

where $n \approx 3$ for the investigated samples (see Table III).

This value of the exponent allows us to assume that the phonon-scattering defects are the grain and block boundaries of the investigated solid crystalline samples. If this is the case, then the thermal conductivity, which is controlled by phonon scattering from boundaries separated by a characteristic distance l, can be estimated by a relation that follows from[14].

$$\frac{1}{\Delta W_{ph}} \approx \frac{1}{3} Cul = 2{,}58 \cdot 10^8 v_m^{-2/3} \Theta_D^{-2} l T_c^3 t^3 \frac{\text{mw}}{\text{cm} \cdot \text{deg}}, \qquad (12)$$

where Θ_D is the Debye temperature and v_m is the molar volume. From (11) and (12) we obtain

$$l \approx 3.88 \cdot 10^{-9} A T_c^{n-3} l^{n-3} v_m^{2/3} \Theta_D^2 \text{ cm}. \qquad (13)$$

The values of l obtained in this manner are given in Table III. They should be regarded as reasonable, since

TABLE III. Certain characteristics of investigated samples

Sample	T_c, K	α/B	Δ_0	$\rho_0 \cdot 10^6$, Ω-cm	AT_c^3	n	$l \cdot 10^3$, cm
Investigated niobium	9,1	870	1.87	1,08	1580	2.84	$2{,}6t^{-0.16}$
Niobium in [13]	9.0	2500	1.89	0.52			
Niobium with 1.7 at.% zirconium	9,5	700	1,80	0.76	1450	2.94	2.3

they correspond approximately to the dimensions of the crystallites and blocks of the investigated samples. The values of ΔW_{ph} depend, naturally, on the correctness with which the temperature dependences of the gaps were extrapolated into the low-temperature region. Therefore the fact that the temperature dependence of ΔW_{ph} satisfies relation (11), and also the fact that estimates of l based on the construction in Fig. 7 turn out to be reasonable, favors the correctness of the values of Δ_0 obtained by extrapolation.

The authors thank V. Z. Dresin for interest in the work and for a discussion, and A. S. Volovik for help with the computer calculations.

[1)]Formula (10) makes it possible to calculate the gap error due to the error in the coefficient B for an arbitrary gap, since $\Omega(b)$, $\Omega'(b)$, and $\Phi'(b)$ are universal functions of b.

[1]B. Ya. Sukharevskiĭ, É. A. Anders, and T. G. Kazanskaya, Sob. trudov FTINT AN UkrSSR (Coll. Papers, Physicotech. Inst. of Low Temp., Ukr. Acad. Sci.), Vol. 4, Khar'kov, 1969, p. 135.
[2]B. Ya. Sukharevskiĭ, É. A. Anders, and I. V. Shestakova, 5th All-Union Conf. on Calorimetry, Expanded abstracts (in Russian), Moscow Univ. Press, 1971, p. 423.
[3]E. M. Savitskiĭ, B. Ya. Sukharevskiĭ, V. V. Baron, É. E. Anders, V. A. Frolov, K. V. Mal'chuzhenko, and I. V. Shestakova, Superconducting Alloys and Compounds, Proc. 6th All-Union Conf. on the Problem of Superconducting Materials (in Russian), Nauka, 1972, p. 125.
[4]E. M. Savitskiĭ, B. Ya. Sukharevskiĭ, V. V. Baron, É. E. Anders, V. A. Frolov, and I. V. Shestakova, Dokl Akad. Nauk SSSR **203**, 1044 (1972) [Sov. Phys.-Dokl. 17, 390 (1972)].
[5]B. T. Geĭlikman and V. Z. Kresin, ZhETF Pis. Red. 5, 271 (1967) [JETP Lett. 5, 220 (1967)].
[6]B. T. Geĭlikman and V. Z. Kresin, Fiz. Tverd. Tela 9, 3111 (1967) [Sov. Phys.-Solid State 9, 2453 (1968)].
[7]J. Bardeen, G. Rickaysen, L. Tewordt, Phys. Rev. **113**, 982 (1959).
[8]B. T. Geĭlikman, Zh. Eksp. Teor. Fiz. 34, 1042 (1958) [Sov. Phys.-JETP 7, 721 (1958)].
[9]B. T. Geĭlikman and V. Z. Kresin, Dokl. Akad. Nauk SSSR 123, 259 (1958).
[10]B. Muhlshlegel, Zs. Phys. **155**, 313 (1959).
[11]E. Bucher, F. Heiniger, J. Muller. Low Temperature Physics LT9 (part a), N. Y., 1965, p. 616.
[12]E. R. Dobbs, J. M. Pers. Proc. Phys. Soc. (London), **A296**, 113 (1967).
[13]S. M. Wasim, N. H. Lebowni, Phys. Rev. **187**, 539 (1969).
[14]P. G. Klemens, in: Low Temperature Physics, Hand. Physik Vol. XIV-XV, 1956; W. H. Keesom and N. Pearlman, ibid.

Translated by J. G. Adashko
182