Critical velocities for vortex formation by an oscillating disc in helium II and relaxation processes during its rotation

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The critical velocity for vortex formation in rotating helium II as a function of its temperature is investigated by the oscillating-disc technique. It is shown that in a state of rotation the critical velocity for vortex formation is larger than in a state of rest. Relaxation processes occurring in helium II due to its sudden rotation, to variation of its rotational velocity, or to cooling of rotating helium I below T_λ are studied by the same method.

1. Two experimental investigations connected with the change in the state of motion of helium II are joined in this paper. In the first of these, the critical velocities for vortex formation due to oscillations of a disc in rotating or non-rotating helium II have been measured; in the second, the relaxation times for rotation of the helium II have been studied.

2. As is known (see, for example[1]), the critical velocities for vortex formation, observed in channels with width $d > 10^{-3}$ cm, agree relatively well with the formula of Feynman[2]

$$v_c = \frac{\hbar}{md}\ln\frac{d}{a_0} \tag{1}$$

(with some variation in the coefficients for $\hbar/md$ and d/a_0), where m is the mass of the helium atom, $a_0 \approx 3 \times 10^{-8}$ cm is the dimension of the tube of the quantized vortex.[2]

In oscillations of solids in helium II with axial symmetries, the region where relative motion of the normal and superconducting components exists has a width of the order of the penetration of the viscous wave λ:

$$\lambda = (\nu_n\theta/\pi)^{1/2}, \tag{2}$$

where $\nu_n = \eta_n/\rho_n$ is the kinematic viscosity of the normal component and θ is the period of the oscillations.

The superconducting component does not generally take part in motion along with the solid at sufficiently small amplitudes. Upon increase in the amplitude of the oscillations, the appearance of a dependence of the damping on the amplitude is associated with the setting of the superconducting component into oscillation. It is assumed that it is in turn connected with vortex formation, and the maximum velocity on the periphery of the vibrating body, which it has at the critical amplitude φ_c, is identified as the critical velocity for vortex formation:

$$v_c = \varphi_c R\Omega,$$

where R is the radius of the vibrating body, $\Omega = 2\pi/\theta$ the cyclotron frequency of the oscillations. The critical velocity for vortex formation was measured in this fashion for an oscillating disk and a stack of discs by Hollis-Hallet[3] and by Gamtsemlidze,[4] for the oscillations of a cylinder by Dash and Taylor,[5] and for oscillations of a sphere by Benson and Hollis-Hallet.[6]

3. In the experiments, we used a "heavy" roughened disk. For measurement of the damping of the oscillations, we used the chronometer method.[7] The measurements were carried out on a new, completely automatized apparatus, which made it possible to record the results automatically. These were then analyzed on a high-speed computer. The disk was made of brass and had a diameter of 30.0 mm, thickness 1 mm and was covered by a single layer of sand particles with a linear diameter of ~ 50 μ. The disk was suspended by means of a phosphor-bronze elastic wire (length ~ 120 mm, diameter 25 μ) in a glass container of diameter 42 mm and height h = 80 mm and, in simultaneous equilibrium with the glass, carried out rotational oscillations with a period of θ = 19.99 sec.

To carry out the experiment in a glass vessel isolated from the helium bath, helium was condensed and in what followed, the quantity of liquid in it did not change. The container was put into rotation with a specified velocity and after a lapse of time of 15—20 min, measurements were begun of the logarithmic damping decrement of the oscillations of the disk. The results of the measurements are shown in Fig. 1. The black circles indicate the critical velocity in nonrotating helium. These results are in agreement with the data of other authors (for example, with[3,4]), who also observed an increase in the critical velocity with temperature. The open circles in Fig. 1 refer to rotation velocities ω_0 = 0.012 sec^{-1} and the crosses to velocities ω_0 = 0.029 sec^{-1}. An increase in critical velocity with increase in the speed of rotation was observed at all temperatures.

As we have already remarked, in oscillations of solids in helium II, the region in which relative motion of the normal and superconducting components exists has a width λ. For reasonable frequencies of oscillation used experimentally, λ changes in the range 10^{-1}—10^{-2} cm. If we recognize in this case that the oscillating body

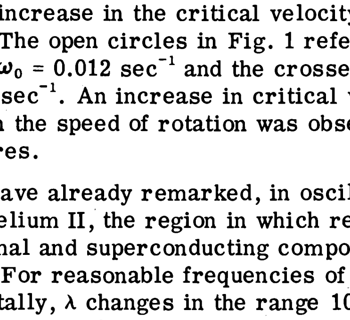

FIG. 1. Temperature dependence of the critical velocities, measured for various angular velocities of rotation: ● – motionless helium, ○ – ω_0 = 0.012 sec^{-1}; × – ω_0 = 0.029 sec^{-1}. (v, cm/sec)

usually has a radius 10—20 mm, then the picture is very close to one in which a narrow channel (of width λ) moves, while the normal component of the liquid moves relative to the fixed superfluid component (in contrast with experiments on the measurement of the critical velocities in narrow pores, in which, on the contrary, the normal component is at rest and the superfluid component moves). It can therefore be expected that in oscillatory experiments Eq. (1) is also valid, in which we set λ in place of d. According to the relation (2), λ increases with decrease in temperature (like $\sqrt{\nu_n}$), which, by the Feynman formula, ought to cause a decrease in the critical velocity, which is observed experimentally.

Thus, in contrast with the experiments in narrow slits and pores, where the width of the channels naturally does not change with change in temperature, in oscillatory experiments, the characteristic dimension of the channel depends on the temperature. We rewrite (1) in the form

$$\lambda v_c = \frac{\hbar}{m} \ln \frac{\lambda}{a_0}, \tag{1'}$$

where the right side is almost constant in view of the presence of the logarithm, in which $\lambda \gg a_0$ ($\lambda \approx 10^{-2} \gg 3 \times 10^{-8}$). Substituting the values of the constants, we obtain $v_C\lambda \approx 2.5 \times 10^{-3}$ cm^2-sec^{-1}. Figure 2 shows the dependence of $v_C\lambda$ on the temperature for similar experiments of different authors. It is seen from consideration of this drawing that the product $v_C\lambda$ is of the same order of magnitude as the calculated value of the quasiconstant in the Feynman formula. However, the values of $v_C\lambda$ in different experiments differ more widely than can be due to the logarithm of different λ (for example, for our results $\lambda_{max} = 6.75 \times 10^{-2}$ cm, $\lambda_{min} = 3.05 \times 10^{-2}$ cm, ln ($\lambda_{max}/\lambda_{min}$) = 0.79, and the observed difference $v_C\lambda_{max}/v_C\lambda_{min} = 1.5$). Second, we see that the quasiconstant character of the product $v_C\lambda$ is quite relative (25% for a roughened disk from the data of Hallis-Hallet[3] and our experiment, and 40% for a smooth disk). It can therefore be concluded that in oscillatory experiments, the use of the penetration depth as a characteristic dimension of the problem is not completely accurate.

The increase in the critical velocity for rotation cannot be explained by the penetration depth. Actually, as is shown in[8], in the case of rotation the penetration depth of a viscous wave is expressed by the formula

$$\lambda^{\pm} = \left(\frac{2\nu_n}{|\Omega \pm 2\omega_0|} \right)^{1/2}.$$

It is evident that λ^- is always larger than λ. So far as

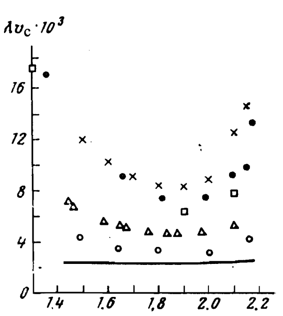

FIG. 2. Temperature dependence of $\bar{v}_c\lambda$ constructed from various experimental data: ● – sphere, θ = 25 sec;[6] X – smooth disc, θ = 14 sec;[4] □ – cylinder, θ = 15 sec;[5] O – disc, [3] Δ – data of present research. Continuous curve – temperature dependence of the quasiconstant in Eq. (1) of Feynman.

λ^+ is concerned, at $\omega_0 = 0.029$ sec^{-1} we have $\lambda/\lambda^+ = 1.09$, since the corresponding ratio $v_C(\omega_0 = 0.029)/v_C(\omega_0 = 0)$ = 1.3 for all temperatures.

In contrast with helium II at rest, in which the superfluid component is motionless, the superfluid component in rotating liquid joins in the rotation of the vessel through the already existing N = 2000 ω_0 quantized vortices per cm^2. What is more, the vortices that exist in rotating helium II and the vortices formed by the oscillating disk are differently oriented. The increase in the critical velocity in the rotational state evidently means that the setting into vortex motion of the fixed superfluid components is easier than the formation of vortices that extend along the surface of the disk, in the case in which vortices perpendicular to this surface already exist in the liquid. The second case is connected with the more important change in the state of motion of helium II, which naturally requires the expenditure of more energy.

4. Relaxation phenomena observed in the rotation of helium II were studied long ago. In particular, back in 1948, Andronikashvili[9] and later Andronikashvili and Kaverkin[10] observed the kinetics of the development of the meniscus of this liquid. They showed that the setting of the helium II into rotation begins at the wall of the vessel, and is then propagated toward the axis of rotation. As is now known, in rotation with subcritical velocity of the container of the helium II, quantized Onsager-Feynman vortices are formed in the superfluid component.[2] In this way, the relaxation times connected with the initial subcritical rotational state are due to the kinetics of the accumulation of quantized vortices and their interaction with one another and with the normal component of the liquid helium. Evidently, the vortices created at the walls, are redistributed until an equilibrium vortex lattice is formed.

Subsequently, the relaxation processes associated with the onset of the subcritical state of motion of helium II have been studied by many authors, both by measurement of the local velocity fields (in the work of Pellam,[11,12] who used a test body), and by other methods: the variation time of the velocity of a freely suspended container with liquid helium,[13] the additional damping of the waves of second sound.[14] Table I gives some of the results, obtained for T ~ 2° by different authors. It is seen from this table that in these experiments the angular velocities of rotation ω_0 differ from one another by three orders of magnitude, the characteristic dimensions of the containers differ by a factor of two, and the relaxation time differs by more than 20 times.

Our purpose, along with the communication of new experimental data, is the connection of all the results obtained in the study of the setting of helium II into rotation in cylindrical containers by a universal dependence of the form

$$v = c u^{1/2} R, \tag{3}$$

where v is the velocity of propagation of the vortex front,

TABLE I.

t_0, sec	d, cm	ω_0, sec^{-1}	Method of measurement
1800	1.40	0.16	test body[12]
1000	1.25	0.22	change in velocity[13]
300	1.0	0.104	Rayleigh disc[11]
120	1.25	30	depth of meniscus[10]
80	1.2	1.76	damping of second sound[14]
410	2.0	0.029	damping of disc oscillations (present research)

u the linear velocity of the surface of the glass ($u \equiv \omega_0 R$), R its radius, and c some constant. It is natural that the applicability of Eq. (3), which is semi-empirical, is limited to intervals of parameter values that are close to those given in Table I (in particular, formula (3) is no longer valid as $R \rightarrow \infty$ and $\omega_0 \rightarrow \infty$).

5. We have applied the method of damping of the free oscillations of a disk, which was first used by Andronikashvili et al.[15] By means of the apparatus described above in Sec. 3, the time was measured from the beginning of rotation to the establishment of equilibrium damping of the oscillations of the disk. Figure 3 gives the dependence of the relaxation time on the temperature for the rotation of initially motionless helium II to a velocity $\omega_0 = 0.29\ \mathrm{sec}^{-1}$ and for a change in the velocity of its rotation from 0.012 to 0.029 sec^{-1}. Upon approach to T_λ, the relaxation times diminish, which is evidently connected with the increased role of the viscosity in relation to the increase in the amount of the normal component.

The next drawing (Fig. 4) shows that as the velocity of rotation decreases, the relaxation time t_0 increases nonlinearly, as was observed earlier[16] at higher velocities of rotation.

In contrast to the described experiments, for which a sudden change takes place in the state of rest (or of velocity of rotation) of the container with helium II, we also measured the relaxation time in the case in which uniformly rotating helium I was quickly cooled below T_λ. Here the critical state sets in throughout the volume of the liquid and the relaxation times should not depend on the geometry of the container. Figure 5 gives the results of such experiments (small circles). The liquid was cooled from 2.22 to 2.142°K for different velocities or rotation in a glass with radius R = 2.1 cm. Figure 5 also shows the results of a previous work,[14] where the temperature of the liquid was measured in the same temperature interval, but the method of second sound was used. The experiments were carried out in a radial resonator with a gap width of 1.2 cm. Complete agreement of the results of both experiments (in spite of the difference in the volumes of the liquid) is obvious.

We now give unpublished data obtained by Cheremisina and one of the authors (Dzh. Tsakadze). The time for the establishment of the depth of the meniscus in the rotations of helium II at constant temperature T = 2.15°K and velocity of rotation $\omega_0 = 30\ \mathrm{sec}^{-1}$ was studied in various ring gaps formed by the surfaces of coaxial cylinders (the diameter of the internal cylinder was varied—see Fig. 6). As is seen from Table II, the relaxation time decreases with decrease in the gap width.

It was shown previously[16] that the relaxation time t_0 corresponding to the formation of an equilibrium (for a given velocity of rotation) number of vortices, depends on the linear velocity $u \equiv \omega_0 R$ of the walls of the cylinder contained in the following way:

$$t_0 \infty u^{-1/3}.$$

If we have a container with the characteristic dimension R, then the velocity of propagation of the vortex front will be $v = R/t$, from which (3) follows.

Figure 7 shows the dependence of v on $u^{1/3}R$ (continuous curve) from the empirical formula (3) and the experimental data according to Table I. The results are in excellent agreement.

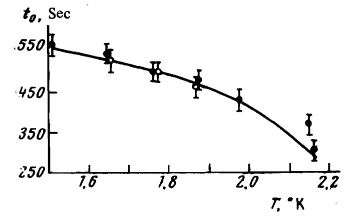

FIG. 3.

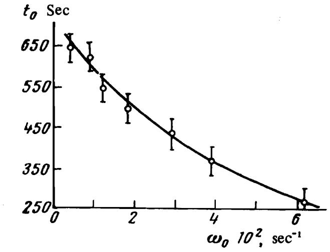

FIG. 4.

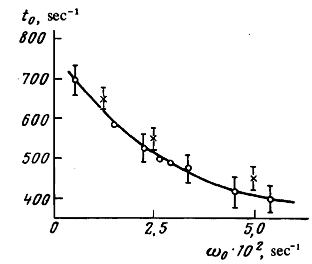

FIG. 5.

FIG. 3. Dependence of the relaxation time on the temperature of the liquid: ● – twisting of the apparatus from rest to a velocity of $\omega_0 = 0.029\mathrm{sec}^{-1}$, ○ – change in velocity of rotation of the container from $\omega_0 = 0.012\mathrm{sec}^{-1}$ to $\omega_0 = 0.029\mathrm{sec}^{-1}$.

FIG. 4. Dependence of the relaxation time on the velocity of rotation for T = 1.765°K.

FIG. 5. Dependence of the relaxation time on the velocity of rotation in the cooling of rotating liquid helium from 2.22 to 2.142°K: ○ – data of present experiment; X – data of [14].

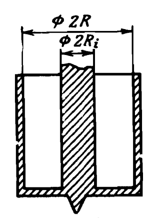

FIG. 6.

TABLE II.

R_i, cm	$R - R_i$, cm	t_0, sec
0	2.1	165
0.25	1.95	150
0.60	1.50	120
0.90	1.20	80
1.30	0.80	40

Here R = 2.1 cm

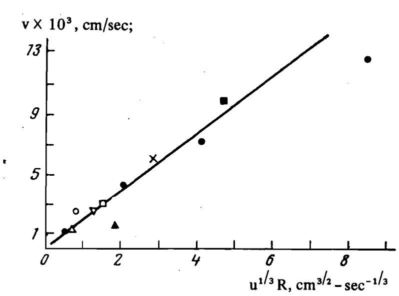

FIG. 7. Dependence of the velocity of the vortex front on $u^{1/3}$ R. The continuous curve is the result of a calculation from Eq. (3). The experimental points were obtained from the researches: ■ – [10], □ – [11], ▲ – [12], Δ – [13], X – [14], ● – [16], ○ – result of present experiment.

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