Resistance of superconductors near the critical field strength H_{c2}

A. I. Larkin and Yu. N. Ovchinnikov
L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences
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It is shown that near the superconducting transition temperature the conductivity depends nonlinearly on the magnetic field strength and may considerably exceed that of a normal metal in fields near to the critical value. Several regions exist with significantly different dependences of the conductivity on magnetic impurity concentration, temperature and magnetic field strength.

1. INTRODUCTION

The resistance of type-II superconductors in fields sufficiently close to H_{c2} is usually obtained by a series expansion in the order parameter Δ[1-3]. At temperatures close to critical, there exists a wide range of magnetic fields

$$1 \gg 1 - H/H_{c_2} \gg (1 - T/T_c), \tag{1}$$

in which this expansion cannot be used to calculate the resistance. However, this expansion can be used to find the thermodynamic quantities. As will be shown below, in the range of fields defined by formula (1), the conductivity increases like $(H_{c2} - H)^{3/2}$, and can become much larger than the conductivity of the normal metal.

The existence of this region is connected with the strong nonlinearity of the Ginzburg-Landau equation for the time-dependent order parameter Δ[4].

2. LINEAR RESPONSE IN A SUPERCONDUCTOR WITH SMALL MEAN FREE PATH

We consider a type-II superconductor in a strong constant magnetic field $H = \operatorname{curl} \mathbf{A}$ and a weak alternating field. Let us find the expressions for the current density in the approximation linear in the alternating electromagnetic field.

The equations for the Green's function in the linear approximation in the alternating field can be obtained by linearizing the Gor'kov equations[5]. For very "dirty" superconductors, these equations reduce to a system of differential equations for the Green's functions at coinciding points[6].

In the approximation linear in the alternating field, these equations take the form

$$\begin{gathered} -i\omega_+\tau_z G_1 + i\omega G_1\tau_z + (e\varphi - \hat{\Delta}^{(1)})G_\omega - G_{\omega+}(e\varphi - \hat{\Delta}^{(1)}) - [\hat{\Delta}^{(0)}, G_1] \\ = -iD[\partial, G_1\partial G_\omega + G_{\omega+}\partial G_1 - ie\mathbf{A}_1 G_{\omega+}\tau_z G_\omega] + eD\mathbf{A}_1(G_{\omega+}\partial G_{\omega+}\tau_z \\ - \tau_z G_\omega \partial G_\omega) + eD\tau_z \frac{\partial \mathbf{A}_1}{\partial r} + \frac{i}{2\tau_s}[\tau_z, G_1\tau_z G_\omega + G_{\omega+}\tau_z G_1], \end{gathered} \tag{2}$$

$$G_{\omega+}G_1 + G_1 G_\omega = 0, \tag{3}$$

where $\omega_+ = \omega + \omega_0$, $\omega_0 = 2\pi T m$, $\omega = \pi T(2n+1)$, $\partial = \partial/\partial r - ie\mathbf{A}\tau_z$; $[,]$ is a commutator, τ_z is a Pauli matrix, $D = ul_{tr}/3$ is the electron diffusion coefficient, φ and $\mathbf{A}_1$ are the amplitudes of the scalar and vector potentials of the alternating electromagnetic field of frequency ω_0, and τ_s is the time of collision with the impurity with spin flip. In the answer, as usual, it is necessary to carry out an analytic continuation in ω_0.

The Green's-function increment linear in the alternating field depends on two frequencies and can be expressed in the form

$$G_1 \equiv G_1(\omega, \omega + \omega_0) = \begin{pmatrix} g_1 & f_1 \\ -f_2 & g_2 \end{pmatrix}. \tag{4}$$

The correction to the order parameter is

$$\hat{\Delta}^{(1)} \equiv \hat{\Delta}^{(1)}_{(\omega_0)} = \begin{pmatrix} 0 & \Delta_1 \\ -\Delta_2 & 0 \end{pmatrix}; \quad \Delta_{1,2} = i\nu|\lambda|\pi T \sum_\omega f_{1,2}, \tag{5}$$

where $\nu = mp/2\pi^2$ is the density of states on the Fermi surface and λ is the interaction constant.

The Green's function G_ω and the order parameter $\hat{\Delta}^{(0)}$ take the following forms in the absence of an external alternating field:

$$G_\omega = \begin{pmatrix} \alpha & -i\beta \\ i\beta^* & -\alpha \end{pmatrix}, \quad \hat{\Delta}^{(0)} = \begin{pmatrix} 0 & \Delta \\ -\Delta^* & 0 \end{pmatrix}. \tag{6}$$

The functions α and β satisfy the system of equations[7,8]

$$\begin{gathered} \alpha^2 + |\beta|^2 = 1, \\ \tfrac{1}{2}D[\alpha(\partial/\partial \mathbf{r} - 2ieA)^2\beta - \beta\partial^2\alpha/\partial \mathbf{r}^2] + \alpha\Delta - \beta\omega = \tau_s^{-1}\alpha\beta. \end{gathered} \tag{7}$$

The order parameter Δ and the current density j are expressed in terms of the function β:

$$\Delta = \nu|\lambda|\pi T \sum_\omega \beta,$$

$$j = -\frac{i\sigma}{2}\pi T \sum_\omega (\beta^*\partial_-\beta - \beta\partial_+\beta^*),$$

where

$$\partial_\pm = \frac{\partial}{\partial \mathbf{r}} \pm 2ie\mathbf{A}, \qquad \sigma = e^2p^2 l_{tr}/3\pi^2$$

is the residual conductivity of the normal metal.

Expressions for the alternating-current density $\mathbf{j}_1$ and for the charge density ρ_1 take the following form in the approximation linear in the alternating field[6]

$$\begin{gathered} \mathbf{j}_1 = \pi\sigma T \sum_\omega \left\{ -\mathbf{A}_1 + \frac{i}{2e}\operatorname{Sp}\tau_z[G_1\partial G_\omega + G_{\omega+}\partial G_1 - ie\mathbf{A}_1 G_{\omega+}\tau_z G_\omega] \right\}, \\ \rho_1 = -e\nu\left\{ 2e\varphi + i\pi T \sum_\omega \operatorname{Sp} G_1 \right\}. \end{gathered} \tag{9}$$

Formulas (2)—(9) describe the behavior of the superconductors in a weak alternating field at arbitrary temperatures. If the temperature is not close to T_c, then there exist two regions of magnetic field intensity. In the region $H_{c2} - H \ll H_{c2}$, one can solve Eqs. (2)—(9) by expanding in the order parameter Δ. This yields for the conductivity the results of[1-3]. In the region $H \ll H_{c2}$, the resistance is determined by the motion of the vortices.

A field region (1) in which the results of[1-3] do not hold is obtained at temperatures close to critical.

3. TEMPERATURES CLOSE TO CRITICAL

At temperatures close to critical, $T_c - T \ll T_c$, we can expand in terms of the parameters Δ/T and eHD/T in Eqs. (2)—(9).

In a constant field, the order parameter Δ satisfies the Ginzburg-Landau equation. To obtain an equation for $\hat{\Delta}^{(1)}$ it is necessary, before expanding in powers of Δ, to carry out an analytic continuation in Eq. (5) with respect to the frequency ω_0, from the values $\omega_0 > 0$. This gives rise to terms of two types, normal and anomalous. The normal term stems from those terms in the sum over ω for which ω and $\omega + \omega_0$ are of equal sign. In these terms, just as in the static case, the function $f_{1,2}$ can be obtained by expanding Eq. (2) in powers of the parameters Δ/T, eHD/T, ω_0/T. For the anomalous term, which comes from the region $-\omega_0 < \omega < 0$, there is no simple expansion in powers of these parameters. After analytic continuation, Eq. (5) reduces to the form

$$\hat{L}\begin{pmatrix}\Delta_1\\ \Delta_2\end{pmatrix}=\frac{i\pi}{2T}e\mathbf{A}_1D\begin{pmatrix}\partial_- & \Delta\\ -\partial_+ & \Delta^*\end{pmatrix}+\frac{\pi\omega_0}{8T}\begin{pmatrix}\Delta_1\\ \Delta_2\end{pmatrix}-\frac{\omega_0}{8T}\int_{-i\infty}^{i\infty}d\omega\begin{pmatrix}f_1(\omega-\delta,\ \omega+\omega_0+\delta)\\ f_2(\omega-\delta,\ \omega+\omega_0+\delta)\end{pmatrix},\tag{10}$$

where

$$\hat{L}=\begin{pmatrix}\dfrac{T_c-T}{T_c}+\dfrac{\pi D}{8T}\partial_-^2-\dfrac{7\zeta(3)|\Delta|^2}{4\pi^2T^2}; & -\dfrac{7\zeta(3)\Delta^2}{8\pi^2T^2}\\ -\dfrac{7\zeta(3)\Delta^{*2}}{8\pi^2T^2}; & \dfrac{T_c-T}{T_c}+\dfrac{\pi D}{8T}\partial_+^2-\dfrac{7\zeta(3)|\Delta|^2}{4\pi^2T^2}\end{pmatrix},\tag{11}$$

δ is an infinitesimally small increment that indicates the rule of going around the singularities. In the derivation of (10) and (11) it was assumed that the concentration of the magnetic impurities is low: $\tau_s T \gg 1$.

Expression (9) for the current density can be represented in similar fashion in the form of a sum of normal and anomalous terms:

$$\mathbf{j}_1=\mathbf{j}_n^{(1)}+\mathbf{j}_a^{(1)},$$

$$\mathbf{j}_n^{(1)}=-\frac{\pi\sigma}{2T}\left\{\mathbf{A}_1|\Delta|^2-\frac{i}{2e}(\Delta_1\partial_+\Delta^*-\Delta_2\partial_-\Delta)-\frac{i}{4e}\frac{\partial}{\partial r}(\Delta\Delta_2-\Delta^*\Delta_1)\right\},$$

$$\mathbf{j}_a^{(1)}=-\omega_0\sigma\mathbf{A}_1+\frac{i\omega_0\sigma}{8T}\int_{-i\infty}^{i\infty}d\omega\left\{\mathbf{A}_1(-1-\alpha\alpha_++\beta_+\beta^*)+\frac{1}{e}\left[f_1\partial_+\beta^*+\beta_+\partial_+f_2-ig_1\frac{\partial\alpha}{\partial r}-i\alpha_+\frac{\partial g_1}{\partial r}\right]\right\},\tag{12}$$

where

$$\alpha_+\equiv\alpha(\omega+\omega_0+\delta),\quad \alpha\equiv\alpha(\omega-\delta),$$
$$\beta_+\equiv\beta(\omega+\omega_0+\delta),\quad \beta\equiv\beta(\omega-\delta).$$

The general formulas (10) and (12) can be further simplified in the case of low alternating-field frequencies, when the first term in the right-hand side of (10) is large in comparison with the second and third terms. In this case the lattice is basically displaced as a unit under the influence of the alternating field.

In the static approximation ($\omega_0 = 0$), Eqs. (10) and (11) can be solved by an invariant shift of the unperturbed solution by a constant vector $\mathbf{b}$:

$$\mathbf{A}_1[\mathbf{Hb}],\quad \Delta_1=\mathbf{b}\partial_-\Delta,\quad \Delta_2=\mathbf{b}\partial_+\Delta,$$
$$\mathbf{j}_1=-\frac{i\pi\sigma}{8eT}\left(\mathbf{b}\frac{\partial}{\partial\mathbf{r}}\right)(\Delta^*\partial_-\Delta-\Delta\partial_+\Delta^*).\tag{13*}$$

In this approximation, the average current is equal to zero.

In the approximation linear in ω_0, the solution of (10) takes the form

$$\begin{pmatrix}\Delta_1\\ \Delta_2\end{pmatrix}=\begin{pmatrix}\mathbf{b} & \partial_- & \Delta\\ \mathbf{b} & \partial_+ & \Delta^*\end{pmatrix}+\frac{\pi\omega_0}{8T}\hat{L}^{-1}\left\{\begin{pmatrix}\mathbf{b} & \partial_- & \Delta\\ \mathbf{b} & \partial_+ & \Delta^*\end{pmatrix}-\int_{-i\infty}^{i\infty}\frac{d\omega}{\pi}\begin{pmatrix}f_1\\ f_2\end{pmatrix}\right\}.\tag{14}$$

In this case it is necessary to make the substitutions $\Delta_1 = \mathbf{b}\partial_-\Delta$ and $\Delta_2 = \mathbf{b}\partial_+\Delta^*$ in Eq. (2) for $f_{1,2}$, and take the limit as $\omega_0 \to 0$.

To determine the effective conductivity, we calculate the power P dissipated per unit volume. Since the operator $\hat{L}$ is Hermitian, we obtain from (12) and (14)

$$-\omega_0^{-1}P=\langle\mathbf{A}_1\mathbf{j}_1\rangle=\langle\mathbf{A}_1\mathbf{j}_a^{(1)}\rangle-\frac{\pi\omega_0\sigma}{8e^2DT}\langle|\mathbf{b}\partial_-\Delta|^2\rangle+\frac{\omega_0\sigma}{16e^2DT}\int_{-i\infty}^{i\infty}d\omega\langle f_1(\mathbf{b}\partial_+\Delta^*)+f_2(\mathbf{b}\partial_-\Delta)\rangle,\tag{15}$$

where the angle brackets $\langle\ldots\rangle$ denote averaging over the cell.

We have left the term $-\langle\mathbf{j}_1\nabla\varphi\rangle$ out of the expression for the power in (15). In the chosen gauge, the average electric field is described by the vector potential $\mathbf{A}_1$, and $\langle\nabla\varphi\rangle = 0$. The contribution of $\nabla\varphi$ to the local value of the electric field may not be small. To estimate the contribution of $\nabla\varphi$ to the dissipative power, we used the relation

$$-\langle\mathbf{j}_1\nabla\varphi\rangle=\langle\varphi\,\mathrm{div}\,\mathbf{j}_1\rangle=\frac{\omega_0}{4\pi}\left\langle\varphi\frac{\partial^2\varphi}{\partial\mathbf{r}^2}\right\rangle.$$

In metals, the Debye radius is small in comparison with the pair dimension ξ. Therefore the scalar potential φ can be obtained by equating the charge density ρ_1 in formula (9) to zero. As a result we find that $\langle\mathbf{j}_1\nabla\varphi\rangle$ contains, in comparison with $\langle\mathbf{j}_1\omega_0\mathbf{A}_1\rangle$, a small factor $\omega_0\sigma^{-1}$.

In the immediate vicinity of the critical field H_{c2}, where $|\Delta| \ll eHD$, the expression (15) can be expanded in powers of Δ. The last term is small in this case, the second term is equal to the Maki correction[1,2], and the first term gives the conductivity of the normal metal and the Thompson correction[3]. The parameter Δ begins to increase with decreasing magnetic field and becomes larger than $T_c - T$. At temperatures close to T_c, the transition to this region occurs at fields close to critical, $H_{c2} - H \ll H_{c2}$. In this region, the Thompson correction becomes relatively small[9] and the Maki correction remains unchanged, but the principal role is assumed by the last term in (15).

4. CONDUCTIVITY OF SUPERCONDUCTOR WITH PARAMAGNETIC IMPURITIES

The magnetic impurities influence the thermodynamic properties of a superconductor only at $\tau_s T \sim 1$, where τ_s is the time between the collisions with spin flip. The resistance depends strongly on the parameter $\tau_s\Delta$. We consider the relatively simple case

$$T\gg\tau_s^{-1}=\Gamma\gg eH_{c2}D,\Delta.\tag{16}$$

In this case the zero-order Green's function (6) can be obtained by expanding Eqs. (7) in powers of Δ:

$$\beta=\frac{\Delta}{|\omega|+\Gamma},\qquad \alpha=\left[1-\frac{|\Delta|^2}{2(|\omega|+\Gamma)^2}\right]\mathrm{sign}\,\omega.\tag{17}$$

Substituting these values in (3), we obtain

$$f_1=\frac{i\Delta}{2}\left(\frac{g_1}{-\omega+\Gamma}+\frac{g_2}{\omega+\Gamma}\right),\qquad f_2=-\frac{i\Delta^*}{2}\left(\frac{g_1}{\omega+\Gamma}+\frac{g_2}{-\omega+\Gamma}\right).\tag{18}$$

We obtain two more equations by calculating respectively the trace and the trace with the matrix τ_z, in both sides of (2). Taking (13), (17), and (18) into account, we have

$$\left(|\Delta|^2-D\frac{\Gamma^2-\omega^2}{2\Gamma}\frac{\partial^2}{\partial\mathbf{r}^2}\right)(g_1+g_2)=\mathbf{b}(\Delta\partial_+\Delta^*-\Delta^*\partial_-\Delta),$$
$$D\frac{\partial^2}{\partial\mathbf{r}^2}(g_1-g_2)=\frac{2\omega}{\Gamma^2-\omega^2}\mathbf{b}\frac{\partial|\Delta|^2}{\partial\mathbf{r}}.\tag{19}$$

It follows from (18) and (19) that the last term in

(15) is large in comparison with the second term in the region

$$\Delta^2\tau_s \gg eH_{c2}D. \quad (20)$$

In the opposite limiting case, the dependence of the conductivity on the magnetic field is determined by the Maki correction[1]. The Thompson correction in superconductors with magnetic impurities is small if the condition (16) is satisfied.

In the region determined by the condition (20), it follows from (19) that

$$g_1 = -g_2 = \frac{\omega}{D(\Gamma^2-\omega^2)} \int K(\mathbf{r}-\mathbf{r}_1)\mathbf{b}\frac{\partial|\Delta(\mathbf{r}_1)|^2}{\partial \mathbf{r}_1} d^2\mathbf{r}_1, \quad (21)$$

where $K(\mathbf{r} - \mathbf{r}_1)$ is the Green's function of the Laplace operator on a plane. We substitute expressions (18) and (21) in (15) and integrate by parts with respect to the coordinates $\mathbf{r}$ and $\mathbf{r}_1$. In a triangular lattice, the conductivity does not depend on the direction of the electric field in a plane perpendicular to the magnetic field. Therefore expression (15) can be averaged over the direction of the electric field. Taking into account the first formula of (13), we obtain

$$\frac{\sigma_{\text{eff}}}{\sigma} = \frac{P}{\sigma\langle \mathbf{E}\rangle^2} = 1 + \frac{\pi\tau_s\langle(|\Delta|^2 - \langle|\Delta|^2\rangle)^2\rangle}{64e^2D^2T\langle \mathbf{H}\rangle^2}. \quad (22)$$

In magnetic fields close to H_{c1}, expression (22) leads to the results obtained in[10]. In strong fields, $H_{c2} - H \ll H_{c2}$, we have

$$\frac{\sigma_{\text{eff}}}{\sigma} = 1 + \pi(\beta_A - 1)\tau_s T\left(\frac{\pi^3}{28\zeta(3)\beta_A}\frac{1 - H/H_{c2}}{1 - {}^1/_2\varkappa^2}\right)^2, \quad (23)$$

where $\beta_A = 1.16$ and κ is the Ginzburg-Landau parameter. Thus, the effective conductivity depends quadratically on the magnetic field in the region under consideration. Since $\tau_s T \gg 1$, the second term in (23) may be larger than the first.

5. CONDUCTIVITY OF SUPERCONDUCTOR WITHOUT MAGNETIC IMPURITIES

At a low concentration of the magnetic impurities, when $eH_{c2}D\tau_s \gg 1$, the magnetic impurities do not influence the conductivity at any value of the magnetic field. In magnetic fields close to H_{c2}, the Maki and Thompson corrections make equal contributions to the conductivity. With decreasing magnetic field, the last term in (15) increases rapidly and becomes predominant in the region

$$\Delta > eH_{c2}D \quad (24)$$

In the region defined by (24), the gradient terms in Eq. (7) are small, while the functions α and β are determined by the local values of the parameter Δ:

$$\alpha = \frac{\omega}{(\omega^2+|\Delta|^2)^{1/2}},\quad \beta = \frac{\Delta}{(\omega^2+|\Delta|^2)^{1/2}},\quad \beta^* = \frac{\Delta^*}{(\omega^2+|\Delta|^2)^{1/2}}. \quad (25)$$

To calculate the integral (14) it is necessary to find the function f on the imaginary axis of the variable ω. The function f has different forms in the regions $|i\omega| > |\Delta|$ and $|i\omega| < |\Delta|$. When the condition (24) is satisfied, the main contribution to the integral (14) is made by the frequency and coordinate region in which $|i\omega| > |\Delta|$.

Let us calculate the trace of Eqs. (2) with τ_z in this frequency region. Taking (25) and (3) into account, we obtain

$$f_1/\Delta = f_2/\Delta^* = [(\omega+\delta)^2 + |\Delta|^2]^{-1/2}\psi,$$
$$D\frac{\partial^2}{\partial \mathbf{r}^2}\psi = 2i\mathbf{b}\frac{\partial}{\partial \mathbf{r}}((\omega+\delta)^2+|\Delta|^2)^{1/2}. \quad (26)$$

Equation (26) was derived by Gor'kov and Kopnin[11] by another method when solving the problem of the motion of one vortex in a constant electric field. The contribution made to the integral (14) by the frequency region $|\omega| > |\Delta|_{max}$ can be obtained by the same method as in the already considered case of magnetic impurities. The region $|\omega| < |\Delta|_{max}$, however, calls for a more detailed study.

To obtain the boundary conditions for (26), it is necessary to consider the region near the threshold $|\omega| = |\Delta|$, where formulas (25) and (26) do not hold. It follows from (7) that the dimension of this region is of the order of $\delta r \sim \xi(eH_{c2}D/\Delta)^{2/5}$. When the condition (24) is satisfied, the dimension of δr is small in comparison with ξ, so that the transition region makes a small contribution to the integral (14), and the boundary condition of Eq. (26), accurate to terms of order $\delta r/\xi$, takes the form

$$\psi = 0 \quad \text{for } |\omega| = |\Delta|. \quad (27)$$

It follows from (2) that the functions f_1 and f_2 decrease rapidly in the region $|\omega| < |\Delta|$ and this region makes a small contribution to the integral (14). It therefore suffices to consider in the unit cell a region S in which $|\omega| > |\Delta|$. On the boundary of the region S, which coincides with the cell boundary, the boundary conditions for Eq. (26) are periodic.

Averaging (15) over the directions of the electric field, we obtain

$$\frac{\sigma_{\text{eff}}}{\sigma} - 1 = \frac{\langle|\Delta|^2\rangle}{4e^2D^2\langle H\rangle^2 T}\int_0^\infty d\omega\, \Phi\left(\frac{\omega}{\langle|\Delta|^2\rangle^{1/2}}\right), \quad (28)$$

where

$$\Phi\left(\frac{\omega}{\langle|\Delta|^2\rangle^{1/2}}\right) = -\frac{1}{\langle|\Delta|^2\rangle}\frac{1}{V} \times \iint_S d^2\mathbf{r}\, d^2\mathbf{r}_1 K(\mathbf{r},\mathbf{r}_1)\left(\frac{\partial\sqrt{\omega^2-|\Delta(\mathbf{r})|^2}}{\partial \mathbf{r}}\frac{\partial\sqrt{\omega^2-|\Delta(\mathbf{r}_1)|^2}}{\partial \mathbf{r}_1}\right) \quad (29)$$

In (29), $K(\mathbf{r}, \mathbf{r}')$ is the Green's function of the Laplace operator for the region S satisfying the same boundary conditions as the function ψ; V is the area of the unit cell.

At $\omega > \Delta_{max}$, the region S coincides with the entire cell. In this case, the Green's function K in (29) can be replaced by the Green's function for a plane, and the integration with respect to $\mathbf{r}_1$ can be extended over the entire plane. Integrating (29) by parts, we obtain

$$\Phi = \langle((\omega^2-|\Delta|^2)^{1/2} - \langle(\omega^2-|\Delta|^2)^{1/2}\rangle)^2\rangle / \langle|\Delta|^2\rangle. \quad (30)$$

At $\omega \ll \Delta_{max}$ in the region S, the value of $|\Delta|$ depends only on $\mathbf{r}^2$. Therefore Eq. (26) can easily be integrated, and we obtain for Φ the expression

$$\Phi = \frac{1}{\langle|\Delta|^2\rangle}\frac{1}{V}\int_S d^2\mathbf{r}\left[(\omega^2-|\Delta|^2)^{1/2} - \frac{1}{V_S}\int_S d^2\mathbf{r}_1(\omega^2-|\Delta(\mathbf{r}_1)|^2)^{1/2}\right]^2, \quad (31)$$

V_S is the area of the region S.

At $H \ll H_{c2}$, the order parameter Δ depends only on $\mathbf{r}^2$ and formula (31) is valid for all ω. In this case formula (28) leads to the expression obtained for the conductivity by Gor'kov and Kopnin[11].

In fields close to H_{c2}, we use for $|\Delta|^2$ the expression[12]

$$|\Delta|^2 = \langle|\Delta|^2\rangle\left\{1 - \frac{1}{3}\left[\cos\frac{2\pi x}{a} + \cos\frac{2\pi y}{a} + \cos\frac{2\pi(x+y)}{a}\right]\right\}, \quad (32)$$

where a is the length of the edge of the cell and x and y are coordinates in an oblique system with axes directed along the sides of the cell. From (30)–(32) we can obtain the function Φ in the limiting cases of large and small ω:

$$\Phi(x)=\begin{cases}\dfrac{1}{24x^2}\left(1+\dfrac{5}{6x^2}\right), & x\gg 1,\\[2mm] \dfrac{x^4}{12\sqrt{3}\pi}\left(1+\dfrac{3x^2}{10}\right), & x\ll 1.\end{cases}\tag{33}$$

When calculating the integral in (28), we shall use the limiting expressions (33) in the entire frequency region. The results will be matched at a point where the limiting expressions for Φ coincide. As a result we obtain for the integral a value $0.051\,\langle|\Delta|^2\rangle^{1/2}$. Thus, in the region $1 \gg 1 - H/H_{c2} \gg (1 - T/T_c)$ the conductivity of the superconductor is equal to

$$\frac{\sigma_{\text{eff}}}{\sigma}=1+0{,}18\left(\frac{T_c}{T_c-T}\right)^{1/2}\left(\frac{1-H/H_{c2}}{1-{}^1/_2\varkappa^2}\right)^{3/2}.\tag{34}$$

At near-critical temperature, the second term in this formula can become larger than unity. We note that formula (34) is also valid for alloys with magnetic impurities, if the condition $\tau_s\Delta \gg 1$ is satisfied.

6. CONCLUSION

Near the transition temperature, at fields close to H_{c2}, there exist several regions with different dependences of the resistance on the magnetic field and on the temperature:

$$\frac{\sigma_{\text{eff}}}{\sigma}-1=\begin{cases}5x, & \text{I}\\ 2{,}5x, & \text{II}\\ 0{,}32\,(\tau_s T)\,x^2 & \text{III}\\ 0{,}2\,(1-T/T_c)^{-1/2}x^{3/2}, & \text{IV}\end{cases},$$

where $x = (1 - H/H_{c2})/[1 - (\kappa^2/2)]$. We have $x < (1 - T/T_c)$ and $\tau_s(T_c - T) \gg 1$ in region I; $x < (\tau_s T)^{-1}$ and $\tau_s(T_c - T) \ll 1$ in region II, $(\tau_s T)^{-1} < x < [\tau_s^2 T_c(T_c - T)]^{-1}$ in region III, and $x > (1 - T/T_c)$, $[\tau_s^2 T_c(T_c - T)]^{-1}$ in region IV. In the immediate vicinity of H_{c2}, the resistance depends linearly on the magnetic field[1-3]. With decreasing magnetic field or with decreasing temperature at a constant magnetic field, the growth of the conductivity becomes faster and is described by formulas (23) and (34). We note that to observe this effect it is necessary to have temperatures close to T_c, since the numerical coefficient in the Maki-Thompson formula is relatively large. At these temperatures the conductivity of the superconductor can also be significantly larger than the conductivity of the normal metal in fields close to H_{c2}, when $x \ll 1$.

At $x \sim 1$, formulas (35) are continuations, in order of magnitude, of the results obtained by Gor'kov and Kopnin[10,11] for fields close to H_{c1}.

The conductivity of the superconductor is sensitive to the concentrations of the magnetic impurities at $\tau_s T \gg 1$, when the thermodynamic quantities depend little on τ_s. Therefore formulas (35) can be used for an experimental determination of τ_s; this may be useful for the interpretation of experiments on fluctuation conductivity.

The results obtained above (formulas (22), (23), (28), (34)) are valid not only for bulky samples, but also for films of arbitrary thickness in a perpendicular magnetic field. For thin films ($d \ll \xi$) in an oblique field, the longitudinal component of the magnetic field has the same effect as magnetic impurities. It is then necessary to replace τ_s^{-1} in all formulas by $\tau_s^{-1} + e^2H_{\parallel}^2Dd^2/6$.

Formulas (23) and (34) are valid for frequencies satisfying the condition

$$\omega \ll (eHD/\Delta)^2 \max\{\Delta, \tau_s^{-1}\}.$$

At these frequencies, the vortex lattice moves mainly as a unit, and Eq. (10) can be solved by iteration with respect to ω_0.

In the derivation of (23) and (34) it was assumed that the amplitude of the lattice vibrations is small in comparison with its period. This limits the region of applicability of (23) and (34) on the low-frequency side. It seems natural to us that in weak electric fields formulas (23) and (34) should be valid for arbitrarily low frequencies, but a limit exists on the amplitude of the electric field:

$$E \ll H\xi \max\{\omega, \tau_\epsilon^{-1}\},$$

where τ_ϵ is the energy relaxation time. On the other hand, the electric field should be strong enough to be able to disregard the pinning forces.

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*$A_1\,[Hb] \equiv A_1 \cdot H \times b$.

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