Excitation of oscillations of charge carrier density in a tunnel junction

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Inelastic scattering of tunneling electrons in a metal-insulator-semiconductor system by volume plasma oscillations is taken into account. General expressions are obtained for the interference and inelastic contributions to the tunnel current. It is shown that information on some features of the plasmon spectrum can be derived from the second derivative of the current with respect to voltage.

It is known that consistent consideration of the long-range Coulomb interaction between electrons not only strongly influences single-particle behavior, but also leads to the appearance of excited states that can be interpreted as bound states of a quasiparticle and a quasihole. An example of such excitations is found in plasma waves—relative oscillations of regions with excesses and deficiencies of electrons. In principle, it is sufficient to disturb the local-concentration equilibrium in the carrier gas in order to excite such oscillations. Thermal excitation of plasmons (plasma-oscillation quanta) and excitation at the expense of the energy of individual electron motions are not possible in the long-wave limit in the electron plasma of a solid, since this requires an energy much larger than the energy of the electron (see, for example,[1]). This condition is easily met if we consider the passage of an extraneous charged particle through the plasma. The experimentally measured energy losses of fast electrons after their passage through a thin film or after reflection from the surface of a solid constitutes the basic proof of the existence of plasmons.

It is interesting that a similar situation can also be realized in a tunnel junction, when the injected electron has sufficient excess energy that can be given up to the collective oscillations. It is characteristic that this introduces not fast particles, but energy into the sample. Therefore information on the energy spectrum of the collective excitations in solids can also be obtained from the tunnel effect. If we consider the tunneling of electrons neglecting their electrostatic interaction with one another, then the mechanism of the release or absorption of the energy in tunneling can be, for example, absorption or emission of a quantum of lattice vibrations. This circumstance becomes appropriately manifested in the current-volatge characteristic of the junction[2,3]. It is possible to investigate analogously the channel where tunneling with participation of plasmons takes place. It is necessary, however, to separate the self-energy effects from effects connected with inelastic scattering of tunneling electrons with emission (absorption) of plasma oscillations. The tunnel current of "dressed" (as a result of interaction with plasmons) electrons was calculated by Duke et al.[4]. The excess current connected with the inelastic tunneling of the electrons exciting the surface plasmons in a metal-insulator-semiconductor (MIS) junction has been determined in[5].

In the present paper we calculate the I(V) dependence (I is the tunnel current and V is the junction voltage) with allowance for the generation of long-wave volume oscillations of the carrier density in the superconducting electrode by the tunneling electrons. It should be noted that the contribution made to the current by the generation of the volume plasmons is at least of the same order as the self-energy effects considered in[4], but the shape of the $d^2I/d(eV)^2$ curve in the vicinity of $eV \sim \omega_p^S$ (ω_p^S is the plasma frequency) is entirely different. It is possible that this may be the cause of the discrepancy between theory and experiment discussed by Duke et al.[4].

GENERAL RELATIONS

We consider an MIS tunnel junction. We assume that the metallic electrode occupies the half-space $z < -d/2$, and the semiconductor the half-space $z > d/2$ (d is the thickness of the insulating layer, and the z axis is perpendicular to the plane of the junction).

For electrons situated in the periodically spatially-varying field of the ion cores, the Hamiltonian takes the usual form

$$H=\sum_i\left[\frac{p_i^2}{2m}+U(\mathbf{r}_i)\right]+\frac{1}{2}\sum_{i\neq j}V(\mathbf{r}_i;\mathbf{r}_j), \qquad (1)$$

where the function $U(\mathbf{r}_i)$ determines the periodic potential of the field of the ion cores of the junction, and the second term takes into account the energy of the Coulomb interaction of the electrons with one another.

According to[6], it is possible to introduce the particle-field operators $\Psi_\alpha^+(\mathbf{r})$ and $\Psi_\alpha(\mathbf{r})$, expressed in the form of the sum of the operators $\{\Psi_{1\alpha}^+;\ \Psi_{2\alpha}^+\}$ and $\{\Psi_{1\alpha};\ \Psi_{2\alpha}\}$, which are responsible for the creation (annihilation) of the particle at a point r with spin projection α in the right-hand or left-hand electrode, respectively. Consequently, in terms of the operators $\{\Psi_{i\alpha}^+,\ \Psi_{1\alpha}\}$ ($i = 1, 2$), that part of the Hamiltonian (1) which describes the Coulomb interaction of the electrons in the tunnel junction breaks up into usual Hamiltonians of the two-particle interaction in the left-hand and right-hand electrodes, the energy operator of the electrostatic interaction between the charge densities of the electrodes, and the increment to the single-particle tunnel Hamiltonian

$$H_1'=\sum_{\alpha,\beta}\int d\mathbf{r}\{\Psi_{1\alpha}^+(\mathbf{r})\Psi_{2\alpha}(\mathbf{r})C_{1\beta}(\mathbf{r})+\Psi_{2\alpha}^+(\mathbf{r})\Psi_{1\alpha}(\mathbf{r})C_{2\beta}(\mathbf{r})\}+\text{H.c.},$$

which is responsible for the inelastic tunneling with excitation of the carrier density[1)],

$$C_{i\beta}=\int d\mathbf{r}'\,\rho_{i\beta}(\mathbf{r}')V(\mathbf{r},\mathbf{r}'),\qquad \rho_{i\beta}(\mathbf{r})=\Psi_{i\beta}^+(\mathbf{r})\Psi_{i\beta}(\mathbf{r}).$$

When the tunnel junction is connected to an external circuit, an electric field is produced near the barrier. This field is easiest to calculate by modifying the matrix elements in the tunnel Hamiltonian[7,8]. Introducing the phase shift $\varphi(t)$, which is connected with the electric field $\mathbf{E}$ in the barrier by the equation

$$\frac{\partial \varphi}{\partial t} = e\int_1^2 E\,dz,$$

we can write the resultant tunnel Hamiltonian $\hat{T}$ in the form

$$\hat{T} = H_T \exp[-i\varphi(t)] + \text{H.c.},$$
$$H_T = H_0 + H_1 = \sum a^+_{p\alpha i} b_{g\alpha}\left\{T^{(i)}_{pg} + \int d\mathbf{r}\, V^{(i)}_{pg}(\mathbf{r})\left[\rho(\mathbf{r}) - \rho_0\right]\right\}, \quad (2)$$

where the electron density operator is $\rho = \rho_{1,1} + \rho_{2,2}$ ($\rho_{1,1}$ and $\rho_{2,2}$ are the operators of the electron density in the metal and in the semiconductor) and ρ_0 is the average density connected with allowance for the effect of the homogeneous positively-charged background:

$$T^{(i)}_{pg} = \frac{1}{2\sqrt{mm_i}}\int d\mathbf{r}_\perp \left(\chi^{+*}_{pi}\frac{\partial \chi_g^-}{\partial z} - \chi_g^- \frac{\partial \chi^{+*}_{pi}}{\partial z}\right)\Big|_{z=0};$$
$$V^{(i)}_{pg} = \int d\mathbf{r}\, V(\mathbf{r},\mathbf{r}')\chi^{+*}_{pi}(\mathbf{r})\chi_g^-(\mathbf{r}).$$

In re ation (2) we take into account the fact that the state of an electron moving in a periodic field is characterized not only by the wave vector $\mathbf{p}$, but also by the number i of the allowed energy band. When considering the metallic electrodes, we are interested in the motion of the electron within a single energy band. In the case of the semiconducting electrode, interest attaches to the case when two bands are significant, the valence band and the conduction band. Accordingly, $b_{g\alpha}$ is the annihilation operator for a particle with quasi-momentum $\mathbf{q}$ and spin α in the metallic electrode, $a^+_{p\alpha i}$ is the operator of electron production in the semiconductor, m is the mass of the electron, m_i is the effective mass of the electron in the semiconductor (the subscript i, which indicates the number of the band, will henceforth be omitted for the sake of brevity). The states $\{\chi^\pm_p\}$ are single-particle wave functions for electrons incident from the right (+) and from the left (-) on the barrier.

In the calculation of the tunnel current I, we use the definition of I in terms of the variational derivative of the mean value of the Hamiltonian $\hat{T}$ with respect to $\varphi(t)$

$$I = e\delta\langle\hat{T}\rangle / \delta\varphi(t).$$

After subtracting (2), putting $\varphi = eVt$, we obtain

$$I = -2e\,\mathrm{Im}\int_{-\infty}^{\infty} d\tau\, e^{ieV\tau}\{i\theta(\tau)\langle[H_T^+(\tau)H_T(0)]_-\rangle\}. \quad (3)$$

The angle brackets denote here averaging over the equilibrium ensemble of non-interacting subsystems, $H_T(\tau)$ is the operator H_T in the interaction representation, and $\theta(\tau)$ is a step function.

We express the kernel of the integrand in (3) in a more expanded form:

$$i\theta(t)\langle[H_T^+(t)H_T(0)]_-\rangle = K(t) = K_{00}(t) + K_{10}(t) + K_{01}(t) + K_{11}(t). \quad (4)$$

The meaning of (3) is then obvious. The term with K_{00} in (4) leads to the usual tunnel current I_{00}. The increments containing K_{10} and K_{01} are the result of interference between the different tunneling mechanisms (elastic and inelastic). The current contribution due to these terms will be called the interference contribution and designated I_{10}. Finally, the last term of (4) leads to a purely inelastic contribution to the tunnel current, with plasma excitation (I_{11}). The currents I_{10} and I_{11} are due to allowance for the long-range Coulomb interaction. We emphasize that the bare electron-interaction potential $V(\mathbf{r}_i;\mathbf{r}_j)$ in Eq. (1) is assumed to be unscreened. The final expressions for these terms, however, contain, as shown by subsequent calculations, only the screened potential with allowance for the delay.

INTERFERENCE AND INELASTIC CONTRIBUTIONS TO THE TUNNEL CURRENT

On changing over to Fourier components, we obtain in place of (3) the following relation for the current:

$$I = -2e\,\mathrm{Im}\,K(eV),$$

where the role of the function $K(\omega)$ is assumed by the expression

$$K(\omega) = Q^{-1}\sum_{m,n}\frac{P_{mn}}{\omega + E_n - E_m + i\delta}, \quad (5)$$

with $P_{mn} = |T_{mn}|^2[\exp(-\beta E_m) - \exp(-\beta E_n)]$. Here β is the reciprocal temperature, T_{mn} is the matrix element of the operator $\hat{H}_T$ between states with energy E_m and E_n, and Q is the partition function.

It can be shown in general form (see, for example,[9]) that the function $K(\omega)$ defined by Eq. (5) and the Fourier component $K^c(\omega_0)$ of the causal Green's function, considered on the imaginary time axis from $i\beta$ to $-i\beta$ (the frequencies ω_0 run through the discrete values $\omega_0 = 2i\pi n\beta^{-1}$) are values of one and the same function of the complex variable ω, the function analytic in the upper half-plane; these values are taken on the real axis in the former case and at the points $\omega = \omega_0$ in the latter. Therefore, calculating the function $K^c(\omega_0)$ and continuing it analytically to the real frequencies in such a way that the resultant function has no singularities in the upper half-plane ω, we can find the function $K(\omega)$ that determines the tunnel current.

With this purpose in mind, let us determine the Fourier expansion of the Green's function $K^c(t)$

$$K^c(t) = i\beta^{-1}\sum_{\omega_0} e^{-i\omega_0 t}K^c(\omega_0) = \frac{1}{i}\langle TH_T^+(t)H_T(0)\rangle,$$
$$K^c(\omega_0) = \int_0^{-i\beta} dt\, e^{i\omega_0 t}K^c(t). \quad (6)$$

Here T is the operator of ordering along the imaginary axis, $-\beta \le it \le \beta$.

Calculating $K_{00}(eV)$ in this manner, we obtain the usual expression for the elastic contribution to the tunnel current[10]

$$I_{00} = e\sum_{\mathbf{p},\mathbf{g}}|T_{pg}|^2\int\frac{d\omega_1\, d\omega_2}{2\pi}A^M(\mathbf{g},\omega_1)A^S(\mathbf{p},\omega_2)\left[f(\omega_1) - f(\omega_2)\right] \times \delta(\omega_1 - \omega_2 + eV), \quad (7)$$

where $A^M(\mathbf{g},\omega)$ and $A^S(\mathbf{p},\omega)$ are the spectral intensities for the metal and semiconductor, respectively, and $f(\omega)$ is the Fermi distribution function. It is important that the self-energy effects are included in the spectral intensities in (7). And since they have been considered by Duke et al.[4], we shall not deal with them in the calculation that follows.

Before we proceed to finding relations for the interference and inelastic contributions to the current-voltage characteristic of the junction, we note that we are considering in the present paper only the case when $eV \ll \omega_p$ (ω_p is the plasma frequency of the metal, $\omega_p \gg \omega_p^S$). We therefore do not take into account the corresponding contribution made to I_{10} and I_{11} by the interaction of electrons with the plasmons in the metal. Allowance for this interaction leads only to a screening of the Coulomb potential in the metallic electrode without delay, and consequently simply to a renormalization of the matrix elements T_{pg}. This can readily be seen from the obtained relations for I_{10} and I_{11}. We note also an equation that will be of use later on. Namely, it is easy to show from the definition of $K(\omega)$ that

$$K_{01}^*(\omega+i\delta)=K_{10}(\omega-i\delta). \qquad (8)$$

Because of this equation, it suffices to consider only the term with K_{10}.

1) Using the definition of the operators H_1 and H_0, we express $K_{10}^c(t)$ in terms of the single-particle Green's functions of the metal and semiconductor electrons:

$$K_{10}^c(t)=\sum_{p,g}A_{pg}\int_0^{-i\beta}d\tau\, G^M(g;-t)G^S(p,\tau)\Sigma^S(p;t-\tau); \qquad (9)$$

here

$$A_{pg}=T_{pg}\tilde{T}_{pg}^*, \qquad \tilde{T}_{pg}^*=\int dr\,\chi_p^+(r)\chi_g^{-*}(r).$$

The function $\Sigma^S(p;t)$ is the self-energy part of the electrons in the semiconductor electrode.

We take the Fourier transform of (9):

$$K_{10}^c(\omega_0)=-\beta^{-1}\sum_{p,g,\omega_0'}A_{pg}G^M(g;\omega_0'-\omega_0)G^S(p;\omega_0')\Sigma^S(p;\omega_0'), \qquad (10)$$

the frequency ω_0' assumes here only the odd values $i(2n+1)\pi\beta^{-1}$, while ω_0 assumes the even values $2in\pi\beta^{-1}$.

The self-energy part $\Sigma^S(p;\omega_0')$ breaks up into two parts:

$$\Sigma^S(p;\omega_0')=-\beta^{-1}\sum_{\bar{\omega}_0}\int\frac{dk}{(2\pi)^3}V_s(k,\bar{\omega}_0)\cdot G^S(p-k,\omega_0'-\bar{\omega}_0) = \Sigma_0^S(p;\omega_0')+\Sigma_1^S(p;\omega_0'). \qquad (11)$$

One part of the self energy, $\Sigma^S(p;\omega_0')$, is connected with the exchange:

$$\Sigma_0^S(p;\omega_0')=-\int\frac{dk\,d\omega}{(2\pi)^4}V_s(k;\omega_0'-\omega)f(\omega)A^S(p-k;\omega). \qquad (12)$$

The other part Σ_1^S of the self-energy Σ^S is due to collective processes in the system. We can write for Σ_1^S the expression

$$\Sigma_1^S(p;\omega_0')=2\int\frac{dk\,d\omega\,d\omega_1}{(2\pi)^5}\frac{\nu(-\omega)A^S(p-k;\omega_1)}{\omega_0'-\omega-\omega_1}\operatorname{Im}V_s(k,\omega), \qquad \nu(\omega)=[\exp(\beta\omega)-1]^{-1}. \qquad (13)$$

The definitions (12) and (13) contain the screened Fourier transform of the Coulomb potential

$$V_s(k,\omega)=V(k)\varepsilon^{-1}(k,\omega), \qquad V(k)=4\pi e^2/k^2.$$

Here $\epsilon(k,\omega)$ is the dynamic dielectric constant.

Summing in the usual manner over ω_0' in the expression for $K_{10}^c(\omega_0)$ and making an analytic continuation to the real frequencies with allowance for the equation (8) for the interference contribution made to the tunnel current, we obtain the following relation:

$$I_{10}=-4e\sum_{p,g}\operatorname{Re}A_{pg}\int\frac{d\omega_1\,d\omega_2}{(2\pi)^2}A^M(g;\omega_1)A^S(p;\omega_2)[f(\omega_1)-f(\omega_1+eV)] \times\operatorname{Im}\frac{\Sigma(p;eV+\omega_1+i\delta)}{eV+\omega_1-\omega_2+i\delta}. \qquad (14)$$

2) Repeating the reasoning that had led to expression (9), we obtain for K_{11}^c during the intermediate stage

$$K_{11}^c(t)=(-i)\int\prod_{n=1}^4 dr_n\,V(r_1;r_2)V(r_3;r_4) \times G^M(r_3;r_4;-t)\langle T\delta\rho(r_2;t)\Psi(r_1;t)\Psi^+(r_3;0)\delta\rho(r_4;0)\rangle, \qquad (15)$$

where $\delta\rho$ is the operator of the deviation of the electron-gas density from its equilibrium value ρ_0.

We transform the function (15), using the method described in[11] to obtain approximations for the function g. To this end, we generalize in purely formal fashion the definitions of the two-particle and single-particle Green's function

$$K(r_1;r_2;r_3;t;U)=\frac{\langle TS\delta\rho(r_2;t)\Psi(r_1;t)\Psi^+(r_3;0)\rangle}{\langle T,S\rangle},$$

$$G(r_1;r_2;t;U)=\frac{\langle TS\Psi(r_1;t)\Psi^+(r_2;0)\rangle}{\langle TS\rangle}.$$

The operator S is defined here as follows:

$$S=\exp\Big[-i\int_0^{-i\beta}dr\,dt\,U(r;t)\delta\rho(r;t)\Big],$$

where $U(r,t)$ is a function of space and time. By considering the change of $K(U)$ following a infinitesimally small change of the potential U, we can express the three-particle Green's function in the definition (15) in terms of the variational derivative of $K(U)$ with respect to the potential U. In this case, Eq. (15) reduces to the form

$$K_{11}^c(t)=i\int\prod_{n=1}^4 dr_n\int_0^{-i\beta}d\tau\,V(r_3;r_4) \times C^M(r_3;r_4;-t)\Sigma^S(r_1;r_2;t-\tau)\frac{\delta G^S(r_2;r_3;\tau;U)}{\delta U(r_4;0^-)}. \qquad (16)$$

In the derivation of this relation we also used the smallness of the term that contains the variational derivative $\delta\Sigma/\delta U$. The term with $\delta\Sigma/\delta U$ is smaller by a factor $\lambda=3/4\pi\rho_0 r_D^3$ than the term taken into account in (16) (r_D is the Debye screening radius) in the case of Boltzmann's statistics, and is smaller by a factor $r_S = me^2(3/4\pi\rho_0)^{1/3}$ for Fermi statistics. Consequently, the approximation data are suitable only if $\lambda \ll 1$ or $r_S \ll 1$.

We recognize further that the particles in the Coulomb system move in such a way that they produce a screening field. As shown in[11], in such a system one can regard $K(U)$ as a functional of the effective potential field U_S. Then, characterizing the degree of attenuation of the effective field with the aid of the dielectric function of the reaction $\epsilon^{-1}(r,t;r',t')=\delta U_S(r,t)/\delta U(r',t')$, we obtain for the Fourier component of the Green's function $K_{11}^c(t)$, at the assumed accuracy[2)],

$$K_{11}^c(\omega_0)=-\beta^{-1}\sum_{p,g,\omega_0'}|\tilde{T}_{pg}|^2G^M(g;\omega_0'-\omega_0)G^S(p;\omega_0')\Sigma^S(p;\omega_0')\bar{\Sigma}^S(p;\omega_0'),$$

where

$$\bar{\Sigma}^S(p;\omega_0')=-\beta^{-1}\sum_{\bar{\omega}_0}\int\frac{dk}{(2\pi)^3}V_s(k,\bar{\omega}_0)G^S(p+k;\omega_0'+\bar{\omega}_0).$$

After performing the operation of summation and analytic continuation of the function $K_{11}^c(\omega_0)$, the formula for the inelastic contribution to the current-voltage characteristic is expressed in terms of the spectral intensities:

$$I_{11}=-2e\sum_{p,g}|\tilde{T}_{pg}|^2\int\frac{d\omega_1\,d\omega_2}{(2\pi)^2}A^M(g;\omega_1)A^n(p;\omega_2)[f(\omega_1)-f(\omega_1+eV)]$$

$$\times \operatorname{Im} \frac{\Sigma(\mathbf{p}; eV+\omega_1+i\delta)\bar{\Sigma}(\mathbf{p}; eV+\omega_1+i\delta)}{eV+\omega_1-\omega_2+i\delta}, \quad (17)$$

with

$$\bar{\Sigma}(\mathbf{p}; \omega+i\delta) = \Sigma_0^*(\mathbf{p}; \omega+i\delta) + \Sigma_1(\mathbf{p}; \omega+i\delta).$$

It is interesting to note that the pure-exchange part in (17) leads only to a renormalization of the matrix element T_{pg} in the elastic current (7).

In the direct calculation of the expressions obtained for the current, we confine ourselves in the present paper to the case when the electrode is an intrinsic semiconductor.

We note that if we take the boundary into account in the calculation of Σ^S, then surface charge oscillations can be generated in addition to volume plasmons. The contribution of these oscillations is additive. Since the manifestation of surface plasmons on the current-voltage characteristic of a junction has been considered by Ngai and Economou[5], we shall not take them into account. We use later on for Σ^S an expression corresponding to a bulky semiconductor.

PLASMA WAVES IN A TUNNEL JUNCTION

At sufficiently low temperatures (such that the inequality $\beta^{-1} \ll \Delta/\ln(m_2/m_1)$ is satisfied, where Δ is the gap in the energy spectrum of the semiconductor and m_1 and m_2 are the effective masses of the electrons and holes), the Fermi level ϵ_F of an impurity-free semiconductor lies in the middle of the forbidden band. In the calculation of relations (14) and (17), we can assume that the functions $A^M(g; \omega)$ and $A^S(p; \omega)$ are the spectral intensities of free particles[3)]. In addition, we assume that the electronic excitations in the two electrodes of the junction are described by a quadratic dispersion law. Then

$$A^M(g;\ \omega) = 2\pi\delta(\omega - g^2/2m + \varepsilon_F), \quad A_i^S(p;\ \omega) = 2\pi\delta[\omega + (-1)^i(p^2/2m_i + \Delta/2)],$$

where i indicates the number of the band, i = 1 and 1 = 2 standing for the conduction and valence bands, respectively.

Introducing the summation over the bands in explicit fashion, we obtain for the current I_{10}, after integrating with respect to $d\omega_1$ and $d\omega_2$ in the definition (14),

$$I_{10}(eV) = \sum_i \frac{2eN(0)}{\pi^2} A_i (2m_i)^{1/2} \int_{-\infty}^{\infty} d\xi [f(\xi) - f(\xi+eV)] \int_0^{\infty} d\varepsilon\, F_i(\xi; \varepsilon; \sqrt{\varepsilon}; eV),$$

$$F_i(\xi; \varepsilon; \sqrt{\varepsilon}; eV) = \operatorname{Im} \sqrt{\varepsilon} \frac{\Sigma(\sqrt{2m_i\varepsilon}; eV+\xi+i\delta)}{(-1)^i(\varepsilon+\Delta/2)+eV+\xi+i\delta} \quad (18)$$

Here N(0) is the density of states on the Fermi surface for the metallic electrode and A_i are the matrix elements $\operatorname{Re} A^i_{pg}$ averaged over the angles of the vectors p and g. As usual, we have assumed that the dependences on p and g are slow, and have taken some average value A; outside the integral sign.

The result (18) for I_{10} breaks up into two terms, in accord with the resolution (11) for Σ^S. One of them is due to the nonzero exchange energy Σ_0^S, and the second is connected with the collective excitations in the tunnel junction. It should be noted that the logarithmic divergence that arises as $\beta \to \infty$ does not occur in our case when the exchange energy is taken into account, since the definitions (14) and (17) contain the effective screened potential. We shall henceforth assume that the function Σ^S in (18) does not contain an exchange part without allowance for the delay, since this part of the self-energy leads only to a renormalization of the elastic current.

The additional channel for tunneling with participation of the plasmons, like all other channels for inelastic tunneling, leads to kinks on the current-voltage characteristic of the junction. These kinks are observed at voltages eV near the singularities of the density of states of the collective excitations, and are most clearly pronounced in the form of bursts on the second derivative of I(eV) with respect to the voltage. We shall therefore not determine I(eV) directly. It is easier to calculate $d^2I/d(eV)^2$. We note to this end that the integrand in (18) can be expressed either in terms of the parameter $\xi + eV$ or in terms of the parameter $(-1)^i\epsilon + eV$. Using these properties (one in the first differentiation and the other in the second), we obtain for the second derivative with respect to voltage the simple equation

$$G_{10} = \frac{d^2 I_{10}}{d(eV)^2} = -\frac{eN(0)}{\pi^2} \sum_i A_i (2m_i)^{1/2} \int_{-\infty} d\xi \left[-\frac{\partial f(\xi)}{\partial \xi} \right] W(\xi),$$

$$W(\xi) = \int_0^{\infty} \frac{d\varepsilon}{\sqrt{\varepsilon}} \operatorname{Im} \frac{\partial F_i(\xi; \varepsilon; \sqrt{\varepsilon}; eV)}{\partial \sqrt{\varepsilon}} = (-1)^{i+1} \frac{2\sqrt{2m_i}}{\pi} e^2 \cdot \quad (19)$$

$$\cdot \operatorname{Im} \int_0 dz \left\{ \varepsilon^{-1} \left[\sqrt{2m_i} z; eV + \xi + (-1)^i \left(z^2 + \frac{\Delta}{2} \right) + i\delta \right] \delta_{2i} - \int \frac{d\Omega}{\pi} \frac{\nu(-\Omega) \operatorname{Im} \varepsilon^{-1}(\sqrt{2m_i} z; \Omega + i\delta)}{eV + \xi - \Omega + (-1)^i (z^2 + \Delta/2) + i\delta} \right\} \quad (20)$$

We introduce additional simplifications in (20). To this end, we recognize that the values of eV of interest to us are of the order of ω_p^S, and that in this region the spatial dispersion of the dielectric constant is small. We therefore neglect the dependence of ϵ on k, taking into account only the optical branch of the plasmons. The contribution connected with the acoustic plasma oscillations is small to the extent that the ratio m_1/m_2 is small.

The balance of the calculations is governed by the resonant character of the behavior of the functions $W(\xi)$ and $-(\partial f/\partial\xi)$. The former have a clearly pronounced character near the energy $\xi = -eV + \omega_p^S - (-1)^i\Delta/2$ (the degree of diffuseness of the function $W(\xi)$ is ν, where ν is the effective number of collisions and is connected with the experimentally observed mobility by the equality $\mu = e/m_1\nu$), while the function $(-\partial f/\partial\xi)$ has a maximum characterized by a width β^{-1} at the point $\xi = 0$.

The resonant behavior of the functions $W(\xi)$ and $(-\partial f/\partial\xi)$ enables us to find the variation of G_{10} in two limiting cases:

1) The degree of diffuseness of the function $W(\xi)$ is much larger than the width of the diffuseness of the function $(-\partial f/\partial\xi)$ $(\beta^{-1} \ll \nu)$.

In this limit, we obtain after integrating in (19)

$$G_{10} = \frac{e^3 N(0)\omega_p{}^S}{\pi\varepsilon_0} \sum_i A_i (-1)^{i+1} (2m_i)^2 C_T \times \left[(-1)^{i+1} eV - \frac{\Delta}{2} \right] C_\nu \left[\omega_p{}^S + \frac{\Delta}{2} + (-1)^i eV \right]. \quad (21)$$

We have introduced here the notation

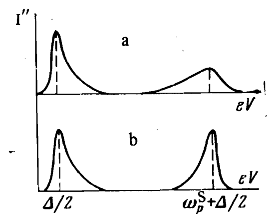
$$C_T(z) = B_1(\beta z)\sqrt{\beta}, \quad C_\nu(z) = B_2(2\nu^{-1}z)\nu^{-1/2},$$

$$B_1(x)=\int_0^\infty dy[1+\mathrm{ch}(y^2-x)]^{-1}, \qquad B_2(x)=\frac{(\sqrt{1+x^2}+x)^{1/2}}{\sqrt{1+x^2}}. \quad (22)$$

As seen from the obtained relations (21) and (22), a sharp spike is observed on the second derivative I'' of the tunnel current in an MIS system if the junction voltage $eV - \Delta/2$ coincides with the frequency of the plasma oscillations in the semiconductor. The ratio of the intensity of the first term in (21) to the second term depends essentially on the effective masses of the carriers,

$$\alpha \sim (m_1/m_2)^2 \times \exp[-d\sqrt{V(0)-\varepsilon_F}(\sqrt{m_1}-\sqrt{m_2})],$$

where $V(0)$ is the height of the barrier at $z = 0$. For typical experiments, α is much larger than unity. Therefore the shape of the spike is determined completely by the first term. A qualitative plot of the voltage dependence is shown in Fig. 1a.



The physical explanation of the result is that at a voltage $eV \approx \omega_p^S + \Delta/2$ the energy of the tunneling electron is such that it is capable of emitting a plasmon. The source energy is then intensively converted into plasma-oscillation energy. On the other hand, the diffuseness ν of the spike can be attributed to the finite lifetime of the Bose excitation. This manifestation of inelastic tunneling is of undoubted interest in connection with the possibility of investigating the spectra of the plasma oscillations in a semiconductor.

2) The other limiting case corresponds to the inequality $\beta^{-1} \gg \nu$. Now it is the function $W(\xi)$ which has a more sharply pronounced maximum. After integration, we obtain

$$G_{10}=\frac{e^3N(0)\omega_p{}^S}{\pi\varepsilon_0}\sum_i(-1)^{i+1}A_i(2m_i)^2 \times C_T\left[(-1)^{i+1}eV-\frac{\Delta}{2}\right]C_T\left[\omega_p{}^S+\frac{\Delta}{2}+(-1)^i eV\right]. \quad (23)$$

The shape of the spike is fully determined in this case by the behavior of the function $C_T(z)$ (Fig. 1b). It is sharper in comparison with the preceding one.

The contribution from the term I_{11} to the second derivative of the tunnel current is smaller than that of I_{10} by a factor λ. In the case of a symmetrical barrier (both electrodes are made of the same substance), the terms with I_{10} will cancel each other, just like the self-energy effects (see, for example,[10]), so that the main contribution will be due to I_{11}.

We note in conclusion that when comparing the results with the experimental data, it is necessary to subtract from the voltage dependence of the second derivative of the current a monotonic contribution due to the variation of the junction transparency with variation of the voltage. This variation can be neglected only when the inequality $V(0) - \epsilon_F \gg eV$ is satisfied. In the investigation of problems connected with the excitation of plasma oscillations, the significant values are $eV \sim \omega_p^S$, and ω_p^S can be either larger or smaller than $V(0) - \epsilon_F$.

[1]As usual, we omit the terms which are of tunneling order of smallness higher than $D \sim |T_{pg}|^2$, where T_{pg} is the effective interaction matrix element.

[2]Since we are finished with the derivation of the functional derivatives with respect to U and U_s, we must put $U = U_s = 0$.

[3]The bending of the bands of the semiconductor electrode near the junction can be neglected, since the plasmons that play the significant role in the problem have a wavelength l much larger than the distortion region near the junction ($l \gg r_D$).

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113