Electromagnetic excitation of sound in alkali metals located in a magnetic field

É. A. Kaner and V. L. Fal'ko
Institute of Radiophysics and Electronics, Ukrainian Academy of Sciences
(Submitted 12 July 1972)
Zh. Eksp. Teor. Fiz. **64**, 1016-1022 (March 1973)

An exact solution is obtained of the problem of the transformation of an electromagnetic wave into a sound wave on the surface of an alkali metal in a normal magnetic field. It is shown that the amplitude of the transverse sound wave, polarized perpendicularly to the external electric field, depends nonmonotonically on the magnetic field strength. The results of the calculation are in agreement with the experimental data.[1,2]

1. The excitation of sound by an electromagnetic wave in metals at low temperatures has been studied in many experimental researches (the literature is given most completely in the paper of Turner et al.[1]). Investigations have been carried out on various metals (tungsten, silver, aluminum, indium, tin, potassium, and others) both in a constant magnetic field **H** and at $H = 0$. It has been found that in a longitudinal magnetic field, whose direction is the same as the direction of propagation of the wave **k** and is normal to the surface of the sample, a transverse sound wave is excited with amplitude u proportional to the magnetic field H. One should distinguish between two geometries of the experiment: in one the displacement vector in the sound wave is parallel to the electric field $\mathbf{E}_0$ (u_E) and in the other the vector **u** is perpendicular to it (u_H). In a strong magnetic field, when spatial dispersion does not play a significant role and at $kR \ll 1$ (R is the cyclotron radius of the electron), generation of the sound wave is due to the Lorentz force. In this case the sound is polarized perpendicularly to the field $\mathbf{E}_0$ and its amplitude increases linearly with increase in the magnetic field H. In weaker fields, when $kR \gtrsim 1$, the picture is considerably more complicated. The force acting on the lattice as a consequence of the deformation interaction of the conduction electrons with the acoustical vibrations becomes comparable with the Lorentz force. In this region one can expect a departure from linear dependence of the sound amplitude $u \propto H$. Observation of such nonlinear dependence $u_H(H)$ in magnetic fields, where $kR \sim 1$, has been reported by Turner et al.[1] Wallace, Gaerttner and Maxfield,[2] have observed a nonmonotonic variation of the sound amplitude u_H. In both experiments, potassium was studied, which has a spherical Fermi surface. In alkali metals, the interpretation of the results is not complicated by the anisotropy effects that arise in the case of a complicated dispersion law $\epsilon(\mathbf{p})$ for the conduction electrons.

The previously published theoretical papers of Quinn[3] and Kazanov[4] do not explain the experimental results[1,2] and are qualitatively in agreement with them only in the region of strong fields $kR \ll 1$. It is therefore of theoretical interest to explain the nonmonotonic dependence of u(H) by solving the problem of the electromagnetic excitation of sound in a longitudinal magnetic field in a metal with a spherical Fermi surface. In the present work we have succeeded in obtaining theoretically the nonmonotonic character of the sound amplitude $u_H(H)$ and in elucidating the reasons for the differences in the experimental results.[1,2]

2. The complete set of equations that describe the electromagnetic and sound waves in a metal consists of Maxwell's equations, the linearized kinetic equation for the conduction electrons, and the equations of the lattice vibrations. We choose a set of coordinates such that the z axis is directed along the normal to the surface of the metal, which fills the half-space $z > 0$; the vectors **H** and **k** are parallel to the z axis; the y axis is parallel to the displacement vector **u**(z). In the different geometries of the experiment the vector of the external alternating electric field $\mathbf{E}_0$ is directed either along the z axis (the polarization u_H) or along the y axis (the polarization u_H) or along the y axis (the polarization u_E). We have

$$\frac{\partial^2 E_\alpha(z)}{\partial z^2}=\frac{4\pi}{c^2}\frac{\partial j_\alpha(z)}{\partial t}\quad(\alpha=x,y), \tag{1}$$

$$\frac{\partial\chi}{\partial t}+v_z\frac{\partial\chi}{\partial z}+\Omega\frac{\partial\chi}{\partial\tau}+\tau_0^{-1}\chi=g(\mathbf{p},z)$$
$$\equiv e\left\{\mathbf{E}(z)+\frac{1}{c}[\dot{\mathbf{u}}\mathbf{H}]\right\}\mathbf{v}+\Lambda_{yz}(\mathbf{p})\dot{u}_{yz}(z), \tag{2}$$

$$j_\alpha(z)=\frac{2e}{(2\pi\hbar)^3}\int m\,dp_z\int_0^{2\pi}d\tau\, v_\alpha(\tau)\chi(\tau,z), \tag{3}$$

$$\rho\ddot{u}=\lambda_{yzzy}\frac{\partial^2 u}{\partial z^2}-\frac{1}{c}j_xH+\frac{\partial}{\partial z}\frac{2}{(2\pi\hbar)^3}\int m\,dp_z\int_0^{2\pi}d\tau\Lambda_{yz}(\mathbf{p})\chi(\mathbf{p},z). \tag{4}$$

Here **E**(z) is the electric field in the metal, j the current density, $f = f_0 - \chi\partial f_0/\partial\epsilon$ the distribution function of the conduction electrons, χ the nonequilibrium contribution, τ the dimensionless time of motion of the electron along the orbit in a magnetic field, e the charge, m the effective mass, **v** and **p** the velocity and momentum of the electron, and $\Lambda_{ik}(\mathbf{p})$ the deformation potential, which, for a metal with a quadratic isotopic dispersion law, takes the form

$$\Lambda_{ik}(\mathbf{p})=\Lambda\left(\frac{v_iv_k}{v^2}-\frac{1}{3}\delta_{ik}\right),\quad \Lambda=-mv^2. \tag{5}$$

On the right side of Eq. (4), the second component represents the induction force and the third the deformation force. The Stewart-Tolman effect has been neglected in Eqs. (2) and (4).

The boundary conditions for this set of equations are the following: 1) reflection of the electrons from the boundary; 2) equilibrium of the forces acting on the surface of the metal; 3) continuity of the tangential components of the alternating electric and magnetic fields. The last two conditions lead to the relation

$$\lambda_{yzzy}\frac{\partial u(0)}{\partial z}+\frac{2}{(2\pi\hbar)^3}\int m\,dp_z\int_0^{2\pi}d\tau\Lambda_{yz}(\mathbf{p})\chi(\mathbf{p},0)=0. \tag{6}$$

The set of equations (1)–(6) can be solved exactly for the case of specular reflection of the electrons from the surface of the metal. The distribution function is of the form

 Copyright © 1974 The American Institute of Physics

$$\chi=\Omega^{-1}\int_{-\infty}^{\tau}d\tau_1 g\left[z+\frac{1}{\Omega}\int_{-\infty}^{\tau_1}v_z d\tau_2\right]\exp\left[\frac{1+i\omega\tau_0}{\Omega\tau_0}(\tau_1-\tau)\right],$$

where Ω is the cyclotron frequency of the electron and τ_0 is the relaxation time. In the expressions for the change in the energy $g(\tau, z)$, there should be left only the component produced by the electric field ($g = e\mathbf{E}(z)\cdot\mathbf{v}$). The remaining terms lead to small corrections proportional to m/M (M is the mass of the ion), which describe the electronic renormalization of the velocity and the damping of the sound.

For solution of the Maxwell equations, it is convenient to introduce the circularly polarized field $E_\pm = E_x \pm iE_y$. Then

$$\frac{\partial^2 E_s(z)}{\partial z^2}=i\beta\int_{-\infty}^{\infty}dt E_s(t)k_{as}\left(\left|\frac{z-t}{l}\right|\right), \tag{7}$$

where the subscript s denotes "plus" or "minus" polarization, l is the free path length,

$$\beta=\frac{8\pi^2\omega}{c^2}\frac{p^2e^2}{(2\pi\hbar)^3},\qquad a_\pm=(\omega\pm\Omega)\tau_0;$$

and k_{as} are the kernels of the conductivity operator:

$$k_{as}(u)=\int_1^\infty dy\left(\frac{1}{y}-\frac{1}{y^3}\right)\exp[-(1+ia_s)yu]. \tag{8}$$

After substitution of Eqs. (3), (5), (7) and simple transformations, the equation of lattice vibrations (4) is rewritten in the form

$$\frac{d^2u(z)}{dz^2}+k^2u(z)=f_1(z)+\frac{d}{dz}f_2(z), \tag{9}$$

(here $k = \omega(\rho/\lambda_{yzzy})^{1/2}$ is the sound wave vector). The first term on the right arises from the induction force, the second is due to the deformation force:

$$f_1(z)=\frac{1}{\lambda_{yzzy}}\frac{H}{c}j_x(z)$$

$$=\frac{H}{\lambda_{yzzy}}\frac{\pi p^2e^2}{(2\pi\hbar)^3}\sum_s\int_1^\infty dy\frac{y^2-1}{y^3}\int_{-\infty}^{\infty}dy\,E_s(t)\exp[-\Gamma_s y|t-z|], \tag{10}$$

$$f_2(z)=\frac{i}{\lambda_{yzzy}}\frac{\pi p^2e^2}{(2\pi\hbar)^3}\frac{\Lambda}{ev}\sum_s\operatorname{sign}s\int_1^\infty dy\frac{y^2-1}{y^4}\int_{-\infty}^{\infty}dt\operatorname{sign}(z-t)E_s(t)$$
$$\times\exp[-\Gamma_s y|t-z|], \tag{11}$$

where $\Gamma_s = (1 + ia_s)/l$.

3. The integro-differential equation (7) is solved by the Fourier method. We continue the field $E_s(z)$ into the region $z < 0$ in even fashion:

$$\mathscr{E}(p)=2\int_0^\infty dz\,E(z)\cos pz;\qquad E(z)=\frac{1}{\pi}\int_0^\infty dp\,\mathscr{E}(p)\cos pz. \tag{12}$$

The solution takes the form

$$E_s(z)=\frac{1}{2\pi}\int_{-\infty}dp\frac{-2E_s'(0)}{p^2+i\beta K_{as}(p)}e^{ipz},\qquad l^{-1}>\operatorname{Im}p>0, \tag{13}$$

$K_{as}(p)$ is the Fourier transform of the conductivity operator (8):

$$K_{as}(p)=K_{as}(-p)=\frac{1}{ip}\left\{2\frac{\Gamma_s}{ip}+\left[1-\left(\frac{\Gamma_s}{ip}\right)^2\right]\ln\frac{1+ip/\Gamma_s}{1-ip/\Gamma_s}\right\},$$
$$-l^{-1}<\operatorname{Im}p<l^{-1}. \tag{14}$$

Now, knowing the right side of the differential equation (9), it is easy to write down its solution. We are interested in the amplitude $u(0)$ of the sound wave on the surface of the metal. It is equal to

$$u(0)=-\frac{1}{ik}\left\{\int_0^\infty f_1(t)e^{-ikt}dt+ik\int_0^\infty f_2(t)e^{-ikt}dt\right\}$$

$$=\frac{i}{\lambda_{yzzy}}\frac{3Ne}{8}\left\{x[S_++S_-]-\frac{\Lambda}{mv^2}[T_+-T_-]\right\}, \tag{15}$$

$$x=\Omega\tau_0/kl=(kR)^{-1},\quad N=p^3/3\pi^2\hbar^3;$$

$$S_s=-i\frac{E_s'(0)}{\pi}\int_{-\infty}^{\infty}dp\frac{K_{as}(p)}{p^2+i\beta K_{as}(p)}\frac{1}{p-k}, \tag{16}$$

$$T_s=-i\frac{E_s'(0)}{\pi}\int_{-\infty}^{\infty}\frac{dp}{p}\frac{4/3-\Gamma_sK_{as}(p)}{p^2+i\beta K_{as}(p)}\frac{1}{p-k} \tag{17}$$

4. Calculation of the integrals (16), (17) is conveniently carried out by transforming to the complex plane. The integrands have two branch points connected with the presence of logarithms in the function $K_{as}(p)$. They are located at $p = \pm(a_s = i)/l$. Moreover, there are poles which are the roots of the equation

$$p_k^2+i\beta K_{as}(p_k)=0. \tag{18}$$

Taking into account all the singularities of the integrand and choosing the integration contour in the upper half-plane, we obtain

$$S_s=E_s'(0)[\psi_{1s}-\Phi_{1s}],\qquad T_s=E_s'(0)[\psi_{2s}-\Phi_{2s}],$$

where ψ_{1s} and ψ_{2s} denote the sums of the residues of the corresponding integrands of (16) and (17), and Φ_{1s} and Φ_{2s} are the integrals over the edges of the cut ($i\Gamma_s\ \infty;\ i\Gamma_s$).

Equation (18) contains the large parameter $\beta \gg 1$ and is solved approximately. Using the asymptotic form $K_{as}(p) \approx \pi/|p|$, which is valid for $|p| \gg 1$, we find $p_k = (\beta\pi)^{1/3}\exp(in\pi/6)$, n = −1, 3, 7. In the upper half-plane, the variable p has the single root $p_1 = (\beta\pi)^{1/3}e^{i\pi/2}$. Consequently,

$$\psi_{1s}=\frac{2i}{3\beta},\qquad\psi_{2s}=\frac{8}{9\pi\beta}+\frac{2}{3\beta}\frac{1+ia_s}{il}\frac{1}{(\beta\pi)^{1/3}}. \tag{19}$$

The integrals along the edges of the cut take the following form after uncomplicated transformations;

$$\Phi_{1s}=\frac{2i}{k^3\mu_s^2}\int_0\frac{dt}{t-i\mu_s}\frac{t^2(1-t^2)}{[1-i\xi_st^3\varphi_1(t)]^2-\xi_s^2t^6\varphi_2^2(t)}, \tag{20}$$

$$\Phi_{2s}=-\frac{2i}{k^3\mu_s^2}\int_0\frac{dt}{t-i\mu_s}\frac{t^3(1-t^2)[4/3i\xi_st^2-1]}{[1-i\xi_st^3\varphi_1(t)]^2-\xi_s^2t^6\varphi_2^2(t)} \tag{21}$$

Here

$$\mu_s=\frac{1+ia_s}{kl}=\alpha\pm ix,\qquad\alpha=(kl)^{-1},$$
$$\xi_s=\frac{\beta}{k^3\mu_s^3},\qquad\varphi_1(t)=2t+(1-t^2)\ln\frac{1+t}{1-t},\qquad\varphi_2(t)=\pi(1-t^2). \tag{22}$$

In the range of magnetic fields of interest to us, $x \lesssim 1$, we have $|\xi_s| \gg 1$. Here the quantity Φ_{1s} is small in comparison with Φ_{2s} ($\Phi_{1s}/\Phi_{2s} \sim |\xi_s|^{-1}$), and Φ_{1s} can be disregarded. Introducing the functions ψ_s and Φ_s, we rewrite the sound-wave amplitude $u(0)$ in the form

$$u(0)=\frac{i}{\lambda_{yzzy}}\frac{3Ne}{8}E_x'(0)\left\{x[\psi_{1+}+\psi_{1-}]-\frac{\Lambda}{mv^2}[(\psi_{2+}-\psi_{2-})-(\Phi_{2+}-\Phi_{2-})]\right\}$$
$$-\frac{1}{\lambda_{yzzy}}\frac{3Ne}{8}E_y'(0)\left\{x[\psi_{1+}-\psi_{1-}]-\frac{\Lambda}{mv^2}[(\psi_{2+}+\psi_{2-})-(\Phi_{2+}+\Phi_{2-})]\right\}. \tag{23}$$

Separating the real and imaginary parts in the expressions for Φ_{2s}, we get

$$\Phi_{2+}-\Phi_{2-}=\frac{1}{\beta}(F_1-iF_2),\qquad\Phi_{2+}+\Phi_{2-}=-\frac{1}{\beta}F_3,$$

$$F_1=\frac{32}{3}\alpha x\int_0^1 dt\frac{t(1-t^2)}{(\varphi_1^2+\varphi_2^2)[(t^2-\alpha^2-x^2)^2+4\alpha^2t^2]},$$

$$F_2=\frac{16}{3}x\int_0^1 dt\frac{(1-t^2)(\alpha^2+x^2-t^2)}{(\varphi_1^2+\varphi_2^2)[(t^2-\alpha^2-x^2)^2+4\alpha^2t^2]},$$
$$F_3=\frac{16}{3}\alpha\int_0^1 dt\frac{(1-t^2)(\alpha^2+x^2+t^2)}{(\varphi_1^2+\varphi_2^2)[(t^2-\alpha^2-x^2)^2+4\alpha^2t^2]} \qquad (24)$$

After substitution of the values of ψ_S and Φ_S and neglecting the small components, the expression for the amplitude of the sound wave takes the form

$$u(0)=-u_{01}E_x'(0)[x-{}^3/_4(F_2-iF_1)]-u_{02}E_y'(0)(0.565+F_3),$$
$$u_{01}=\frac{4}{3\lambda_{yzzy}}\frac{c^2p}{8\pi\omega e},\qquad u_{02}=\frac{3}{4}u_{01}. \qquad (25)$$

The integrals (24) cannot be calculated in explicit form for arbitrary values of the parameters α and s. It is not difficult to obtain their asymptotic expressions, if we assume $\alpha \ll 1$, in the limiting cases $x \gg 1$ and $x \ll 1$. For $x \gg 1$, the functions $F_1(\alpha, x)$, $F_2(\alpha, x)$, and $F_3(\alpha, x)$ fall off rapidly:

$$F_1=0.18\ \alpha x^{-3},\quad F_2=0.283\ x^{-1},\quad F_3=0.283\ \alpha x^{-2}. \qquad (26)$$

The wave amplitude $u_H(0)$ increases in strong fields with increase of the field H (i.e., of x), asymptotically approaching the linear relation

$$|u_H(0)|=u_{01}E'(0)x[1-O(1/x^2)],\qquad x\gg 1, \qquad (27)$$

and the quantity $u_E(0)$ approaches saturation:

$$|u_E(0)|=u_{02}E'(0)[0.565+O(1/x^2)],\qquad x\gg 1. \qquad (28)$$

In the range of small x, when the inequality $x \ll \alpha \ll 1$ is valid, the asymptotic behavior of the integrals (24) is described by the following formulas:

$$F_1=0.534\ \alpha^{-1}x,\quad F_2=0.987\ x,\quad F_3=0.84. \qquad (29)$$

The dependence of the amplitude of the wave on the magnetic field in the limiting case (29) takes the form

$$|u_H(0)|=u_{01}E'(0)x\sqrt{0.16\ \alpha^{-2}+0.068}. \qquad (30)$$
$$|u_E(0)|=u_{02}E'(0)\cdot 1.405. \qquad (31)$$

As is seen from Eqs. (27) and (30), the amplitude of the wave $|u_H(0)|$ increases linearly with increase in the field H, both in the region of weak magnetic fields $x \ll 1$ and in the region of strong fields $x \gg 1$. However, the coefficients of proportionality of the asymptotic forms are different. For $x \gg 1$, the coefficient of porportionality is less than unity. For $x \ll 1$, it depends on the parameter α; it increases as α decreases and can become larger than unity. The amplitude $u_E(0)$ for $x \gg 1$ and $x \ll 1$ tends to constant values, which differ in the cases of strong and weak fields. Thus, in the region $x \sim 1$ and $\alpha \ll 1$, one must expect a departure from the linear dependence and the appearance of nonmonotonicity of the function $u_H(x)$. This conclusion is in agreement with the experimental results.

The functions $F_1(x, \alpha)$, $F_2(x, \alpha)$, and $F_3(x, \alpha)$ were tabulated on a high-speed computer for three values of the parameter α: 0.1, 0.5 and 1. Their plots are shown in Fig. 1. It is seen that in the region of magnetic fields in which $x \gg 1$ the functions F_1, F_2, and F_3 are connected with the deformation forces, are small, and can be disregarded, whereas in weak magnetic fields ($x \lesssim 1$) they can play an important role.

Figures 2 and 3 show the dependences of the displacement amplitudes u_E and u_H on the magnetic field for two values of the parameter kl: 2 and 10. For $kl = 2$, the function $u_H(x)$ departs appreciably from a straight line in the range of values $x \approx 1$, but approaches it at $x \gg 1$. It is natural to compare the experimental data of Turner et al.[1] with the calculated curve. With increasing kl, the value of the deformation force increases and the maxima of the functions $F_1(x)$ and $F_2(x)$ increase along with it. This leads to the appearance of a nonmonotonic dependence of u_H on H, which has been observed by Wallace, Gaerttner and Maxfield.[2] It is seen from a comparison of the $u_H(H)$ curves obtained by them and our curves of Fig. 3 that the value of kl in the experiment is in all probability much greater than assumed in[2]. It is possible that the divergences are due to the nonspecular character of the reflection of the electrons from the surface of the sample. So far as the dependence $u_E(H)$ is concerned, our calculation agrees qualitatively with the experimental data.

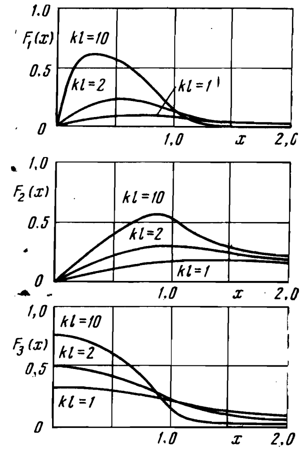

FIG. 1

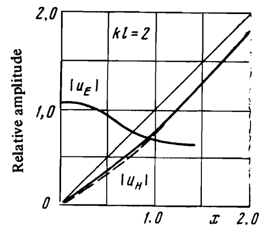

FIG. 2

FIG. 1. Plots of the functions $F_1(x, \alpha)$, $F_2(x, \alpha)$, and $F_3(x, \alpha)$ for values of the parameter $\alpha = (kl)^{-1}$: 0.1, 0.5 and 1.

FIG. 2. Dependence of the displacement amplitudes $|u_E|$ on the magnetic field and $|u_H|$ for the value of the parameter $kl = 2$. The abscissa in each case is the quantity $x = (kR)^{-1}$.

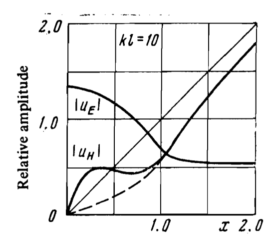

FIG. 3. Dependence of the displacement amplitude $|u_E$ and $|u_H|$ on the magnetic field for the value of the parameter $kl = 10$.

Unfortunately, a more detailed comparison with experiment becomes difficult because insufficient experimental information is contained in[2].

For real metals, one ought generally take into account the nonspecular character of the reflection of the electrons from the boundary. In the case of diffuse scattering of the electrons, the exact solution of Maxwell's equations is obtained by the Wiener-Hopf method and has a much more complicated form than for specular reflection. The expression for the sound-wave amplitude $u(0)$ contains, in comparison with Eq. (15), additional components due both to deformation and to induction forces. It has not yet been possible to find simple asymptotic expressions for them. Computer calculations also turn out to be very cumbersome. In our opinion, allowance for these components should not lead to any significant change in the character of $u_H(H)$ and $u_E(H)$.

Thus, on the basis of the free electron model, it is possible to explain the singularities of the production of ultrasound in potassium in a magnetic field. These singularities are due to the nonomonotonic dependence of the deformation force on the magnetic field, and also to the competition of the induction and deformation mechanisms of transformation of the electromagnetic wave into sound vibrations.

[1] G. Turner, R. L. Thomas, and D. Hsu, Phys. Rev. **B3**, 3097 (1971).
[2] W. D. Wallace, M. R. Gaerttner, and B. W. Maxfield, Phys. Rev. Lett. **27**, 995 (1971).
[3] J. J. Quinn, Phys. Lett. **25A**, 522 (1967).
[4] R. Casanova Alig, Phys. Rev. **178**, 1050 (1969).

Translated by R. T. Beyer
112