Nonlinear effects due to the propagation of sound and electromagnetic waves in conductors in a quantizing magnetic field

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The nonlinear susceptibilities of a degenerate gas located in a quantizing magneti field are determined. The generation of sound second harmonics and the nonlinear direct current in the fields of sound and electromagnetic waves are studied. It is shown that the nonlinear susceptibilities experience oscillations of the giant-quantum or geometric-resonance type. The effects considered can be used to study the electronic spectra of solids.

Propagation of sound and electromagnetic waves in conductors located in a quantizing magnetic field gives rise to a number of pronounced resonance effects which can be observed: giant oscillations of the absorption coefficient, oscillations at geometric resonance, oscillations of the sound velocity in an oblique magnetic field, and so forth (see, for example,[1]). All the listed effects are linear in the sound amplitude. The abundance of such effects is connected with the multi-component character of the electron system in a magnetic field.

In the present work we consider resonance effects that are quadratic in the field of the wave and whose existence is also connected with the multi-component character of the electron system. However, the nonlinear susceptibilities, which describe these effects contain more information on the electron system than the linear susceptibilities. To calculate the effects of interest to us, such as frequency mixing, frequency doubling, and generation of a static current, it is necessary to find the nonlinear contribution to the concentration and to the nonlinear current in a completely degenerate electron gas located in a quantizing field. We assume that the condition $\omega\tau \gg 1$ is satisfied and that the nonlinearity is connected with the resonance distortion of the electron density matrix in the field of a sound or electromagnetic wave. We assume also that the spectrum of the carriers is isotropic and quadratic.

1. To calculate the nonlinear susceptibilities, it is convenient to use the equation of motion for the electron density matrix in the field of an electromagnetic or sound wave.[1)] In second order in H_{int}, the corrections to the current and to the concentration of electrons take the form

$$\langle \mathbf{j}(x)\rangle = i\int_{-\infty}^{t} dt'\langle[\tilde{H}_{int}(t')\tilde{\mathbf{j}}(x)]\rangle - \int_{-\infty}^{t} dt'\int_{-\infty}^{t'} dt''\langle[\tilde{H}_{int}(t'')[\tilde{H}_{int}(t')\tilde{\mathbf{j}}(x)]]\rangle, \quad (1)$$

$$\langle \delta n^{(2)}(x)\rangle = -\int_{-\infty}^{t} dt'\int_{-\infty}^{t'} dt''\langle[\tilde{H}_{int}(t'')[\tilde{H}_{int}(t')\tilde{n}(x)]]\rangle, \quad (2)$$

where $H_{int}(t)$ is the Hamiltonian of the electrons in the field of an electromagnetic or sound wave in the interaction representation; $\tilde{\mathbf{j}}(x)$ and $\tilde{n}(x)$ are the current and density operators in the interaction representation. Averaging in (1) and (2) is carried out over the ground state of the electron system.

To find the nonlinear effects in the propagation of electromagnetic waves, it is necessary to use a gauge with a scalar potential $\varphi = 0$. In this case, we have

$$\tilde{\mathbf{j}}(x) = \sum_{\lambda\lambda'}\left\{\frac{e}{2m}[\varphi_\lambda^*\hat{\mathbf{p}}\varphi_{\lambda'} - \varphi_{\lambda'}\hat{\mathbf{p}}\varphi_\lambda^*] - \frac{e^2}{mc}\varphi_\lambda^*\varphi_{\lambda'}(\mathbf{A}+\mathbf{A}_0)\right\}\exp\{-i(\varepsilon_{\lambda'}-\varepsilon_\lambda)t\}\hat{a}_\lambda^+\hat{a}_{\lambda'}, \quad (3)$$

$$\tilde{H}_{int}(t) = \sum_{\lambda\lambda'}\left\{-\frac{e}{2mc}\left[\mathbf{A}\left(\hat{\mathbf{p}}-\frac{e}{c}\mathbf{A}_0\right)+\left(\hat{\mathbf{p}}-\frac{e}{c}\mathbf{A}_0\right)\mathbf{A}\right]_{\lambda\lambda'} + \frac{e^2}{2mc^2}(\mathbf{A}^2)_{\lambda\lambda'}\right\}\exp\{-i(\varepsilon_{\lambda'}-\varepsilon_\lambda)t\}\hat{a}_\lambda^+\hat{a}_{\lambda'}, \quad (4)$$

where $\mathbf{A}_0(-Hy, 0, 0)$ is the vector potential of the constant magnetic field, H the field intensity, $\mathbf{A}$ the vector potential of the electromagnetic wave, e the charge, m the effective mass of the carriers, c the velocity of light, $\varphi_\lambda(\mathbf{r}) = \varphi_{np_xp_z}(\mathbf{r})$ the wave functions in the magnetic field, $\epsilon_\lambda = \epsilon_n(p_z)$ the energy eigenvalues, and $\tilde{a}_\lambda^+$ and $\tilde{a}_\lambda$ the creation and annihilation operators (Planck's constant is $\hbar = 1$).

We first find the quadratic increment of the current in the field of two plane waves

$$\mathbf{A}(\mathbf{r}, t) = \mathbf{A}_1 e^{i(\mathbf{q}_1\mathbf{r}-\omega_1 t)} + \mathbf{A}_2 e^{i(\mathbf{q}_2\mathbf{r}-\omega_2 t)}.$$

Carrying out the averaging in (1) over the ground state of the electrons and transforming to the $\mathbf{q}$, ω representation, we get the amplitude of the nonlinear current, after rather cumbersome calculations:

$$\begin{aligned} j_\alpha^{(2)}(\mathbf{q}=\mathbf{q}_1+\mathbf{q}_2;\ \omega=\omega_1+\omega_2) &= \frac{e^3}{2\pi^2 m^2 c^2} A_1^\beta A_2^\gamma P(\omega_1\mathbf{q}_1\beta;\ \omega_2\mathbf{q}_2\gamma) \times\sum_{mnn_1}\frac{1}{\gamma m}\exp\left\{\frac{i\gamma}{2}(q_{1x}q_{2y}-q_{2x}q_{1y})\right\} \\ &\times\Bigg\{\int dp_z X^\alpha_{nm}(\mathbf{q}, p_z) X^\beta_{mn_1}(-\mathbf{q}_1, p_z+q_z) X^\gamma_{n_1n}(-\mathbf{q}_2, p_z+q_{2z}) \\ &\times\Bigg[\frac{f[\varepsilon_n(p_z)]-f[\varepsilon_{n_1}(p_z+q_{2z})]}{[\varepsilon_{n_1}(p_z+q_{2z})-\varepsilon_n(p_z)-\omega_2-i\delta][\varepsilon_m(p_z+q_z)-\varepsilon_n(p_z)-\omega-i\delta]} \\ &+\frac{f[\varepsilon_m(p_z+q_z)]-f[\varepsilon_{n_1}(p_z+q_{2z})]}{[\varepsilon_m(p_z+q_z)-\varepsilon_{n_1}(p_z+q_{2z})-\omega_1-i\delta][\varepsilon_m(p_z+q_z)-\varepsilon_n(p_z)-\omega-i\delta]}\Bigg] \\ &+\frac{M_{mn}(-\mathbf{q})}{2\gamma}\delta_{\beta\gamma}\int dp_z\frac{X^\alpha_{nm}(\mathbf{q}, p_z)[f[\varepsilon_m(p_z+q_z)]-f[\varepsilon_n(p_z)]]}{\varepsilon_m(p_z+q_z)-\varepsilon_n(p_z)-\omega-i\delta} \\ &+\frac{M_{nm}(\mathbf{q})}{\gamma}\delta_{\alpha\beta}\int dp_z\frac{X^\gamma_{mn}(-\mathbf{q}, p_z+q_z)[f[\varepsilon_m(p_z+q_{2z})]-f[\varepsilon_n(p_z)]]}{\varepsilon^m(p_z+q_{2z})-\varepsilon_n(p_z)-\omega_2-i\delta}\Bigg\}, \end{aligned} \quad (5)$$

where $P(\omega_1, q_1\beta;\ \omega_2 q_2\gamma)$ is the permutation operator, $\gamma = c/eH$ the square of the magnetic length, $f[\epsilon_n(p_z)]$ the distribution functions of the electrons, and A_1^β and A_2^γ the components of the vector potential in the first and second wave. In obtaining Eq. (5), we used the expression for the matrix elements of the operator

$$\hat{\mathbf{V}}(\mathbf{q}) = \frac{1}{2}\left[e^{-i\mathbf{q}\mathbf{r}}\left(\hat{\mathbf{p}}-\frac{e}{c}\mathbf{A}_0\right)+\left(\hat{\mathbf{p}}-\frac{e}{c}\mathbf{A}_0\right)e^{-i\mathbf{q}\mathbf{r}}\right]$$

$$\hat{V}^\alpha_{\lambda\lambda'} = \delta(p_x-p_x'+q_x)\delta(p_z-p_z'+q_z) \times\exp\{i\gamma p_x q_y + {}^1\!/_2 i\gamma q_x q_y\}X^\alpha_{nn'}(\mathbf{q}, p_z), \quad (6)$$

where λ is the set of electron quantum numbers.

Calculating the quantities $X^{\alpha}_{nm}(q, p_z)$, we get

$$X^z_{nm}(\mathbf{q}, p_z) = M_{nm}(\mathbf{q})(p_z + {}^1/_2 q_z),$$

$$X^y_{nm} = -\frac{q_y}{2}M_{nm}(\mathbf{q}) + i\left(\frac{m+1}{2\gamma}\right)^{1/2} M_{n,m+1}(\mathbf{q}) - i\left(\frac{m}{2\gamma}\right)^{1/2} M_{n,m-1}(\mathbf{q}), \qquad (7)$$

$$X^x_{nm} = -\frac{q_x}{2}M_{nm}(\mathbf{q}) + \left(\frac{m+1}{2\gamma}\right)^{1/2} M_{n,m+1}(\mathbf{q}) + \left(\frac{m}{2\gamma}\right)^{1/2} M_{n,m-1}(\mathbf{q}),$$

$$M_{nm}(\mathbf{q}) = 2^{-|n-m|/2}[\operatorname{sign}(m-n)\gamma^{1/2}q_x - iq_y\gamma^{1/2}]^{|n-m|}L^{|n-m|}_{\min(n,m)}(\rho)e^{-\rho/2},$$

where $\rho = {}^1/_2\gamma(q_x^2 + q_y^2)$ and $L^{|n-m|}_{\min(n,m)}$ is a Laguerre polynomial normalized to unity. Using the Hermitian property of the operator $\hat{p} - e\mathbf{A}_0/c$, we can obtain the following relation:

$$\hat{V}^{\alpha}_{\lambda\lambda'}(\mathbf{q}) = \hat{V}^{\alpha *}_{\lambda'\lambda}(-\mathbf{q}). \qquad (8)$$

Then, with account of (6), we get the symmetry relation for the matrix elements $X^{\alpha}_{nm}(q, p_z)$:

$$X_{nm}{}^{\alpha}(\mathbf{q}, p_z) = X_{mn}{}^{\alpha *}(-\mathbf{q}, p_z + q_z),$$
$$X_{nm}{}^{\alpha}(-\mathbf{q}, p_z) = X_{mn}{}^{\alpha *}(\mathbf{q}, p_z - q_z). \qquad (9)$$

As $H \to 0$, the expression (5) is transformed into the similar formula obtained by Cheng and Miller[3] for the nonlinear current.

To study the nonlinear effects in the propagation of sound, we find $\delta n^{(2)}$ and $\mathbf{j}^{(2)}$ in the field of a longitudinal sound wave. Taking into consideration only the deformation interaction, and neglecting the induction interaction, we write out the Hamiltonian in the form

$$\hat{H}_{int}(t) = \sum_{\lambda\lambda'} \hat{a}_\lambda^+ \hat{a}_{\lambda'} V_{\lambda\lambda'} \exp\{-i(\varepsilon_\lambda - \varepsilon_{\lambda'})t\}, \qquad (10)$$

where $V_{\lambda\lambda'}$ are the matrix elements of the operator Λ div $\mathbf{u}$ over the wave functions of the electron in the quantizing magnetic field, Λ is the constant of the deformation potential, and $\mathbf{u}$ is the displacement vector of the lattice.

Averaging over the ground state of the electron subsystem and transforming to the $\mathbf{q}$, ω representation, we get the following expression for the nonlinear contribution to the current in the field of two plane waves $\mathbf{u}_1\exp[i(\mathbf{q}_1\cdot\mathbf{r} - \omega_1 t)] + \mathbf{u}_2\exp[i(\mathbf{q}_2\cdot\mathbf{r} - \omega_2 t)]$:

$$j_\alpha^{(2)}(\mathbf{q} = \mathbf{q}_1 + \mathbf{q}_2; \omega = \omega_1 + \omega_2) = \frac{e}{2\pi^2}V_1V_2P(\omega_1\mathbf{q}_1; \omega_2\mathbf{q}_2)$$
$$\times \sum_{mnn_1}\frac{1}{\gamma m}\exp\left\{\frac{i\gamma}{2}(q_{1x}q_{2y} - q_{2x}q_{1y})\right\}$$
$$\times \int dp_z X^{\alpha}_{nm}(\mathbf{q}, p_z)M_{mn_1}(-\mathbf{q}_1)M_{n_1n}(-\mathbf{q}_2) \qquad (11)$$
$$\times\left\{\frac{f[\varepsilon_n(p_z)] - f[\varepsilon_{n_1}(p_z + q_{2z})]}{[\varepsilon_{n_1}(p_z + q_{2z}) - \varepsilon_n(p_z) - \omega_2 - i\delta][\varepsilon_m(p_z + q_z) - \varepsilon_n(p_z) - \omega - i\delta]}\right.$$
$$\left.+\frac{f[\varepsilon_m(p_z + q_z)] - f[\varepsilon_{n_1}(p_z + q_{2z})]}{[\varepsilon_m(p_z + q_z) - \varepsilon_{n_1}(p_z + q_{2z}) - \omega_1 - i\delta][\varepsilon_m(p_z + q_z) - \varepsilon_n(p_z) - \omega - i\delta]}\right\}$$

The matrix elements M_{nm} and X^{α}_{mn} in (11) are determined by Eq. (7): $V_{1,2} = \Lambda q_{1,2}u_{1,2}$ and Λ is the constant of the deformation interaction.

For the nonlinear contribution to the concentration, we get similarly[4]

$$\delta n^{(2)}(\mathbf{q} = \mathbf{q}_1 + \mathbf{q}_2; \omega = \omega_1 + \omega_2) = \frac{V_1V_2}{2\pi^2}P(\omega_1\mathbf{q}_1; \omega_2\mathbf{q}_2)$$
$$\times\sum_{mnn_1}\frac{1}{\gamma}\exp\left\{\frac{i\gamma}{2}(q_{1x}q_{2y} - q_{1y}q_{2x})\right\}M_{nm}(\mathbf{q})M_{mn_1}(-\mathbf{q}_1)M_{n_1n}(-\mathbf{q}_2)$$
$$\times\left[\int dp_z\frac{f[\varepsilon_n(p_z)] - f[\varepsilon_{n_1}(p_z + q_{2z})]}{[\varepsilon_{n_1}(p_z + q_{2z}) - \varepsilon_n(p_z) - \omega_2 - i\delta][\varepsilon_m(p_z + q_z) - \varepsilon_n(p_z) - \omega - i\delta]}\right. \qquad (12)$$
$$\left.+\int dp_z\frac{f[\varepsilon_m(p_z + q_z)] - f[\varepsilon_{n_1}(p_z + q_{2z})]}{[\varepsilon_m(p_z + q_z) - \varepsilon_{n_1}(p_z + q_{2z}) - \omega_1 - i\delta][\varepsilon_m(p_z + q_z) - \varepsilon_n(p_z) - \omega - i\delta]}\right]$$

The singularities of the integrands of (11) and (12) correspond to the creation of electron-hole pairs by each phonon, and also by two phonons simultaneously when the laws of conservation of energy and of the z component of the momentum are satisfied. The matrix elements $M_{nm}(\mathbf{q})$ in (11) and (12) describe the approximate conservation of the transverse component of the momentum.

2. We now proceed to the calculation of some concrete nonlinear effects, which arise in the propagation of sound and electromagnetic waves.

We first discuss resonance effects in the generation of the second harmonic of sound. We introduce first the nonlinear susceptibility $\chi^{(2)}(\omega_1 + \omega_2; \mathbf{q}_1 + \mathbf{q}_2)$ with the help of the relation

$$\delta n^{(2)}(\mathbf{r}, t) = \chi^{(2)}V_1V_2\exp\{i[(\mathbf{q}_1 + \mathbf{q}_2)\mathbf{r} - (\omega_1 + \omega_2)t]\}. \qquad (13)$$

Setting $\omega_1 = \omega_2$ and $\mathbf{q}_1 = \mathbf{q}_2 = \mathbf{q}$ in (12), we find

$$\chi^{(2)}(2\omega, 2\mathbf{q}) = -2mq^{-2}[\Pi(\mathbf{q}, \omega) - \Pi(2\mathbf{q}, 2\omega)] \qquad (14)$$

for $\mathbf{q} \parallel \mathbf{H}$; here $\Pi(\mathbf{q}, \omega)$ is the "longitudinal" polarization operator in the quantizing magnetic field. The explicit expression for $\chi^{(2)}(2\omega, 2\mathbf{q})$ is of the form

$$\operatorname{Re}\chi^{(2)}(2\omega, 2\mathbf{q}) = \frac{m^2}{2\pi^2q^3\gamma}\sum_n\left[2\ln\left|\frac{(k_n + q/2)^2 - (m\omega/q)^2}{(k_n - q/2)^2 - (m\omega/q)^2}\right| - \ln\left|\frac{(k_n + q)^2 - (m\omega/q)^2}{(k_n - q)^2 - (m\omega/q)^2}\right|\right],$$
$$\operatorname{Im}\chi^{(2)}(2\omega, 2\mathbf{q}) = \frac{m^2}{2\pi q^3\gamma}\sum_n\{2(f[\varepsilon_n(p_{1z})] - f[\varepsilon_n(p_{1z} + q)]) - (f[\varepsilon_n(p_{2z})] - f[\varepsilon_n(p_{2z} + 2q)])\}, \qquad (15)$$

where k_n is the maximum momentum of the electron the n-th tube, and p_{1z} and p_{2z} in (15) are determined from the conservation laws

$$\varepsilon_n(p_{1z} + q) - \varepsilon_n(p_{1z}) - \omega = 0,$$
$$\varepsilon_n(p_{2z} + 2q) - \varepsilon_n(p_{2z}) - 2\omega = 0. \qquad (16)$$

In the absence of a magnetic field, it is not difficult to obtain the following expression from (14) by expanding the corresponding expressions for $\Pi(\mathbf{q}, \omega)$ in $q/2p_0$ at $q/2p_0 \ll 1$:[5]

$$\chi^{(2)}(2\omega, 2\mathbf{q}) = -3m^2/2\pi p_0. \qquad (17)$$

If the synchronism conditions are satisfied, then, as is well known, the amplitude of the second harmonic will increase linearly:

$$u^{(2)} = 2\lambda^{-1}\,|\,\chi^{(2)}(2\omega, 2\mathbf{q})\,|\,\Lambda^3(qu^{(1)})^2x; \qquad (18)$$

where λ is the elasticity modulus.

The dimensionless parameter that characterizes the amount of the nonlinearity is given by

$$\beta = 2\lambda^{-1}\,|\,\chi^{(2)}(2\omega, 2\mathbf{q})\,|\,\Lambda^3. \qquad (19)$$

For metals, $p_0 \sim 10^8$ cm^{-1}, $m \sim 10^{-27}$g, and $\Lambda = 10$ eV. In this case $\beta \sim 10$, i.e., the electronic nonlinearity is of the same order as the lattice nonlinearity (see[6]), and consequently both nonlinearity mechanisms must be taken into account at the same time. It can be shown that under these conditions the nonlinear terms that arise in the expansion of the energy of the electron in powers of the deformation tensor above the first lead to corrections of the same order as the terms considered above.

In a quantizing field at $qR \ll 1$, when the resonance condition is satisfied only for transitions $\Delta n = 0$, the principal contribution to $\chi^{(2)}(2\omega, 2q)$ is made by the difference of the imaginary parts of $\Pi(\omega, q)$ and $\Pi(2\omega, 2q)$. In that $(\mathbf{q}, \omega)$ region where the conditions (16) are satisfied, we get from (15)

$$\beta = m^2\Lambda^3 / \pi\gamma q^3\hbar^3\lambda. \tag{20}$$

At the usual values of the parameters entering into (20), the nonlinearity β exceeds the lattice nonlinearity by many orders of magnitude. The temperature broadening of the distribution function and the broadening of the spectrum due to collisions greatly decrease the value of β. In pure metals and semimetals, where the temperature broadening predominates, it is necessary for the determination of β to substitute $\operatorname{Im}\Pi(\omega, q)$ at a finite temperature in (15):

$$\operatorname{Im}\Pi(\omega,\mathbf{q}) = -\frac{m^2\Omega}{4\pi q_z}\operatorname{sh}\left(\frac{\omega}{2T}\right)\operatorname{ch}^{-1}\left[\frac{(m\omega/q - q/2)^2 - k_n^2}{4mT}\right] \times\operatorname{ch}^{-1}\left[\frac{(m\omega/q + q/2)^2 - k_n^2}{4mT}\right]. \tag{21}$$

Expanding $\operatorname{Im}\Pi(\omega, \mathbf{q})$ and $\operatorname{Im}\Pi(2\omega, 2\mathbf{q})$ in ω/T, we get the additional factor $(\omega/T)^3$ in (20). Estimates show that for $q = 10^{-5}\ \mathrm{cm}^{-1}$, $H = 10^5$ Oe, $T \sim 1^\circ$K, $m \sim m_e$, and $\Lambda \sim 10$ eV the nonlinear parameter is $\beta \sim 10^2$–10^3.

Another nonlinear effect connected with the propagation of sound is the appearance of a constant electric current. Formulas for the current $j_z^{(2)}$ in the direction of the magnetic field and the current $j_y^{(2)}$ perpendicular to **H** are obtained from (11) at $\mathbf{q}_1 = -\mathbf{q}_2 = \mathbf{q}(q_x, 0, q_z)$ and $\omega_1 = -\omega_2 = \omega$. Replacing $[\epsilon_m(p_z + q_z) - \epsilon_n(p_z) - \omega - i\delta]$ by $-i\nu$ ($\nu = 1/\tau$ is the collision frequency), we get

$$j_z^{(2)} = \frac{2e}{\nu}\frac{q_z}{m}V^2\operatorname{Im}\Pi(\omega,\mathbf{q}), \tag{22}$$

where

$$\operatorname{Im}\Pi(\omega,\mathbf{q}) = \frac{-m}{2\pi\gamma q_z}\sum_{nn_1}|M_{nn_1}|^2(f[\varepsilon_n(p_z+q_z)] - f[\varepsilon_{n_1}(p_z)]) \tag{22$'$}$$

is the imaginary part of the polarization operator; p_z in (22′) is determined from the conservation laws.

In obtaining the expression for $j^{(2)}$ from (11), it is necessary to keep the following in mind. The matrix element X_{nm}^y is pure imaginary ($q_y = 0$), and M_{mn_1} and M_{n_1n} are real. Therefore, it is clear that the current will be different from zero whenever the integral with respect to p_z has an imaginary part. Using (6) and (7), we get

$$j_y^{(2)} = -\frac{|e|V^2}{\pi q_z}\sum_{nn_1}\left\{\left(\frac{n}{2\gamma}\right)^{1/2}M_{n-1,n_1}(-\mathbf{q})M_{n_1n}(\mathbf{q}) - \left(\frac{n_1}{2\gamma}\right)^{1/2}M_{n_1-1,n}(\mathbf{q})M_{nn_1}(-\mathbf{q}) + \left(\frac{n+1}{2\gamma}\right)^{1/2}M_{nn_1}(-\mathbf{q})M_{n_1,n+1}(\mathbf{q}) - \left(\frac{n_1+1}{2\gamma}\right)^{1/2}M_{n_1n}(\mathbf{q})M_{n,n_1+1}(-\mathbf{q})\right\}\{f[\varepsilon_n(p_z)] - f[\varepsilon_{n_1}(p_z+q_z)]\}. \tag{23}$$

In the region of wave vectors $qR < 1$ (R is the Larmor radius), where the contribution to the damping is made only by transitions with $\Delta n = 0$, we get from (23)

$$j_y^{(2)} = \frac{-|e|q_x}{\Omega m}2V^2\operatorname{Im}\Pi(\mathbf{q},\omega). \tag{24}$$

We have taken into account here the relation

$$\left(\frac{n+1}{2\gamma}\right)^{1/2}M_{nn+1}(\mathbf{q}) + \left(\frac{n}{2\gamma}\right)^{1/2}M_{nn-1}(\mathbf{q}) = \frac{q_x}{2}M_{nn}(\mathbf{q}),$$

which follows from the fact that $X_{nn}^x(\mathbf{q}) = 0$ (see below).

It is easy to understand the physical meaning of Eqs. (22) and (24) by separating the following factors: the factor $2V^2\operatorname{Im}\Pi$, which determines the number of phonons absorbed per unit time; the factor $q_z/m\nu$—the distance traversed by an electron with the additional momentum q_z in the z direction; the factor $q_x/m\Omega$—the displacement of the center of the orbit in the x direction following absorption of a phonon with momentum q_x and carrier charge e.

The nonlinear current $j_y^{(2)}$ at $qR \ll 1$ has been observed experimentally[7] for sound propagation in Bi in a quantizing field. The current oscillations correspond entirely to the giant oscillations in the sound absorption (see [8]).

In the absent of a magnetic field, the factor

$$\operatorname{Im}\Pi(\mathbf{q},\omega,H=0) = \frac{m^2\omega}{2\pi q}\theta(2p_0 - k), \tag{25}$$

should appear in (22), where $\theta(x)$ is the theta function (see[2]).

According to Eqs. (22)–(24), the oscillations of the current amplitude following a change in the quantity H, and also following a change of the angle between **q** and **H**, will be synchronous with the oscillations of the absorption coefficients.

It must be noted that in a two-component plasma the oscillations of the current are due to resonance absorption by carriers of different types belonging to a definite Landau level, and the direction of the current j_z will change if the signs of the carriers are different and resonance does not begin simultaneously. Measurement of the current can give more information on the spectrum of the carriers than the giant oscillations of the absorption coefficient. In a nonquantizing field, the currents associated with the absorption of phonons by electrons and holes cancel each other in the z direction and are additive in the y direction.

If the angle between **q** and **H** is close to $\pi/2$ ($\cos\theta < s/v_F$), then one can have not only the oscillations of $j^{(2)}$ that are connected with the threshold in the production of electron-hole pairs, but also oscillations of the geometric-resonance type. These oscillations are described by the factors $|M_{nn}(q)|^2$ in (24). With the help of (23), it is not difficult to show that current oscillations can be observed at $\mathbf{q} \perp \mathbf{H}$ if the conditions of cyclotron resonance $\omega = n\Omega$ are satisfied.

In a nonquantizing magnetic field, obviously, an abrupt change of the static current ought to be observed if the resonance condition is satisfied for the electrons and holes (the tilt effect):

$$v_F^{e,h}\cos\theta = s,$$

where $v_F^{e,h}$ is the Fermi velocity, s the sound velocity, and θ the angle between **q** and **H**.

Of course, to observe all the enumerated effects, it is necessary that the sound damping over the length of the sample be small.

We did not take into account above the fact that the electrons interacting resonantly with a longitudinal wave can be trapped by the wave at sufficiently high sound intensities; this corresponds to strong distortion of the distribution function in the region of resonance momenta. Interesting papers have been devoted to the consideration of the corresponding effects.[9,10] The intensity threshold at which effective capture of the carriers begins is determined by the condition

$$\omega_0\tau \gg 1, \quad \omega_0 = q\sqrt{\Lambda m^{-1}\,\mathrm{div}\,\mathbf{u}}. \tag{26}$$

Upon satisfaction of the condition (26), the absorption coefficient can be appreciably decreased;[10] in a quantizing field, this leads to a decrease in the amplitude of the resonance oscillations. If $\tau\omega_0 \lesssim 1$, then electron capture can be disregarded. At sufficiently high sound amplitudes, one nonlinear effect is still possible in a quantizing field—the change in the number of filled levels in the field of the wave. This effect should also lead to a weakening of the oscillations. Simple estimates show that these effects are important at sound intensities $\gtrsim 10^{-1} - 1$ W/cm² if $\omega \sim 10^9 - 10^{10}$, $\tau = 10^{-9}$ sec, and $H \sim 10^5$ G.

Heating of the carriers can also lead to such a blurring of the oscillations. To take this heating into account, we need to introduce the temperature T_{eff}. It was shown in[10] that heating of the carriers by the sound wave is usually negligible without a magnetic field. Calculation of T_{eff} in a quantizing field is a very complicated problem. The first experiments on the nonlinear sound effects in a quantizing field[7,8] show that oscillations of the nonlinear characteristics are clearly evident at powers on the order of $10^{-1} - 1$ W/cm².

Thus, it follows from numerical estimates and experimental data, that our theory is valid at intensities $\lesssim 10^{-1} - 1$ W/cm².

3. We now find the constant current which arises upon propagation of low-frequency electromagnetic waves, for example, helicons, which interact in resonant fashion with the electrons (this effect was considered in[11] for a nonquantizing field). From Eq. (5), at $\omega_1 = -\omega_2 = \omega$ and $q_1 = -q_2 = q$, we get the current density created by the transverse wave having vector-potential components A^β and propagating along the magnetic field:

$$j_z^{(2)} = \frac{2eq_z}{mc\nu}\,\mathrm{Im}\,A^\beta A^\gamma \sigma_{\beta\gamma}(\mathbf{q},\omega), \tag{27}$$

where $\sigma_{\alpha\beta}$ is the conductivity tensor ($j_\alpha^{(1)}(\mathbf{q},\omega) = \sigma_{\alpha\beta}(\mathbf{q},\omega)A_\beta$). It follows from (27) that in an isotropic metals, where transitions with $\Delta n = 1$ are realized, we should observe a burst of current at the boundary of the Doppler-shifted cyclotron resonance. The current oscillations associated with the transitions $\Delta n = 1$ are difficult to resolve.

If the wave is propagating at an angle to **H** in an isotropic metal, or parallel to **H** in an anisotropic one, then $j_z^{(2)}$ will oscillate even under the conditions of giant quantum oscillations ($\Delta n = 0$).

Now let the electromagnetic wave propagate in the (x, z) plane. We calculate the constant current flowing along the y axis. The electromagnetic wave propagating at an angle to the magnetic field is elliptically polarized, and one of the axes of the ellipse is perpendicular to the plane of q and H, i.e., it is directed along the y axis. Consequently, the component A_y of the vector potential can be taken to be imaginary, and the remaining components A_x and A_z to be real. Inasmuch as $q_y = 0$, the quantities $M_{nm}(q)$ are real and consequently $X_{nm}^x(q)$ and $X_{nm}^z(q, p_z)$ are real. The matrix element $X_{nm}^y(q)$ is pure imaginary. It then follows that $A^\beta X_{nm}^\beta(q, p_z)$ and $A^{*\beta} X_{nm}^\beta(q, p_z)$ are real quantities. Then, as follows from Eq. (5), all the components of the current $j_y^{(2)}$ are pure imaginary if the integrals with respect to p_z have no imaginary parts, i.e., if ω and q do not lie in the region of collisionless damping. Consequently, if the wave is not damped, then the constant current $j_y^{(2)} = 0$. With account of (9), after some transformations, one can then obtain an expression for $j_y^{(2)}$. We give the formula here only for the case in which the transitions with $\Delta n = 0$ are possible:

$$\begin{aligned} j_y^{(2)} = \frac{2e^3}{\pi mc^2 q_z} A^\beta A^\gamma \sum_n \Big\{ & \left(\frac{n}{2\gamma}\right)^{1/2} X^\beta_{n-1,n}(-\mathbf{q}, p_z) X^\gamma_{nn}(-\mathbf{q}, p_z) \\ & - \left(\frac{n+1}{2\gamma}\right)^{1/2} X^\beta_{n,n+1}(-\mathbf{q}, p_z) X^\gamma_{nn}(-\mathbf{q}, p_z) \\ & - \frac{i\delta_{y\beta}}{2\gamma} M_{nn}(-\mathbf{q}) X_{nn}(-\mathbf{q}, p_z) \Big\} \{ f[\varepsilon_n(p_z)] - f[\varepsilon_n(p_z - q_z)] \}, \\ & \varepsilon_n(p_z) - \varepsilon_n(p_z - q_z) - \omega = 0. \end{aligned} \tag{28}$$

In obtaining these formulas, we have also taken it into account that $X_{nn}^x(q) = 0$. It is not difficult to establish this. Since $X_{nn}^x(q)$ is a real quantity, it follows from (9) that it is an even function of q_x. It follows from the explicit expression (7) for $X_{nn}^x(q)$ that $X_{nn}^x(q)$ is also an odd function of q_x. Hence $X_{nn}^x(q) = 0$.

1) Similar calculations can also be carried out with the aid of the method used in [2] to calculate the linear conductivity. In our case, it is necessary to take into account the next terms in the expansion of the S matrix.

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