Theory of field effect in metallic films

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The problem of variation of the conductivity of a metallic film under the action of an electric field perpendicular to the film surface is solved. It is found that in thin films changes in the surface roughness induced by an external field determine the magnitude and sign of the effect.

The field effect, i.e., the variation of the longitudinal resistance of thin plates under the influence of an electric field normal to the surface, is frequently used to study the surface of a semiconductor. At the same time, this method is hardly ever used for metals. The reason is the significant difference between the carrier densities in the metal and in the semiconductor. The field effect in a semiconductor is mainly connected with carriers induced by the external field. It can be easily seen that in a metal the number of carriers induced by an external field that is weaker than the intra-atomic field can become comparable with the number of the intrinsic carriers only in films consisting of a small number of monoatomic layers. Such an effect therefore becomes insignificant in sufficiently thick metallic films.

There is however, another mechanism whereby the external field influences the resistance of the metal film. If the film thickness is smaller than the electron mean free path, then scattering by the surface becomes the predominant scattering mechanism. A change in the potential $\varphi(z)$ near the surface, under the influence of an electric field normal to the surface, can change the conditions for electron scattering on the surface, and this leads to a change in the resistance of the entire film.

The theory of the field effect in metallic films[1,2] is based on a solution of the classical kinetic equation with the Fuchs boundary conditions for the distribution function[3].

The errors of[1] were demonstrated in[2]. The kinetic equation was solved in[2] neglecting the action of the transverse field on the non-equilibrium carriers. In the effect obtained in that paper, the non-equilibrium increment to the electron distribution function, caused by the drawing field at a concentration on the order of the surface concentration, is transported into the volume to a distance on the order of the volume mean free path l_V. The relative change in the conductivity $\Delta\sigma/\sigma$ is proportional to the ratio of the change ΔN in the number of carriers in the film to the total number of carriers N. In terms of the surface potential φ_S, the film thickness a, the Fermi energy E_F, and the Debye radius r_D, we obtain $\Delta\sigma/\sigma \sim (\varphi_S/E_F)(r_D/a)$. However, neglect of the strong transverse field while taking into account the weak drawing field in the region where the external field acts is not justified. It is apparently justified only if there is no field effect when the drawing field is neglected in the region r_D. This is correct only if the surface-diffuseness coefficient does not depend on the energy and angle of incidence of the electrons on the surface. On the other hand, if the diffuseness coefficient is not constant, then the change of the electron velocity at the surface as a result of the external field produces a change in the diffuseness. Since the change of the diffuseness changes the mobility of all the electrons in the film, one should expect this effect to be of the order of φ_S/E_F, i.e., larger than that taken into account in[2].

However, description of the field effect in metals on the basis of the classical kinetic equation is subject to serious doubts. The external magnetic field penetrates into the metal to a distance on the order of the screening length r_D. Usually r_D is of the order of the electron wavelength $\lambda \sim \pi/p_F$ in the metal (we put henceforth $\hbar = m = 1$, and p_F is the Fermi momentum). The use of the kinetic equation at distances on the order of λ from the surface is not legitimate.

In the present paper we solve the quantum problem. The metallic film is placed in a homogeneous electric field **E** perpendicular to the plane of the film. It is assumed that the potential produced by this field in the film is small in comparison with the Fermi energy; the depth of penetration of the field into the film is much smaller than the film thickness a; we consider films with thickness smaller than the carrier mean free path in the volume l_V. The problem breaks up into two parts: determination of the field distribution in the film and determination of the change of the resistance in this field.

PENETRATION OF THE FIELD INTO THE METAL

We neglect first the polarization of the internal shells, and retain only the contribution made to the polarization by the free electrons. We assume that the wave function of the electrons on the surface is equal to zero. In the linear approximation, the field is described by the diagram equation of Fig. 1, corresponding to

$$\frac{d^2\varphi}{dz^2} = -4\pi\int K(z,z')\varphi(z')dz',$$

$$K(z,z') = 2e^2\sum\frac{\psi_p^*(z)\psi_{p'}^*(z')\psi_p(z')\psi_{p'}(z)}{E_p - E_{p'}}f(E_p) + \text{c.c.}$$

Here $\psi_p = \sqrt{2/a}\exp[i(\boldsymbol{\rho}_\parallel\cdot\boldsymbol{\rho})]\sin p_z z$, $p = (\mathbf{p}_\parallel, p_z)$, $\boldsymbol{\rho} = (x, y)$, E_p is the electron dispersion law, and $f(E_p)$ is the Fermi function. This equation can be solved by a variational method. The varied potential is

$$\int\left(\frac{d\varphi}{dz}\right)^2 dz - 4\pi\int\varphi\hat{K}\varphi\,dz,$$

and we use $\varphi(z)$ in the form $\varphi_S e^{-\kappa z}$. The result of the calculations is shown in Fig. 2. As seen from the

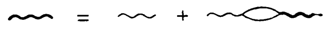

FIG. 1

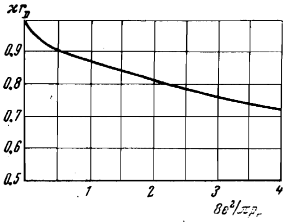

FIG. 2

figure, κ differs quite little from the reciprocal Debye screening radius $r_D^{-1} = (4e^2p_F/\pi)^{1/2}$. This means that the boundary condition for the wave function on the surface exerts little influence on the screening near the surface.

The polarization of the inner shell must be taken into account if the product of the metal-ion polarizability by its atomic density is comparable with unity. Allowance for the polarization leads, generally speaking, to two characteristic dimensions of the penetration of the external field. One of these dimensions, which is connected with the polarization of the cores, coincides with the interatomic distance in the case of non-close packing. At this distance, the field varies periodically and is attenuated in oscillatory fashion in the interior. In the case of close packing, when the inner shells are in contact, the field attenuates exponentially near the surface to a depth $r_D^0 = (2e)^{-1}(\pi/3n)^{1/6}$ determined by the concentration of the ions on the inner shell. At distances larger than interatomic, however, allowance for the polarization of the inner shells is equivalent to introducing a dielectric constant ϵ and an average field $E = E_{ext}\epsilon^{-1}\exp(-z/r_D)$. The Debye radius $r_D = (\pi\epsilon/4e^2p_F)$ is determined by the concentration of the free carriers and by the electric constant of the ionic cores. For the static dielectric constant of the core one can use the infrared limit of the frequency dependence of the real part of the dielectric constant, obtained from experiments on light reflection (see, for example,[4]). Although generally speaking the field does not vary exponentially in the interior of the metal, the exponential approximation can be regarded as satisfactory, since it is exact in a number of limiting cases. We shall henceforth assume that the potential takes the form $\varphi = \varphi_S e^{-\kappa z}$.

VARIATION OF THE FILM RESISTANCE

Specular scattering of the carriers is apparently possible only from atomically-pure and atomically-smooth surfaces. The finite diffuseness of the surface can be connected either with foreign atoms adsorbed on the surface, or with irregularity of the surface itself.

These irregularities can be divided into two types. If an incompletely-finished monolayer is produced on the surface during the film growth, then the atoms of this monolayer, just like individual "foreign" atoms, form a two-dimensional gas. It is natural to describe a sufficiently large number of incomplete monolayers as geometrical irregularities of the surface.

In thin films, the main contribution to the conductivity is made by a small group of electrons that travel almost parallel to the surface[5]. We shall assume that the characteristic angle of incidence of electrons on the surface satisfies the relation $\cos\theta = p_z/p \ll \lambda/R$, where R is the characteristic height of the irregularities of H or the dimension of the surface atom. If the work function U_0 exceeds the transverse electron energy $p_z^2/2$, then the Hamiltonian of the interaction with the surface defect listed above takes the form

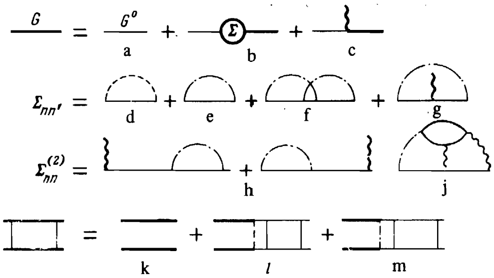

FIG. 3

$$H_{es} = \frac{1}{2}\int \frac{\partial\psi^+}{\partial z}\delta(z)u(\rho)\frac{\partial\psi}{\partial z}d^3r, \tag{1}$$

where $\psi(r)$ is the wave function of the electrons and $u(\rho)$ for geometric defects is the z-coordinate of the surface at the point ρ. This Hamiltonian was cited without proof in[6], and a proof of its applicability can be found in[7].

The residual conductivity can be expressed with the aid of the Greenwood-Peierls formula in terms of the average product of the Green's function of the ensemble of randomly distributed defects and the random function $u(\rho)$[8]. The quantity $u(\rho)$ is assumed to be a random Gaussian function with a correlator

$$W(\rho-\rho') = \langle u(\rho)u(\rho')\rangle = \frac{W_0}{\pi L^2}\exp\left[-\frac{|\rho-\rho'|^2}{L^2}\right]. \tag{2}$$

We assume henceforth that the correlation radius $L \to 0$, i.e., the heights of the surface irregularities at neighboring points are independently distributed, $W(\rho - \rho') = W_0\delta(\rho - \rho')$. The mean-squared height H of the irregularities is connected with W_0 by the relation $H^2 = W_0/\pi L^2$.

A diagram technique describing the averages of the Green's functions and their product is illustrated in Fig. 3. A thin solid line represents the bare Green's function G^0, which in the representation of the eigenfunctions of an ideal film takes the form

$$G_{nn'}{}^0(\mathbf{p}-\mathbf{p}') = \delta_{nn'}\delta(\mathbf{p}-\mathbf{p}')(2\pi)^2[E-\mathbf{p}^2/2-\pi^2n^2/2a^2 - i\delta\,\mathrm{sign}\,(E-E_F)]^{-1}.$$

A thick solid line denotes the average Green's function $G_{nn'}(p)$. A dashed line corresponds to the potential of the volume defect, and the dash-dot line, given by

$$\pi^4a^{-6}n_1n_2n_3n_4W_0\delta(\mathbf{p}-\mathbf{p}')(2\pi)^2,$$

describes the interaction with the surface defects. The wavy line denotes interaction with the external field $\varphi_S e^{-\kappa z}$. The matrix element $V_{nn'}$ of the interaction is given by

$$V_{nn'}(\mathbf{p}-\mathbf{p}') = \frac{8}{a}\frac{\varphi_s\varkappa(\pi n/a)(\pi n'/a)(2\pi)^2\delta(\mathbf{p}-\mathbf{p}')}{[\varkappa^2+\pi^2(n-n')^2/a^2][\varkappa^2+\pi^2(n+n')^2/a^2]}. \tag{3}$$

The square in the figure represents a product of two Green's functions. Since we are seeking only the response linear in the external pole, only diagrams with one wavy line are taken into account in Fig. 3.

If we confine ourselves to scattering by short-range centers in the volume, then the problem of calculating the conductivity becomes much simpler. Diagrams of the type shown in Fig. 3l and 3m make no contribution to the electric conductivity, and the Greenwood-Peierls formula takes the form

$$\sigma = \frac{e^2}{4\pi a} \sum_{nn'} \int \frac{d^2p\, dE}{(2\pi)^2} p^2 \tilde{G}^2_{nn'}(\mathbf{p}) \frac{\partial f_0}{\partial E}. \qquad (4)$$

Here $\tilde{G}_{nn'}(p)$ is the difference between the retarded and advanced Green's functions determined by the diagrams of Fig. 3, and $f_0(E)$ is the Fermi distribution function.

The Dyson equation (Fig. 3) connects the Green's function with the self-energy part $\Sigma_{nn'}(\mathbf{p})$. In first-order approximation, the field-independent part of Σ takes the form $\Sigma^{(0)}_{nn'}(p) = \Sigma^S + \Sigma^V$[9], where

$$\Sigma^s_{nn'}(\mathbf{p}) = \frac{1}{a^2} \sum_{n_1} \int \frac{d^2p_1}{(2\pi)^2} \left(\frac{\pi}{a}\right)^4 nn'n_1^2 G_{n_1}{}^0(\mathbf{p}_1) W(\mathbf{p}-\mathbf{p}_1),$$

$$\Sigma^V_{nn'}(\mathbf{p}) = \frac{i}{2\tau_V} \delta_{nn'},$$

Σ^S is the contribution made to the self-energy part by the scattering from the surface. The volume scattering is described by a relaxation time τ_V without specifying the model concretely.

We shall need to know only the imaginary part of Σ^S. In the limits of short-range centers ($Lp_F \ll 1$), we have

$$\operatorname{Im} \Sigma^s_{nn'}(\mathbf{p}) = \frac{W_0}{6\pi a} p_z p_z' (2E_F)^{3/2}, \qquad p_z = \frac{\pi n}{a}. \qquad (5)$$

The next higher approximation gives rise to diagrams with encompassing and intersection of dashed and dash-dot lines. Such diagrams in the case of volume scattering are small in comparison with diagrams without intersection in a ratio $(E_F\tau_V)^{-1}$, i.e., they are of the order of smallness usually proposed in the theory of kinetic phenomena[8].

The enclosures and intersections of the dash-dot lines diverge as a result of summation over large n. The divergences due to the inapplicability of the model (the Hamiltonian (1)) for values $p_z > \min(\pi/R, \sqrt{2U_0})$. To estimate the contribution of the divergent terms, we can use the results of[10]. The expressions obtained in that reference for the enclosures and intersections are finite in magnitude. For them to be small in comparison with the first-order diagrams, it suffices to have the height of the irregularities H (or in the general case the quantity $\sqrt{W_0/L^2}$) small in comparison with the correlation radius L.

The change of the scattering on the surface as a result of the external field gives rise to an increment to Σ, corresponding to the diagram g of Fig. 3. After simple calculations we obtain

$$\operatorname{Im} \Sigma^{(1)} = \frac{p_z p_z'}{a} \frac{W_0 \varphi_s \varkappa}{\pi} \left[\frac{2\sqrt{2E_F}}{\varkappa} - \operatorname{arctg} \frac{2\sqrt{2E_F}}{\varkappa} \right] \qquad (6)$$

The probability of scattering by volume centers situated in the surface-charge layer, being lower than the probabilities of scattering by all volume centers by a factor r_D/a, are all the more smaller than the probabilities of surface scattering. Therefore the change of scattering by volume defects can be disregarded.

As will be shown below, the main contribution to the conductivity is made by the diagonal elements of the Green's function $G_{nn}(p)$. In addition to the diagrams taken into account in $\Sigma_{nn}(p)$, a contribution to $G_{nn}(p)$ is made by diagram h of Fig. 3. When these diagrams are taken into account, $G_{nn}(p)$ takes the form

$$G_{nn}(\mathbf{p}) = [E - p^2/2 - \pi^2 n^2/2a^2 - V_{nn} - \Sigma_{nn}(\mathbf{p}) - \Sigma^{(2)}_{nn}(\mathbf{p})]^{-1}$$

where

$$\Sigma^{(2)}_{nn}(\mathbf{p}) = 2\frac{1}{a^2} \sum_{n_1 \neq n} V_{nn_1} \Sigma^{(s)}_{n_1 n}(\mathbf{p}) G_{n_1}{}^0(\mathbf{p}).$$

The imaginary part of $\Sigma^{(2)}_{nn}$ near $E = p^2/2 + \pi^2 n^2/2a^2$ is expressed in terms of $\Sigma^{(S)}_{nn}$ in the following fashion:

$$\operatorname{Im} \Sigma^{(2)}_{nn}(\mathbf{p}) = -\frac{4\varphi_s}{4p_z^2 + \varkappa^2} \operatorname{Im} \Sigma_{nn}{}^s(\mathbf{p}). \qquad (7)$$

The seeming exageration of the accuracy, due to taking into account terms proportional to the small external field in the Green's-function denominator, is insignificant if all the results are expanded in terms of this field, as will be done subsequently. It is easy to understand the meaning of the increments to the frequency of the collisions with the surface, $\Sigma^{(1)}$ and $\Sigma^{(2)}$, which are connected with the external field, by making a comparison with the limiting case when the field becomes classical in the space-share region ($p_z r_D \gg 1$). We can use here the representation of the diffuseness constant, a representation that depends on the transverse momentum and the kinetic energy of the electrons near the surface[9]. In collisions with the surface, the relaxation time is given by

$$\tau_s^{-1}(p_z, \mathbf{p}) = (p_z^2 - 2\varphi_s)^{1/2} (E_F - \varphi_s')^{3/2} W_0 T^{-1},$$

where T is the period of the transverse motion of the particle. Assuming φ_S to be small, we see that $\Sigma^{(1)}$ corresponds to an expansion in powers of φ_S/E_F, and is consequently connected with the change of the kinetic energy, and $\Sigma^{(2)} \sim \varphi_S/p_Z^2$ and is connected with the change of the transverse momentum. The contribution made to the conductivity by the diagonal elements is

$$\sigma = \frac{e^2}{\pi a} \sum_n \int \frac{d^2p}{(2\pi)^2} dE p_\parallel^2 \frac{(\operatorname{Im}\Sigma)^2 \partial f/\partial E}{[(E - p^2/2 - \pi^2 n^2/2a^2 - V_{nn} - \operatorname{Re}\Sigma)^2 + (\operatorname{Im}\Sigma)^2]^2} \qquad (8)$$

The summation over n, taking into account the fact that we are interested in the case of a classical film, should be replaced by integration with respect to p_Z:

$$\frac{1}{a} \sum_n \to \frac{1}{\pi} \int_0^\infty dp_z.$$

In addition, σ must be linearized in $\Sigma^{(1),(2)}$ and V_{nn}. This yields

$$\sigma = \sigma_0 + \sigma_1 + \sigma_2,$$

$$\sigma_0 = \frac{e^2}{(4\pi)^2} \int_0^1 (1-x^2)(2E_F)^{3/2} [\operatorname{Im}\Sigma^0(E_F, x)]^{-1} dx,$$

$$\sigma_1 = -\frac{e^2}{(4\pi)^2} \int_0^1 (1-x^2) V_{nn}(E_F, x) \frac{\partial}{\partial E_F} (E_F{}^{3/2}/\operatorname{Im}\Sigma^0(E_F, x))\, dx, \qquad (9)$$

$$\sigma_2 = \frac{e^2}{(4\pi)^2} \int_0^1 (1-x^2)(2E_F)^{3/2} [\operatorname{Im}\Sigma^{(0)}(E_F, x)]^{-2} \operatorname{Im}[\Sigma^{(1)}(x) + \Sigma^{(2)}(x)]\, dx;$$

$x = p_Z/p$, σ_0 is the conductivity of the film without an external field, σ_1 describes the change in conductivity due to the change in the electron density and to the change of the period of motion in the film, while σ_2 describes the change in the scattering by the surface under the influence of the field. If the scattering from the surface is described with the aid of a diffuseness coefficient, then the last term is the result of taking into account the dependence of the diffuseness coefficient on the transverse field.

Substituting in (9) the corresponding expressions from formulas (3), (5), (6), and (7), we obtain

$$\sigma_0=\frac{3e^2a}{16\pi W_0E_F}I_1(A),$$

$$\sigma_1=-\frac{3e^2a}{16\pi W_0E_F}\frac{\varphi_s}{a\varkappa E_F}\left[I_2\left(A,\ \frac{\varkappa}{2p_F}\right)\right.$$

$$\left.-\left(\frac{5}{2}A-A^2\frac{(2E_F)^{5/2}W_0}{3\pi a}\frac{\partial\tau_V}{\partial E_F}\right)\frac{\partial}{\partial A}I_2\right], \qquad (10)$$

$$\sigma_2=\frac{3e^2a}{16\pi W_0E_F}\frac{\varphi_s}{E_F}\left[\frac{\varkappa}{2p_F}\left(\frac{2p_F}{\varkappa}-\operatorname{arctg}\frac{2p_F}{\varkappa}\right)\frac{\partial}{\partial A}I_1-\frac{\partial}{\partial A}I_2\right];$$

Here

$$I_1(A)=\int_0^1\frac{1-x^2}{A+x^2}dx, \qquad I_2\left(A,\ \frac{\varkappa}{2p_F}\right)=\int\frac{x^2(1-x^2)\,dx}{(A+x^2)\,(\varkappa^2/4p_F{}^2+x^2)}.$$

The parameter $E = 3\pi a/(2E_F)^{5/2}W_0\tau_V = a/\eta l_V$ characterizes the ratio of the effective film thickness a/η to the mean free path in the volume; η is the surface diffuseness coefficient for electrons moving normally towards the surface.

It is seen from (10) that the contribution σ_1 to the conductivity is proportional to the quantity $(\kappa a)^{-1}$ or λ/a, and is small in comparison with σ_2. It must be taken into account only in the case of quantum films ($a \sim \kappa^{-1} > \lambda$), but in this case the formulas given above are incorrect, since it is impossible to change over in (8) from summation to integration. The most interesting case is of films in which the classical size effect is observed ($l_V > a/\eta > \lambda$). We then obtain for σ_0 the characteristic square-root dependence of the conductivity on the film thickness:

$$\sigma_0=(e^2/4\pi)\,(2E_F)^{5/4}(\tau_Va/\eta)^{1/2}. \qquad (11)$$

The form of the increment due to the change produced in the surface scattering by the external field depends on the depth of penetration κ^{-1} of the field into the conductor. If κ is not only smaller than p_Z, but also smaller than the characteristic transverse momentum $p_Z = p_F(a/\eta l_V)^{1/2}$ (this corresponds to large Debye radii), then

$$\sigma_2=\sigma_0\frac{\varphi_s}{E_F}\frac{\eta l_V}{a}\left[1+\frac{a}{\eta l_V}\right]. \qquad (12)$$

If $1 \gg \kappa/2p_F \gg (a/\eta l_V)^{1/2}$, then

$$\sigma_2=\sigma_0\frac{\varphi_s}{E_F}\left(\frac{2p_F}{\varkappa}\right)^2\left[1+\left(\frac{\varkappa}{2p_F}\right)^2\right]. \qquad (13)$$

In (12) and (13), the first term is connected with the change of the transverse momentum in the external field, and the second with the change of the kinetic energy. Their ratio is larger than unity. The reason for this is that $(p_Z) \ll E_F$.

At a small depth of penetration of the external field ($\kappa \gg p_F$), the influence of this depth on the conductivity decreases by a factor $(p_F/\kappa)^2$:

$$\sigma_2=\sigma_0\frac{\varphi_s}{E_F}\left(\frac{2p_F}{\varkappa}\right)^2\left[1+\frac{\varkappa}{6p_F}\right]. \qquad (14)$$

The smallness of the effect of the external field on the scattering can be easily understood in this case, for even in the case of a repulsive bending of the bands the electrons reach the surface by tunneling if $r_D < \lambda$.

The expressions obtained above do not take into account the contribution made to the conductivity by the off-diagonal elements of the Greens function

$$\tilde{G}_{nn'}(\mathbf{p})=2\operatorname{Im}\Sigma_{nn'}(\mathbf{p})\,/\,[\,(E-E_n(p))\,(E-E_{n'}(p))\,].$$

This contribution can be easily calculated and it turns out to be small even in the case of complete diffuseness of the surface ($W_0E_F^2 \sim 1$):

$$\sigma_{\text{off-diag}}/\sigma \leqslant (\lambda/a)^{3/2}\ln(\tau_V\dot{E}_F)\,(\tau_VE_F)^{-1}.$$

The increment to $\sigma_{\text{off-diag}}$, which is proportional to the field, is of the order of $\varphi_S\kappa^{-1}W_0^3E_F^{-1/2}\ln^2(\tau_V\kappa^2)$, and is small in comparison with σ_2.

We did not take into account diagrams of the type of Fig. 3j in the calculation. These diagrams describe the renormalization of the external-field potential because the boundary is not ideal. An estimate shows that the contribution of such diagrams to the self-energy part is of the order of $p_Z^3W_0e^2\varphi_S/ap_F$. When the condition $e^2p_Z/p_F^2 \ll 1$ is satisfied, this contribution can be neglected in comparison with the contribution $\Sigma^{(1)}$. In particular, if $\kappa^2 = e^2p_F \ll p_F^2$, these diagrams need not be taken into account for all angles of incidence of the electrons in the surface. If $\kappa \sim p_F$, then the diagrams of the type of Fig. 3j can be neglected in the case of a thin film, when the main contribution to the conductivity is made by electrons travelling at small angles p_Z/p to the surface.

DISCUSSION

As follows from the results, the field effect connected with the energy at angular dependences of the surface scattering exceeds the possible value of the effect when the diffuseness coefficient is constant. The change of conductivity has a sign opposite to that of the change in the number of carriers, i.e., the sign of the field effect is negative.

The most important limitations used in the paper are the following:

1) the effective thickness $a\eta^{-1}$ of the film is smaller than the mean free path in the volume;

2) the field penetrates into the conductor to a distance much smaller than the film thickness;

3) the characteristic transverse momentum $(p_Z)_{\text{char}} = p_F(a/\eta l_V)^{1/2}$ is smaller than the reciprocal height of the irregularities (than the dimension of the surface atom);

4) the dimension of the irregularities is much smaller than the depth of penetration of the field;

5) the surface potential φ_S is small in comparison with the Fermi energy E_F.

All the calculations were performed for zero temperature. This condition is immaterial, however, At temperatures T above the Debye temperature T_D the scattering of the electrons by the phonons in the metal is isotropic, and is therefore equivalent to scattering in the model of the short-range centers, if l_V is defined as the volume mean free path corresponding to the given temperature. At $T < T_D$, the scattering becomes anisotropic. If the characteristic angle for the scattering of an electron by a phonon $\sim T/T_D$ is small in comparison with $(a/l_V\eta)^{1/3}$, then l_V should be taken to be the transport mean free path, i.e., the length that determines the conductivity of a bulky sample. If $T/T_D > (a/l_V\eta)^{1/2}$, then any collision with a phonon increases effectively the transverse momentum of the electron, which leads to the increase of the frequency of the collisions with the surface, predicted by Azbel' and Gurzhi[11]. It is then necessary to use for l_V the nontransport mean free path, which is determined by the total scattering cross section.

The effective-mass approximation used in the present paper is apparently well satisfied only for materials with low Fermi energy, alkali metals and semimetals such as Bi and Se. The results, however, are valid qualitatively also for metals with large Fermi energy. It is easy to show that when account is taken of the exact form of the wave function of the electrons that make the main contribution to the film conductivity, the result is only a renormalization of $u(\rho)$ in expression (1) for the Hamiltonian of the interaction of the electrons with the surface defects, while the sign and order of magnitude of the matrix element of the external-field potential (3) remain unchanged. Thus, the expected sign of the field effect for noble metals is likewise negative if the conditions listed above are satisfied.

The field effect in metallic film was investigated experimentally in[12,13]. The external field used in[12] was of the order of 10^5 V/cm. At this value of the field the surface potential in ordinary metals is $10^{-3}-10^{-2}$ V. At $E_F \sim 2-5$ eV, the parameter φ_S/E_F is small. In[13], the field was produced in the metal by polarizing a ferroelectric on which the investigated metal was evaporated. The induction attained in this experiment amounted to 3×10^8 V/cm. The corresponding surface potential is comparable with the Fermi energy, so that our results are not directly applicable to a description of this experiment.

A negative sign of the field effect was indeed observed in[12] in n-Sb films. It should be noted, however, that in antimony there are two types of carriers, making it difficult to compare the theory with experiment. A positive field effect was observed in a gold film in contradiction to formula (10). The reason may be that in[12] there was no special quality control of the film surface, so that the principal role in the gold films might have been played by scattering from surface irregularities of larger dimension than the Debye radius. When a field is applied to such a surface, the scattering from the surface changes negligibly, since the field inside the metal duplicates the shape of the surface. The entire effect is determined by the change in the number of carriers in the film and by the change of the effective thickness of the film. Its value turns out to be much smaller than the possible value of the field effect due to variation in the scattering. Another possible explanation is advanced in[12], where all the changes in the conductivity are attributed to electrons localized on surface states.

It seems that the simplest and most unique experiment for the observation of the dependence of surface scattering on the external field would be to measure the width of the magnetic surface level[6] as functions of the transverse electric field.

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