Magnetophonon oscillations of the conductivity of semiconductors with hot electrons

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Oscillations of the conductivity of semiconductors in quantizing magnetic fields due to interaction with optical phonons are studied under conditions of heating of the electron gas. It is shown that the conductivity maxima are shifted toward lower magnetic field strengths relative to the magneto-phonon resonance points. The contribution to the conductivity associated with scattering by optical phonons can be negative near the maxima.

1. A number of theoretical and experimental papers[1-4] have been devoted to the study of oscillations of the conductivity of semiconductors in quantizing magnetic fields, due to the resonance interaction of electrons with optical phonons which were predicted by Gurevich and Firsov.[5] In the work of Pomortsev and Kharus,[6] it was shown that the inelasticity of the scattering of electrons by optical phonons also leads to an oscillating dependence of the power transferred from the electrons to the optical phonons on the magnetic field. In the case in which there is a heating of the electron gas, this circumstance can be the reason for the oscillation of the electron temperature, which makes easier the conditions for observation of the magnetophonon oscillations of the conductivity.[7]

The present work is devoted to the study of magnetophonon oscillation of the conductivity of semiconductors with account of the nonequilibrium of the electron gas. This conductivity can be connected both with the heating action of the static electric field and with the effect of electromagnetic radiation. A similar situation has been investigated experimentally, for example, in the works of Stradling et al.[8,9] As was observed by Stradling and Wood,[8] under conditions of the heating of the electrons by the static field, the maxima of the transverse magnetoresistance, and consequently the conductivity, is shifted relative to the point of magnetophonon resonance $N\omega_c = \omega_0$ (ω_c is the cyclotron frequency, ω_0 is the limiting frequency of optical phonons) toward smaller magnetic field strengths. The authors attributed this fact to the presence of electron transitions not to the ground Landau level, but to impurity levels.

In the present research, it is shown that such a shift in the location of the maxima should take place even in the absence of transitions to impurity levels. Here, there should be a minimum in the conductivity near the maximum (see[10]). The presence of minima in the conductivity in the vicinity of the magnetophonon resonance point was discovered experimentally in the work of Aksel'rod et al.[11] Their appearance was explained by the fact[11] that the contribution made to the conductivity of a nonequilibrium electron gas by scattering by optical phonons near the magnetophonon resonance points can be negative.[10] A more rigorous consideration of this problem is the purpose of this paper.

2. For simplicity, let us consider the case of a nondegenerate semiconductor with a quadratic and isotropic dispersion law. We shall assume that the condition of quasidiscreteness of the energy spectrum is satisfied:

$$\omega_c\tau \gg 1, \tag{1}$$

where τ is the characteristic relaxation time of the momentum of the electrons.

We can then write down the following expression for the conduction current due to the interaction with optical phonons directed transverse to the magnetic field;

$$j \sim I = \sum_{NN'} \int_{-\infty}^{\infty} dy\, y \exp\left(-\frac{y^2}{2}\right) P_{NN'}(y)$$
$$\times\left[(N_0+1)\int_{\varepsilon_+}^{\infty}\frac{d\varepsilon f_{N'}(\varepsilon)}{\varepsilon^{1/2}(\varepsilon-\Delta_{NN'}(y))^{1/2}} - N_0\int_{\varepsilon_-}^{\infty}\frac{d\varepsilon f_N(\varepsilon)}{\varepsilon^{1/2}(\varepsilon+\Delta_{NN'}(y))^{1/2}}\right]. \tag{2}$$

Here $N_0 = [\exp(\omega_0/T_0) - 1)]^{-1}$ is the number of optical phonons, $f_N(\epsilon)$ is the distribution function of the electrons,

$$\Delta_{NN'}(y) = (N-N')\omega_c + \omega_0 - Fy,$$
$$\varepsilon_\pm = \max\{0, \pm\Delta_{NN'}(y)\},$$
$$F = eEL,\quad L = (c/eH)^{1/2},\quad \hbar = k = 1,$$

T_0 is the temperature of the lattice (we neglect heating of the optical phonons).

The presence in (2) of the functions $\exp(-y^2/2)P_{NN'}(y)$ is connected with the dependence of the matrix elements of interaction of the electrons in the magnetic field with the phonons on the quantum numbers of the electrons and the momentum of the phonons. Here we have neglected the dependence of the functions $P_{NN'}(y)$ on the energy of motion of the electrons along the magnetic field. In Eq. (2), we do not take into account the broadening of the Landau levels because of collisions and we neglect the dispersion of the optical phonons.

3. The explicit form of the distribution function $f_N(\epsilon)$ is determined by the competition between the processes of heating of the electrons (by the static electric field or by radiation), relaxations on phonons, and electron-electron interaction. If the frequency of electron-electron collisions ν_{ee} exceeds the characteristic frequency of interaction with optical phonons ν_0, then the distribution function of the electrons has a Maxwellian shape with effective temperature T. We estimate the quantity ν_0 in the considered case. For this purpose, we note that the effect on the distribution function of interaction with the optical phonons is connected both with the processes of emission of the phonons and with processes of their absorption. The probability of absorption of a phonon by an electron situated, for example, at the zeroth Landau level is of order $\tau_0^{-1}N_0f_0(\epsilon)$, where $\tau_0^{-1} \sim \alpha_L\omega_0$, α_L is the constant of the interaction, and the probability of emission of a phonon by an electron situated at the N-th level is of order

$$\tau_0^{-1}(N_0+1)f_N(\varepsilon) = \tau_0^{-1}N_0\exp(\omega_0/T_0)f_N(\varepsilon).$$

Here

$$f_N(\varepsilon) \leqslant f_0(\varepsilon) \exp(-\omega_0/T_0).$$

Therefore, $\nu_0 \sim N_0 \tau_0^{-1}$. We compare the quantities ν_{ee} and ν_0. According to the work of Zlobin and Zyryanov,[7]

$$\nu_{ee} \sim \frac{2\pi m n e^4}{(2mT)^{3/2}} \ln\left(\frac{r_D^2}{L^2}\right) \exp\left(-\frac{\omega_c}{T}\right),$$

where n is the concentration of the electrons and r_D is the Debye radius, This expression is valid for $r_D \gg L$ and $\omega_c > T$. Let $m \sim 10^{-2} m_0$, $T \sim 100°K$, $\alpha_L = 0.014$, $\omega_0 \sim 3 \times 10^{13}$ sec.$^{-1}$. Then, for $\omega_0 > T_0$, we have the condition

$$\nu_{ee} > \nu_0 \sim \tau_0^{-1} \exp(-\omega_0/T_0)$$

if $n > 10^{14}$ cm^{-2}. However, ν_0 increases appreciably near the magnetophonon resonance points.

For $\omega_0 > T_0$ in the more realistic case when $\nu_{ee} \ll \nu_0$, the behavior of the distribution function, as was shown by Levinson and Mažhuolite,[12] can be essentially different in the active region (energy of the electron greater than the energy of the optical phonon) and in the passive region (energy of the electron less than the energy of the optical phonon). Upon fulfilment of the conditions

$$\delta\nu_0 \ll \nu_{ee} \ll \nu_0, \tag{3}$$

if the heating mechanism is not too intense, the distribution function in the active region can be expressed in terms of the distribution function in the passive region, which in turn is Maxwellian with an effective temperature T.

The parameter δ which enters into the relation (3) characterizes the intensity of the processes of composite scattering, in which the electron absorbs an optical phonon with subsequent emission of a phonon. In the case considered, this parameter is determined either by the dispersion of the optical phonons ($\delta \sim (m/M)^2 (\omega_0/T_0)$, M is the mass of the nucleus) or by the inelasticity connected with the shift in the centers of the Larmor electron orbits in the scattering ($\delta \sim F/T_0$). Since the dispersion of the optical phonons is weak (we shall neglect it), the value of the parameter δ in our case for not too weak electric fields is determined by the second mechanism.

We shall assume that

$$\omega_0 > T_0, \quad T - T_0 \leqslant T_0, \tag{4}$$

where T is the effective temperature of the electrons in the passive region and the condition (3) is also satisfied. The latter is possible if

$$F \ll T_0. \tag{5}$$

Two cases are possible, depending on the intensity of the heating. In the first case, the frequency ν_T, which characterizes the energy transfer rate from the source of the heating to the electrons, is much smaller than the characteristic frequency ν_0 of interaction with the optical phonons, In the second case these frequencies are of the same order.

For Joule heating of the electron gas due to conductivity associated with scattering from impurities and (or) acoustic phonons, we have $\nu_T \approx F/T_0\tau_{i,ac}$, where $\tau_{i,ac}$ is the relaxation time of the momentum on impurities and (or) acoustic phonons. If the heating of the electrons is generated by radiation of frequency $\Omega \gg \tau^{-1}$, then $\nu_T \sim \Omega\alpha_R/T_0\tau_{i,ac}$ where $\alpha_R < 1$ is the parameter of interaction of the electrons with the radiation. Actually, in collisions of an electron with an impurity or acoustic phonon, its kinetic energy is increased in the mean by an amount of the order of F or Ω. Recognizing that such processes take place with frequencies $\tau_{i,ac}^{-1}$ and $\alpha_R\tau_{i,ac}^{-1}$, respectively, we obtain the estimates given above.

4. We now consider the case in which[1)]

$$\nu_T \ll \nu_0. \tag{6}$$

Then the effect of the heating mechanism is small in the passive region in comparison with processes of interaction with optical phonons. We neglect quantities of the order of ν_T/ν_0. After this, the distribution function in the active region, as was pointed out above (see[12]), is expressed in terms of the distribution function in the passive region. Taking into account only transitions from the zeroth Landau level to the N-th resonance and the reverse transitions, which can be done if

$$\omega_c > T, \tag{7}$$

we have

$$(\varepsilon + N\omega_c > \omega_0),$$
$$f_N(\varepsilon) = \exp(-\omega_0/T_0) f_0(\varepsilon + N\omega_c - \omega_0). \tag{8}$$

Since the right side of (8) contains the distribution function in the passive region i.e., the Maxwellian function, the expression (8) can be rewritten in the following form:

$$f_N(\varepsilon) = \exp\left[\omega_0\left(\frac{1}{T} - \frac{1}{T_0}\right) + \frac{\mu - N\omega_c - \varepsilon}{T}\right]. \tag{9}$$

Here $\mu = \zeta - \frac{1}{2}\omega_c$, is the Fermi energy. In obtaining Eq. (8) and (9), we have neglected terms of the order of $(F/T_0)^2$ (see footnote 1).

The expression (9) differs from the expression obtained for $f_N(\epsilon)$ by use of the effective temperature approximation, in both the passive and the active regions, by the factor

$$\exp[\omega_0(1/T - 1/T_0)] < 1.$$

We substitute Eq. (9) in Eq. (2). Taking into account the conditions (4) and (7), we obtain

$$I = N_0 e^{\mu/T} \sum_N \int_{-\infty}^{\infty} dy\, y \exp\left(-\frac{y^2}{2}\right) P_N(y) \cdot$$
$$\times \left[\exp\left(\frac{\Delta_N}{T}\right) \int_{\tilde\varepsilon_+}^{\infty} \frac{d\varepsilon\, e^{-\varepsilon/T}}{\varepsilon^{1/2}(\varepsilon - \Delta_N + Fy)^{1/2}} - \int_{\tilde\varepsilon_-}^{\infty} \frac{d\varepsilon\, e^{-\varepsilon/T}}{\varepsilon^{1/2}(\varepsilon + \Delta_N - Fy)^{1/2}}\right]. \tag{10}$$

Here $\tilde\epsilon_\pm = \max\{0, \pm(Fy - \Delta_N)\}$, $\Delta_N = \omega_0 - N\omega_c$, $P_N(y) \equiv P_{0N}(y)$.

Carrying out integration in (10) over $d\epsilon$, we obtain the relation

$$I = \frac{F}{2T} N_0 e^{\mu/T} \sum_N \exp\left(\frac{\Delta_N}{2T}\right) \int_{-\infty}^{\infty} dy\, y^2 \exp\left(-\frac{y^2}{2}\right) P_N(y) K_0\left(\frac{|Fy - \Delta_N|}{2T}\right) \tag{11}$$

where $K_0(x)$ is the Macdonald function.

In the vicinity of the magnetophonon resonance, we get from (11)

$$I \approx I_N \approx \frac{F}{T} N_0 \exp\left(\frac{2\mu - \Delta_N}{2T}\right) a_N \ln\frac{4T}{|F - \Delta_N|},$$
$$a_N = \int_{-\infty}^{\infty} dy\, y^2 \exp\left(-\frac{y^2}{2}\right) P_N(y) \sim 1. \tag{12}$$

In obtaining (12), we used the asymptotic expression for the function $K_0(x)$:

$K_0(x) \approx \ln(2/x)$ for $x \ll 1$.

It is seen directly from (12) that the role of the cutoff factor is played by the quantity $F/4T$. The latter is natural, since we have neglected above the broadening of the Landau levels and the dispersion of the optical phonons.

We note that Eq. (12) differs from the expression obtained in the effective temperature approximation (in the active and passive regions) which, for $T - T_0 \ll T_0^2/\omega_0$ and $F < |\Delta_N| \ll T$, takes the form

$$I_N \approx \frac{F}{T} N_0 e^{\mu/T} \ln\frac{4T}{|\Delta_N|}\left[1 + \frac{\omega_0}{T_0}\frac{(T-T_0)}{\Delta_N \ln(4T/|\Delta_N|)}\right]. \quad (13)$$

5. We now assume that the frequency ν_{ee}, is not too small in comparison with ν_0. In a such a case, the effect of heating on the shape of the distribution function is more important. Since the heating leads to an increase in the number of electrons in the active region, the distribution function in the active region in the given case (cf. with Eqs. (9)) will be equal to

$$f_N(\varepsilon) > \exp\left[\omega_0\left(\frac{1}{T} - \frac{1}{T_0}\right) + \frac{\mu - N\omega_c - \varepsilon}{T}\right]. \quad (14)$$

Moreover, the difference between $f_N(\epsilon)$ and the expression on the right side of (14) is of the order ν_{ee}/ν_0. With account of (14), we obtain from Eq. (2) for the contribution ΔI to the expression (11),

$$\Delta I = N_0 e^{\mu/T} \sum_N \exp\left(\frac{\Delta_N}{T}\right) \cdot \times \int_{-\infty}^{\infty} dy\, y \exp\left(-\frac{y^2}{2}\right) P_N(y) \int_{\varepsilon_+}^{\infty} \frac{d\varepsilon\, g_N(\varepsilon)\, e^{-\varepsilon/T}}{\varepsilon^{1/2}(\varepsilon - \Delta_N + Fy)^{1/2}}. \quad (15)$$

Here we have introduced the notation

$$g_N(\varepsilon) = f_N(\varepsilon) \exp\left[-\omega_0\left(\frac{1}{T} - \frac{1}{T_0}\right) - \frac{\mu - N\omega_c - \varepsilon}{T}\right] - 1.$$

Carrying out the integration over $d\epsilon$, we get

$$\Delta I = N_0 e^{\mu/T} \sum_N g_N \exp\left(\frac{\Delta_N}{2T}\right) \cdot \times \int_{-\infty}^{\infty} dy\, y \exp\left(-\frac{y^2}{2} + \frac{Fy}{2T}\right) P_N(y) K\left(\frac{|Fy - \Delta_N|}{2T}\right). \quad (16)$$

Here $g_N \sim \nu_{ee}/\nu_0$, the function $K(x)$ has the same character with regard to singularities as the function $K_0(x)$, This is connected with the fact that the character of the singularities of the integral over $d\epsilon$ in (15) is determined by the denominator of the integrand of the expression, since the functions $e^{-v/T}$ and $g_N(\epsilon)$ are smooth ($|g_N'(\epsilon)|/g_N(\epsilon) \sim T^{-1}$). Therefore, for $x \ll 1$, we have $K(x) \approx \ln(2/x)$.

In the immediate neighborhood of the magnetophonon resonance points, $|\Delta_N| < F$, we have from (16)

$$\Delta I \approx \Delta I_N \approx \frac{F}{2T} e^{\mu/T} b_N \ln\frac{4T}{|F|}, \quad (17)$$

where $b_N \sim \nu_{ee}/\nu_0$.

In the range $F < |\Delta_N| \ll T$, the formula (16) reduces to

$$\Delta I_N \approx F N_0 e^{\mu/T} \frac{\Delta_N}{\Delta_N^2} b_N. \quad (18)$$

Here we have used the fact that for $x \ll 1$ we have $K'(x) \approx -x^{-1}$.

We note that, according to Eq. (18), $\Delta I_N < 0$ for $\Delta_N < 0$.

We now compare the absolute values of the quantity ΔI_N for magnetophonon resonance (Eq. (17)) and for $|\Delta_N| > F$ (Eq. (18)). We have

$$\frac{(\Delta I_N)_{\rm res}}{|\Delta I_N|} \approx \frac{|\Delta_N|}{T} \ln\frac{4T}{|F|}. \quad (19)$$

It then follows that $(\Delta I_N)_{\rm res}/|\Delta I_N| < 1$ for sufficiently small value of $|\Delta_N|/T$. Therefore, the contribution ΔI_N has a maximum not at the points of resonance but at some value $\Delta_N > 0$. Thus the maxima of ΔI_N are shifted toward weaker magnetic fields. For $\Delta_N < 0$, the quantity ΔI_N has a minimum at which $\Delta I_N < 0$.

We now compare the absolute values of the quantity I_N for $|\Delta_N| < F$, and ΔI_N for $F < |\Delta_N| \ll T$. Using the expressions (12) and (18), we obtain

$$\frac{(I_N)_{\rm res}}{|\Delta I_N|} \sim \frac{|\Delta_N|}{T} \ln\frac{4T}{|F|}\left(\frac{\nu_T}{\nu_0}\right). \quad (20)$$

It is seen from the last relation that for not too small values of the frequency ν_{ee} the values of I_N and ΔI_N can be equal in order of magnitude. The estimate (19) turns out to be valid also for $\nu_{ee} \gtrsim \nu_0$. Therefore, the location of the maxima of the conductivity is shifted somewhat toward weaker magnetic fields. Moreover, for $\Delta_N < 0$, the total conductivity has a minimum at which it can be negative.

The dependence of the effective temperature of the electrons in the passive region on the relation between ω_c and ω_0 can have an effect on the oscillations of the conductivity. The effective temperature in the passive region is determined from the equation of energy balance in this region. Further, the character of its behavior near the magnetophonon resonance depends on the channel through which the passive electrons give up their energy. The energy of the passive electrons can (if we neglect dispersion of the optical phonons) be given up to the acoustic phonons and the active electrons with subsequent transfer to the optical phonons. The corresponding frequencies are equal to δ_{ac}/τ_{ac} and $\nu_{ee}\exp(-\omega_0/T_0)$ in order of magnitude, where $\delta_{ac} \sim (s/TL)^2$ is the parameter of inelasticity of the scattering from acoustic phonons, and s the sound velocity.

The compound scattering from optical phonons can have a significant effect on the energy balance. In each act of this scattering, the energy of the electron is changed by an amount of the order of F. These processes (see above) take place with frequency $\delta\nu_0$. If the frequencies δ_{ac}/δ_{ac} or $\nu_{ee}\exp(-\omega_0/T_0)$ exceed the frequency $\delta\nu_0$, then the effective temperature in the passive region depends weakly on the relation between ω_c and ω_0. In the opposite case, the behavior of the effective temperature near the magnetophonon resonance can have an oscillatory character.[2)] This limit requires special consideration, however, for to find the distribution function in such a case, it is necessary to take into account the effect of the electric field on the interaction of the electrons with the optical phonons. The latter complicates the problem appreciably. We note only that for $\Delta_N < 0$ the temperature is obviously higher than for $\Delta_N > 0$. This fact can lead to a deepening of the conductivity minima and a broadening of its maxima.

6. The singularities in the behavior of the conductivity as a function of the cyclotron frequency (i.e., the magnetic field), which we have noted, are connected with the nonequilibrium electron gas and have a simple physical interpretation. Under the heating conditions the frequency of transitions with emission of optical

phonons exceeds the rate of transitions with absorption. For such transitions, a change takes place both in the kinetic and in the potential energy of the electrons. The latter is connected with the shift in the centers of the electron orbits in scattering along or opposite to the electric field. If $\Delta_N < 0$, then, since transitions are most probable between states in which the energy of motion of the electron along the magnetic field is equal to zero, the difference in the change of the kinetic energy and the energy of the optical phonon in its emission goes into an increase in the potential energy of the electron. Such processes make a negative contribution to the conductivity. If $\Delta_N > 0$, then during the scattering a decrease takes place in the potential energy of the electron. The contribution from such processes is positive. This also leads to a shift in the maxima and to the appearance of additional minima in the conductivity. A similar situation holds in the nonequilibrium two-dimensional electron gas.[13]

[1)] The conductivity associated with scattering by optical phonons also leads to heating of the electron gas. The characteristic frequency of such a process is $\nu_T \sim F\nu_0/T_0 \sim \delta\nu_0$. Because of the inequality (5), the relation (6) is always satisfied in this case (see (3)). Hence it follows, in particular, that, in finding the distribution function, one cannot take into account the effect of the electric field on the interaction of electrons with optical phonons.

[2)] If the distribution function has a Maxwellian shape in both energy regions, then the difference $T-T_0$ tends to zero logarithmically as $N\omega_c \to \omega_0$. [6]

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