Quantum theory of Crowdions at low temperatures

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A quantum theory of crowdion motion at low temperatures is developed on the basis of the defecton model [1,2]. It is shown that the crowdion changes into a quasi-particle (the crowdion wave) characterized by its quasi-momentum and a dispersion law. The fundamental characteristics of the crowdion wave are found and its interaction with long-wave phonons is considered.

At low temperatures point defects cannot be considered as localized at definite sites of the crystal lattice. In view of the possibility of quantum tunneling, they change into quasi-particles–defectons. As a rule, a point defect leads to a volume deformation of the crystal only in a small region, substantially displacing only atoms of the nearest coordination spheres. We shall call such a defect a "defect of small radius" and the corresponding quasi-particle, a "defecton of small radius." The first semiphenomenological theory of defectons of small radius was developed by A. F. Andreev and I. M. Lifshitz[1] and the microscopic theory was constructed by this author [2]. There are, however, cases when the implanted interstitial atom is in a completely different type of configuration called the "crowdion." The deformation of the crystal then occurs mainly along one of the crystallographic directions. Thus, the extra atom is in a more or less close-packed row in which even atoms far away from the impurity are displaced from their equilibrium positions in the ideal lattice (Fig. 1). The crowdion configuration can move only along this row. In this case the motion of the crowdion is accomplished owing to small displacements of its atoms and is not connected with an actual migration of the extra atom to the center of the new configuration. As a rule the crowdion is observed in complex lattices along the direction with the minimum repetition period.

The energy of formation of a crowdion is lower than the energy of formation of an isolated interstitial atom. Also relatively small is the effective potential barrier separating two neighboring equilibrium crowdion positions, and this is another important difference between the crowdion and a defect of small radius. Therefore, the methods used to analyze defectons of small radius and, in particular, the strong-coupling approximation, are not applicable here.

We shall use for the description of a crowdion in a crystal a model similar to the Frenkel'-Kontorova model[3] used to describe dislocations. We shall assume that crowdion atoms interact via elastic forces with their nearest neighbors in the chain and are located in a periodic field produced by the remaining part of the crystal. If, at the same time the crystal is deformed, then the effect of this deformation amounts to a change in the period and amplitude of this field.

The Lagrangian of such a system can be written in the form

$$L=\frac{m}{2}\sum_n \dot{\xi}_n'^2-\sum_n\left[\frac{\alpha}{2}(\xi_{n+1}'-\xi_n')^2+W\left(\frac{\xi_n'-u_n}{1+\Delta_n}\right)\right],$$

where ξ_n' is the displacement of the n-th atom of the crowdion chain from the equilibrium position in the ideal

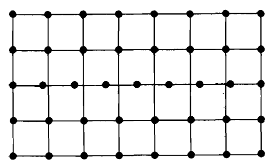

FIG. 1

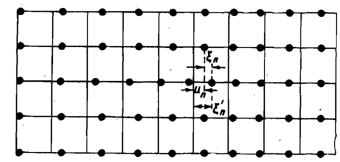

FIG. 2

FIG. 1. Arrangement of crowdion atoms in a crystal.
FIG. 2. Arrangement of crowdion atoms in a crystal deformed by an acoustic wave propagating along the x axis.

crystal; u_n is the x-th component of the deformation vector **u** at the place of the n-th site of the ideal chain (in the absence of the implanted atom) (Fig. 2); $\Delta_n \equiv u_{n+1}-u_n$; $W(\xi)$ is the periodic force field that is produced by the crystal matrix and has the period of the lattice. We shall assume that it has the form

$$W(\xi)=A(\Delta)(1-\cos 2\pi\xi) \tag{1}$$

(the lattice constant $a=1$).

Since the atoms of a crowdion are arranged along the direction with the minimum lattice period, it is natural to suppose that the interaction between the crowdion atoms is much stronger than their interaction with the other atoms of the crystal, i.e., that $\alpha \gg A$.

Let us introduce the variables:

$$\xi_n=\xi_n'-u_n.$$

Then the Lagrange function for the crowdion chain in the crystal field assumes the form

$$L=\frac{m}{2}\sum_n\dot{\xi}_n^2-\sum_n\left[\frac{\alpha}{2}(\xi_{n+1}-\xi_n)^2+W\left(\frac{\xi_n}{1+\Delta_n}\right)\right] \tag{2}$$

$$+m\sum_n\left[\dot{\xi}_n\dot{u}_n+\frac{\alpha}{m}\xi_n(\Delta_n-\Delta_{n-1})\right]+\frac{1}{2}\sum_n[\dot{u}_n^2-\alpha(u_{n+1}-u_n)^2].$$

Let us first consider a static deformation of the crystal. Then

$$L=\frac{m}{2}\sum_n\dot{\xi}_n^2-\sum_n\left[\frac{\alpha}{2}(\xi_{n+1}-\xi_n)^2+W\left(\frac{\xi_n}{1+\Delta}\right)\right]+\alpha\sum_n(\Delta_n-\Delta_{n-1})\xi_n. \tag{3}$$

If the magnitude of the deformation vector **u** varies over distances λ larger than the crowdion length l, then the last sum in (3) can be neglected. Thus, we obtain

$$L=\frac{m}{2}\sum_n\dot{\xi}_n^2-\sum_n\left[\frac{\alpha}{2}(\xi_{n+1}-\xi_n)^2+W\left(\frac{\xi_n}{1+\Delta_n}\right)\right]. \tag{4}$$

The equilibrium positions of the atoms of a crowdion in a deformed crystal can be found from the system of equations $\partial L/\partial\xi_n=0$, i.e.,

$$\alpha[\xi_{n+1}-\xi_n-(\xi_n-\xi_{n-1})]-\frac{\partial W}{\partial \xi_n}=0. \quad (5)$$

Since we are interested in the case when the transition from the displaced ($\xi=1$) to the undisplaced ($\xi=0$) region encompasses a large number of atoms, we can consider ξ_n as a continuous function of the coordinate n and go over in (5) from the system of difference equations to a single differential equation

$$\alpha\frac{d^2\xi(n)}{dn^2}-\frac{\partial W}{\partial \xi}=0. \quad (6)$$

Let us introduce the new variables

$$\zeta(n)=\xi(n)/[1+\Delta(n)], \quad \Delta(n)=\partial u/\partial n.$$

Then (6) can be written in the form

$$\alpha(1+\Delta(n))\frac{d^2}{dn^2}\{\zeta(n)(1+\Delta(n))\}=\frac{dW}{d\zeta}, \quad (7)$$

and its solution should satisfy the conditions

$$\zeta(-\infty)=1,\ \zeta(\infty)=0,\ \frac{d\zeta}{dn}\bigg|_{\pm\infty}=0. \quad (8)$$

The quantity $\Delta(n)$ changes over distances of the order of λ, whereas $\zeta(n)$ changes over distances of the order of the crowdion length l. Therefore, in the most interesting case, when $\lambda \gg l$, the derivatives of Δ can be neglected. Thus, Eq. (7) becomes

$$\alpha(1+\Delta(n))^2\frac{d^2\zeta}{dn^2}=\frac{dW}{d\zeta}. \quad (9)$$

In the case of the sinusoidal field (1) the solution of Eq. (9) satisfying the conditions (8) has the form

$$\zeta(n,x)=\frac{\xi(n,x)}{1+\Delta(n)}=\frac{2}{\pi}\operatorname{arctg}\left\{\exp\left(\frac{x-n}{\mathscr{L}}\right)\operatorname{tg}\left[\frac{\pi}{2}\zeta(x)\right]\right\}, \quad (10)$$

$$\mathscr{L}=\frac{1+\Delta(x)}{2\pi}\sqrt{\frac{\alpha}{A(\Delta)}}$$

($\mathscr{L}$ is the length of a crowdion in a deformed crystal, while x is the number of the atom displaced through the distance $\zeta(x)$). We shall henceforth denote by x the number of the atom at the crowdion center. Then $\zeta(x)=1/2$, and, consequently,

$$\zeta(n,x)=\frac{2}{\pi}\operatorname{arctg}\exp\left(\frac{x-n}{\mathscr{L}}\right). \quad (11)$$

Decomposing $A(\Delta)$ in powers of the small deformation Δ and retaining only the linear terms, we obtain

$$A(\Delta)=A[1-2B\Delta_x-2C(\Delta_y+\Delta_z)],\ \Delta_\sigma=\frac{\partial u_\sigma}{\partial\sigma}\bigg|_{y=z=0},$$

$$B=-\frac{1}{2A}\frac{\partial A}{\partial\Delta_x}\bigg|_{\Delta=0},\quad C=-\frac{1}{2A}\frac{\partial A}{\partial\Delta_y}\bigg|_{\Delta=0}.$$

(We assumed for simplicity that the crystal is elastically isotropic in the $x=0$ plane.) Then

$$\mathscr{L}=[1+(1+B)\Delta_x+C(\Delta_y+\Delta_z)]l/\pi,$$

where $l=\frac{1}{2}\sqrt{\alpha/A}$ is the length of the crowdion in the undeformed crystal.

If we displace the atom x through a small distance η_x, then a rearrangement of the atoms occurs, as a result of which the center of the crowdion goes over into the new point $x'=x+\eta_x l$. In fact, if we substitute $\zeta(x)=\frac{1}{2}+\eta_x$ into (10), then the new configuration is given by the function

$$\tilde{\zeta}(n,x)=\frac{2}{\pi}\operatorname{arctg}\exp\left(\frac{x-n+\eta_x l}{\mathscr{L}}\right)=\zeta(n,x+\eta_x l).$$

The transfer of the center of the crowdion to the new site is then not connected with an actual migration of the extra atom to that point. Since $l\gg 1$, very small displacements of the individual atoms of the chain are sufficient for the displacement of the crowdion. Taking account of the foregoing, we shall assume that x is a continuous coordinate that characterizes completely the location of the crowdion.

Let us find the static energy of the crowdion in the crystal field. For this purpose let us substitute (11) into the expression for the potential energy:

$$U=\sum_n\left[\frac{\alpha}{2}(\xi(n+1,x)-\xi(n,x))^2+W\left(\frac{\xi(n,x)}{1+\Delta(x)}\right)\right].$$

Similar sums have been repeatedly considered in papers dealing with the Frenkel'-Kontorova model. The method used to compute them is well expounded in [4,5]. Therefore, we shall only quote the results here:

$$U=U_0(\Delta)+\frac{1}{2}U_1(\Delta)\cos 2\pi x+O(e^{-2\pi l}),$$
$$U_0(\Delta)=U_0(1+(1-B)[\Delta_x+\gamma(\Delta_y+\Delta_z)]),$$
$$\gamma=C/(1-B),\quad U_0=2mc^2/\pi l,$$
$$U_1(\Delta)=6mc^2(1+\Delta_x)^2\exp(-\pi^2\mathscr{L}),$$

where $c=\sqrt{\alpha/m}$ is the velocity of sound in the crowdion chain.

Thus, we see that besides the constant term $U_0(\Delta)$, which has the meaning of the average energy of the crowdion, the potential energy also has a periodic part whose amplitude $U_1(\Delta)$ depends exponentially on the crowdion length.

Let us now turn to the equations of motion that follow from the Lagrange function (2). In the continuum approximation, they have the form

$$m\frac{\partial^2\xi}{\partial t^2}=\alpha\frac{\partial^2\xi}{\partial n^2}-\frac{\partial W}{\partial\xi}. \quad (12)$$

We shall be interested in the solutions for which the crowdion moves with constant velocity v, i.e., we shall assume that the sought-for solution has the form

$$\xi(n,t)=\xi(n-vt). \quad (13)$$

Substituting (13) into (12), we obtain

$$(c^2-v^2)\frac{d^2\xi}{dn^2}=\frac{\partial W}{\partial\xi}. \quad (14)$$

Except for notation, this equation coincides with (6). Consequently, the solution of interest to us has the form

$$\xi(x,n,t)=(1+\Delta_x)\frac{2}{\pi}\operatorname{arctg}\exp\left(\frac{x+vt-n}{\mathscr{L}_v}\right).$$

It is quite significant that the length of a moving crowdion depends on its velocity:

$$\mathscr{L}_v=\mathscr{L}\sqrt{1-v^2/c^2}.$$

Then the total crowdion energy

$$E_0=\frac{m}{2}\sum_n\dot{\xi}^2(x,n,t)+\sum_n\left[\frac{\alpha}{2}(\xi(n+1,x,t)-\xi(n,x,t))^2+W\left(\frac{\xi(n,x,t)}{1+\Delta_x}\right)\right]=\frac{U_0(\Delta)}{\sqrt{1-v^2/c^2}} \quad (15)$$
$$+\frac{1}{2}U_1(\Delta)\left(1-\frac{v^2}{c^2}\right)\exp\left\{-\pi^2\mathscr{L}\sqrt{1-\frac{v^2}{c^2}}\right\}\cos(x+vt).$$

The formula (15) is valid for velocities for which

$$l\sqrt{1-v^2/c^2}\gg 1,$$

since on the basis of this condition, on going over from the difference equations to the differential equation, we restricted ourselves to only second derivatives. The coefficient in front of the leading derivative in Eq. (14)

vanishes when $v = c$, and, consequently, we must take higher-order derivatives into account in order to consider the region $v \sim c$.

If the crowdion velocity is small compared to the velocity of sound in the chain, then (15) can be represented in the form of a sum of kinetic and potential energies:

$$E_0 = U_0(\Delta) + \frac{1}{2} U_1 \cos 2\pi x + \frac{mv^2}{\pi l}[1 + (1 - B)(\Delta_x + \gamma(\Delta_y + \Delta_z))].$$

Thus, we see that a crowdion behaves like a particle of mass

$$\mu(\Delta) = \frac{2m}{\pi l}[1 + (1 - B)(\Delta_x + \gamma(\Delta_y + \Delta_z))],$$

located in a periodic field of exponentially small depth. As the crystal expands, $U_0(\Delta)$ and $\mu(\Delta)$ increase, while $U_1(\Delta)$ decreases.

Of great interest is the case when the deformation of the crystal is caused by the propagation of an elastic wave whose wavelength $\lambda \gg l$. In this case, eliminating from (2) the total time derivatives

$$\frac{d}{dt}(\xi_n \dot{u}_n) = \dot{\xi}_n \dot{u}_n + \xi_n \ddot{u}_n,$$

we obtain

$$L = \frac{m}{2}\sum_n \dot{\xi}_n^2 - \sum_n \left[\frac{\alpha}{2}(\xi_{n+1} - \xi_n)^2 + W\left(\frac{\xi_n}{1 + \Delta_n}\right)\right] - \sum_n \xi_n [m\ddot{u}_n - \alpha(\Delta_n - \Delta_{n-1})].$$

The quantities u_n correspond to the displacements of the centers of mass of the crystal-lattice cells along the crowdion chain and become, in the case of a continuous elastic medium, the x component of the deformation vector $\mathbf{u}$ along the crowdion axis. Let us write down the equations of the theory of elasticity

$$\frac{M}{V_0}\ddot{u}^\sigma = C_{\sigma\mu\nu\rho}\frac{\partial^2 u^\rho}{\partial x_\mu \partial x_\nu},$$

where $C_{\sigma\mu\nu\rho}$ is the matrix of the elastic moduli, $M = \Sigma m_s$ is the mass of the cell, and V_0 is the cell volume. In complex lattices the quantities M and $C_{\sigma\mu\nu\rho}$ can differ widely from m and α which describe the oscillations of the crowdion chain. Consequently, the speed s of sound in the crystal does not, generally speaking, coincide with the speed of sound in the crowdion chain. In those cases[1)] when $s \ll v \ll c$ the deformation caused by the elastic wave can be assumed to be quasistatic, and the results obtained above can be used.

In the opposite limiting case when $v \ll s \ll c$ the crowdion has time to adjust itself to the deformation, but it cannot undergo any substantial displacement during one period. In this case the crowdion behaves as if it is in a variable field. The crowdion mass, the mean energy $U_0(\Delta)$, and the amplitude of the periodic potential oscillate with the frequency of this field.

THE CROWDION WAVE

Let us consider the quasi-static case. Then the Hamiltonian for a crowdion in a crystal can be written in the form

$$H = \frac{p^2}{2\mu} + \frac{1}{2} U_1 \cos 2\pi x + U_0(\Delta).$$

If $\lambda \gg l$, then Δ is not only small, but it is a slowly varying function of its argument as well. Therefore, we can regard Δ as a parameter and write the Schrödinger equation in the form

$$\frac{d^2\psi}{dx^2} + \frac{2\mu}{\hbar^2}\left(E - \frac{1}{2} U_1 \cos 2\pi x\right)\psi = 0, \tag{16}$$

where

$$E = \varepsilon - U_0(\Delta).$$

After the substitution $\pi x = z$, it reduces to the Mathieu equation:

$$\frac{d^2\psi}{dz^2} + (\varkappa - \delta \cos 2z)\psi = 0, \quad \delta = \frac{\mu U_1}{\pi^2\hbar^2}, \quad \varkappa = 2\mu E / \pi^2\hbar^2.$$

According to the Bloch theorem, the wave function of the crowdion excitation can be written in the form

$$\psi_k(x) = N^{-1/2} e^{ikx} u_k(x), \tag{17}$$

where $u_k(x)$ is a periodic function with the period of the lattice. Generally speaking, the eigenvalue spectrum of Eq. (16) that allows solutions in the form (17) is quite complicated. It however gets simplified greatly if account is taken of the fact that in all physically reasonable cases $U_1 \ll \pi^2\hbar^2/\mu$ and, consequently, $\delta \ll 1$. Then the dependence of the energy on the wave vector can be determined from the equation[6]

$$\cos k = \cos \pi\varkappa^{1/2} + \frac{\pi^2\delta^2}{4(1 - \varkappa)}\frac{\sin \pi\sqrt{\varkappa}}{\pi\sqrt{\varkappa}}$$

For small values of the energy ($E \ll \hbar^2/\mu$) we obtain from this

$$\varepsilon(k) = U_0(\Delta) + \frac{\mu U_1^2}{(2\pi\hbar)^2} + \frac{\hbar^2 k^2}{2\mu^*}, \tag{18}$$

where the effective mass

$$\mu^* = \mu(1 - \mu^2 U_1^2 / 2\pi^4\hbar^4).$$

Thus, the dispersion law for the lowest band of the crowdion excitation is quadratic. In spite of the presence of the potential barriers, owing to tunneling, the crowdion wave moves through the crystal as a free particle of mass close to the crowdion mass.

The wave function of the crowdion excitation for $k = 0$ has the form

$$\psi_0(x) = N^{-1/2}\,\mathrm{ce}\,(x, \delta),$$

where $\mathrm{ce}(x, \delta)$ is the nodeless Mathieu function. For $\delta \ll 1$ it varies slowly and for $\delta = 0$, $\mathrm{ce}(x) = 1$. Consequently, the wave functions of the low-lying crowdion excitations can be approximated by plane waves.

INTERACTION WITH PHONONS

The crowdion is a one-dimensional formation in a three-dimensional crystal. Therefore, the crowdion wave has properties which fundamentally distinguish it from the other quasi-particles in a solid (electrons, excitons, magnons, etc.). It propagates only in a definite direction and is localized on one line. Its interaction with phonons takes place via deformations occurring in the immediate vicinity of the crowdion chain. Therefore, the wave function of the crowdion excitation in the continuum approximation can be written in the form

$$\psi_k(\mathbf{r}) = N^{-1/2} e^{ikx} u_k(x)\delta(y)\delta(z),$$

where N is the number of atoms in the crystal.

Let us consider the scattering of a crowdion wave by long-wave acoustic phonons in the quasi-static case. Let us for this purpose use the deformation potential method. The crowdion excitation-phonon interaction energy can be obtained from the local dispersion law (18). If we neglect the exponentially small correction $\sim U_1^2$, then it is equal to

$$U_0(\Delta)-U_0=\varepsilon_0(\Delta_x+\gamma(\Delta_y+\Delta_z)),\quad \varepsilon_0=(1-B)U_0. \qquad (19)$$

In the elastic continuum approximation the deformation vectors can be written in the form

$$\mathbf{u}(\mathbf{r})=\frac{1}{\sqrt{N}}\sum_{\mathbf{q}}\frac{\mathbf{e}_{\mathbf{q}}}{\sqrt{2M\omega_{\mathbf{q}}}}(a_{\mathbf{q}}e^{i\mathbf{q}\mathbf{r}}+a_{\mathbf{q}}^{+}e^{-i\mathbf{q}\mathbf{r}}),$$

where $a_{\mathbf{q}}^{+}$ and $a_{\mathbf{q}}$ are the creation and annihilation operators for phonons with momentum $\mathbf{q}$, $\mathbf{e}_{\mathbf{q}}$ are the polarization vectors and $\omega_{\mathbf{q}}=sq$ is the dispersion law for long-wave phonons. Then

$$\Delta_\sigma=\frac{i}{\sqrt{N}}\sum_{\mathbf{q}}\frac{e_{\mathbf{q}}^{\sigma}q_\sigma}{\sqrt{2M\omega_{\mathbf{q}}}}(a_{\mathbf{q}}e^{iq_xx}-a_{\mathbf{q}}^{+}e^{-iq_xx}).$$

Since we are considering only longitudinal acoustic waves, $e_{\mathbf{q}}^{\sigma}q_\sigma=q_\sigma^2/q$ and, consequently,

$$\Delta_\sigma=\frac{i}{\sqrt{N}}\sum_{\mathbf{q}}\frac{q_\sigma^2}{q\sqrt{2M\omega_q}}(a_{\mathbf{q}}e^{iq_xx}-a_{\mathbf{q}}^{+}e^{-iq_xx}).$$

According to the general principles of quantum mechanics, the matrix element for the crowdion excitation transition from the k- to k′-state under the action of the perturbation (19) is equal to

$$M_{kk'}=\frac{i\varepsilon_0}{N^{1/2}\sqrt{N}}\sum_{\mathbf{q}}\frac{q_x^2+\gamma q_\perp^2}{q\sqrt{2M\omega_{\mathbf{q}}}}\int(a_{\mathbf{q}}\exp\{i(q_x-k+k')x\}$$
$$-a_{\mathbf{q}}^{+}\exp\{-i(q_x+k-k')x\})\,dx.$$

The matrix element $M_{kk'}$ is different from zero only in the case when the momentum conservation law: $k'=k+q_x$, or $k'=k-q_x$, is fulfilled. These processes can be interpreted as absorption and emission of the x-th phonon component. The corresponding probabilities are equal to

$$W_\pm=\frac{\varepsilon_0^2}{2\pi N\hbar}\int\frac{(q_x^2+\gamma q_\perp^2)^2}{2p_0q^3}\begin{Bmatrix}n_q\\ n_q+1\end{Bmatrix}\delta(\varepsilon(k\pm q_x)-\varepsilon(k)\mp\hbar\omega_q)\,dq_y\,dq_z,$$

where

$$n_q=[e^{\hbar\omega_q/T}-1]^{-1},\quad p_0\equiv\frac{Ms}{\hbar}.$$

Let us transform the δ-function to the form

$$\delta(\varepsilon(k+q_x)-\varepsilon(k)-\hbar sq)=\frac{q_x(2k+q_x)}{\mu s^2}\,\delta\left[q_\perp^2+q_x^2-q_x^2\left(\frac{2k+q_x}{2p}\right)^2\right]$$

where $p=\mu s/\hbar$. After an elementary integration we obtain

$$W_+=\frac{\varepsilon_0^2}{2\hbar Ms^2N}\left(\frac{q_x}{2k+q_x}\right)^2\left[1-\gamma+\gamma\left(\frac{2k+q_x}{2p}\right)^2\right]^2$$
$$\times\left[\exp\left\{\frac{\varepsilon'(k)}{T}\left|\frac{2q_x}{k}\left(1+\frac{q_x}{2k}\right)\right|\right\}-1\right]^{-1},$$

$$\varepsilon'(k)=\hbar^2k^2/2\mu.$$

Similarly,

$$W_-=\frac{\varepsilon_0^2}{2\hbar Ms^2N}\left(\frac{q_x}{2k-q_x}\right)^2\left[1-\gamma+\gamma\left(\frac{2k-q_x}{2p}\right)^2\right]^2$$
$$\times\left[1-\exp\left\{-\frac{\varepsilon'(k)}{T}\left|\frac{2q_x}{k}\left(1-\frac{q_x}{2k}\right)\right|\right\}\right]^{-1}.$$

If the wave propagates along the axis of the crowdion, then it follows from the energy and momentum conservation laws that

$$(2k\pm q_x)/2p=-1,$$

$$W_+\approx W_-=\frac{\varepsilon_0^2}{2\hbar Ms^2N}\left(\frac{q_x}{2p}\right)^2\left[\exp\left(\frac{\hbar sq_x}{T}\right)-1\right]^{-1}.$$

As was to be expected, the probability of emission of such a phonon does not depend on γ. Since in this case phonons with wave vectors $q_x\sim k$ are emitted and absorbed,

$$\hbar sq_x/T\sim\hbar sk/T\sim(\mu s^2/T)^{1/2}.$$

Thus, for $T\gg\mu s^2$

$$W=\frac{\varepsilon_0^2}{8Ms^2N}\frac{q_x}{\mu s}\frac{T}{\mu s^2}.$$

If, on the other hand, oscillations with all possible directions $\mathbf{q}$ are excited in the crystal, then $q\sim T/\hbar s$, whereas $q_x\sim k\sim\sqrt{2\mu T}/\hbar$. Consequently,

$$q_x/q\sim(\mu s^2/T)^{1/2}\sim s/v\ll 1.$$

It can be seen that, in the main, phonons in the directions almost perpendicular to the crowdion axis will be emitted and absorbed. In this case the phonon energy can be of the order of the energy of the crowdion excitation itself, and, therefore, the process cannot be assumed to be elastic.

The author expresses his profound gratitude to I. M. Lifshitz for a discussion of the paper and for valuable hints.

[1)]Such a situation arises, for example, in a complex lattice consisting of light and heavy particles, with the crowdion chain formed by the light particles.

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