Effect of optical action on the "surface elasticity" of solid media

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(Submitted July 5, 1972)

Zh. Eksp. Teor. Fiz. **64**, 301-305 (January 1973)

The influence of interference of optical radiation on the spectrum of the surface waves of transparent condensed dielectrics is considered. The corresponding dispersion equations are given. It is shown that under certain conditions the oscillatory regime of the surface waves in a liquid can be disrupted because of the vanishing of the effective surface tension. Numerical estimates are given.

When radiation is incident on the interface between two transparent dielectrics with different dielectric constants ϵ_1 and ϵ_2, the pressure of the ponderomotive forces experiences a jump equal to

$$f=\frac{\varepsilon_2-\varepsilon_1}{8\pi}\left[\left(\frac{\varepsilon_2}{\varepsilon_1}-1\right)E_\perp{}^2+E^2\right], \tag{1}$$

where $\mathbf{E}$ is the field intensity in the medium with the dielectric constant ϵ_2, and $\mathbf{E}_\perp$ is the component of this field normal to the interface; it is necessary to take into account in (1) only the low-frequency component (averaged over the period of the field)[1)].

We wish to examine below the conditions under which this pressure jump can lead to significant changes in the spectrum of surface waves (SW) traveling over the surfaces of condensed media (capillary waves in the case of a liquid, elastic surface waves in a solid).

If we place the interface in the field of two interfering light waves along planes of equal intensity, then, depending on the position of the level of the surface relative to the extrema of the interference pattern, the frequency of the wave of displacements of the boundary surface $\zeta = \zeta_0\exp(i\mathbf{q}\cdot\mathbf{r} - \Omega t)$ should experience a shift, the magnitude and sign of which are determined by the parameters of the interference picture (here $\mathbf{q} = \{q_x, q_y\}$ and $\mathbf{r} = \{x, y\}$ are two-dimensional vectors in the plane $z = 0$). Under certain definite conditions, the oscillatory regime may stop altogether, as a result of which the motion of the surface becomes unstable.

Let us consider the simplest case, when two plane electromagnetic waves with equal frequencies are normally incident in opposition to each other (along the z axis) on an interface coinciding with the (x, y) plane[2)]. We first estimate the order of magnitude of the effect, starting from qualitative considerations. In the interference field of the waves in question (with intensities I_1 and I_2), the gradient of the pressure jump

$$\partial f/\partial z \sim (I_1I_2)^{1/2}k_0/c,$$

and the change of the electromagnetic energy per unit interface surface area, due to the existence of a surface wave with amplitude $\zeta_0 \ll 1/k_0$, is equal to $\Delta W \sim (I_1I_2)^{1/2}k_0\zeta_0^2/c$ ($k_0 = \omega_0/c$ is the wave number of the electromagnetic waves and c is the speed of light in vacuum; it is assumed that $\Delta\epsilon = |\epsilon_2 - \epsilon_1| \sim 1$).

Obviously, the effect of the radiation field on the dispersion properties of the surface wave is significant in the situation under consideration if ΔW becomes comparable with the average surface-wave energy $\mathscr{E} \sim \rho(\Omega\zeta_0)^2/q$ (ρ is the density of the medium, q is the wave number of the surface wave, and Ω is its frequency). From the condition $\Delta W \sim \mathscr{E}$ we obtain an estimate for the characteristic radiation intensity:

$$(I_1I_2)^{1/2} \sim \rho\Omega^2c/qk_0. \tag{2}$$

We proceed now to a more rigorous quantitative analysis. The electromagnetic waves incident on the interface (along the z axis) have identical linear polarization (along the y axis) and can be represented in the form

$$E_1 = |\mathbf{E}_1|e^{i\varphi_1}\exp[i(-n_1k_0z - \omega_0t)],$$
$$E_2 = |\mathbf{E}_2|e^{i\varphi_2}\exp[i(n_2k_0z - \omega_0t)],$$

where $n_i = \epsilon_i^{1/2}$. It is assumed that the relations $|k_0\zeta| \ll 1$ $|q\zeta| \ll 1$, and $q < n_ik_0$ are satisfied, making it possible to neglect the contribution of the scattered radiation field and the pressure jump (1).

Using the Fresnel formulas for a flat interface, it is easy to obtain an expression for the first unvanishing term in the expansion of f powers of the displacements of the boundary surface:

$$f_\zeta = 4\zeta k_0(n_2 - n_1)c^{-1}(n_1n_2I_1I_2)^{1/2}\sin(\varphi_1 - \varphi_2), \tag{3}$$

where

$$I_i = cn_i|\mathbf{E}_i|^2/8\pi.$$

The solution of the polarized Navier-Stokes equations with the boundary conditions for a free liquid surface ($n_1 = 1$), perfectly analogous to the usual capillary problem[4], but with allowance for the pressure jump (3) in the boundary conditions, leads to the following dispersion equation for surface waves on the surface of a liquid

$$\frac{\alpha_{\text{eff}}}{\rho}q^3 + (2\nu q^2 - i\Omega)^2 = (2\nu q^2)^2\left(1 - i\frac{\Omega}{\nu q^2}\right)^{1/2}$$
$$\alpha_{\text{eff}} = \alpha - (I_1I_2)^{1/2}k_0\beta/cq^2, \tag{4}$$

here α is the surface-tension coefficient, ρ is the density of the liquid, $\nu = \eta/\rho$ is its kinematic viscosity, and

$$\beta = 4(n_2 - 1)n_2^{1/2}\sin(\varphi_1 - \varphi_2).$$

It is seen from (4) that the character of the influence of the radiation field and the considered case of interference of two waves on the dispersion properties of the surface waves is determined not only by the intensity of the interfering waves, but also by the difference between their phases (by the sign of β). At $\beta > 0$, the effect of surface tension decreases, and at $\beta < 0$, to the contrary, it increases. We consider first the latter case as applied to a low-viscosity liquid, when $4q\eta^2/(\rho|\alpha_{\text{eff}}|) \ll 1$). Then Eq. (4) has the roots

$$\Omega_{1,2} = \pm(\alpha_{\text{eff}}q^3/\rho)^{1/2} - 2i\nu q^2. \tag{5}$$

It follows therefore that at

$$(I_1I_2)^{1/2} = \frac{c\alpha q^2}{k_0|\beta|} = \frac{\rho\Omega_0{}^2c}{k_0q|\beta|} \tag{6}$$

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the frequency of the capillary wave $\Omega_0 = (\alpha q^3/\rho)^{1/2}$ with a given wave vector increases by a factor $\sqrt{2}$ (cf., (2)). For a frequency shift $\Delta\Omega \approx \pm 2\nu q^2$ we obtain accordingly

$$(I_1I_2)^{1/2} = \frac{4\nu q^2}{\Omega_0}\frac{\rho\Omega_0^2 c}{k_0 q|\beta|}. \qquad (7)$$

Greatest interest attaches to the case $\beta > 0$, when "concellation" of the surface tension and a transition to negative values of α_{eff} take place. To analyze this case, we rewrite (4) in the form

$$\alpha_{\text{eff}}\rho / q\eta^2 + (2+x)^2 = 4(1+x)^{1/2}, \ x = -i\Omega/\nu q^2. \qquad (8)$$

We obtain the solution of this equation in the vicinity of the point $\alpha_{\text{eff}} = 0$.

The equation $(2+x)^2 = 4(1+x)^{1/2}$ has two roots, $x_1 = 0$ and $x_2 = -0.95$. The root x_2 corresponds to a rapidly damped solution $(\Omega_2 \approx -i095\nu q^2)$ and is therefore of no interest. For x_1 we obtain from (8) the first nonvanishing term in the small parameter $\delta \equiv |\alpha_{\text{eff}}|\rho/q\eta^2$, and as a result we get

$$\Omega = i\mu, \quad \mu = \frac{1}{2\nu q^2}\left[(I_1I_2)^{1/2}\frac{k_0 q}{\rho c}\beta - \Omega_0^2\right]. \qquad (9)$$

Thus, at $\beta > 0$, with increasing intensities of the interfering fields, the frequency of the traveling capillary waves refers to the decreases, followed by a "stoppage" of the oscillatory motion of the surface at $\alpha_{\text{eff}} = 0$, when the quantity $(I_1I_2)^{1/2}$ is determined from formula (6), and finally instability sets in at $\alpha_{\text{eff}} < 0$ $((I_1I_2)^{1/2} > c\alpha q^2/k_0\beta)$:

$$\zeta = \zeta_0 e^{\mu t} e^{iqr}, \ \mu > 0.$$

For waves on the surface of the solid, the problem can be solved in similar fashion, i.e., it reduces to a solution of the linear equations of elasticity theory with the boundary conditions on a free surface[5], in which the pressure jump (1) must be taken into account. For an isotropic body this leads to the dispersion equation

$$(q^2 + \varkappa_t^2)^2 - 4q^2\varkappa_t\varkappa_l + \frac{\varkappa_l\Omega^2}{\rho c_t^4}(I_1I_2)^{1/2}\frac{k_0}{c}\beta = 0;$$

$\varkappa_t = (q^2 - \Omega^2/c_t^2)^{1/2}$, $\varkappa_l = (q^2 - \Omega^2/c_l^2)^{1/2}$, where c_t and c_l are respectively the velocities of the transverse and longitudinal sound waves. In this case it is advantageous to seek the small corrections to the frequency of the Rayleigh wave, $|\Delta\Omega| \ll \Omega_0$. A simple calculation leads to

$$\Delta\Omega = (I_1I_2)^{1/2}\frac{k_0\beta}{4\rho c c_t}\cdot$$
$$\times \xi_0\left(1 - \frac{\xi_0^2 c_t^2}{c_l^2}\right)^{1/2} \Big/ \left[2 - \xi_0^2 - \left(\frac{1 - \xi_0^2 c_t^2/c_l^2}{1 - \xi_0^2}\right)^{1/2} - \frac{c_t^2}{c_l^2}\left(\frac{1 - \xi_0^2}{1 - \xi_0^2 c_t^2/c_l^2}\right)^{1/2}\right] \qquad (10)$$

where the dimensionless parameter ξ_0, which enters in the dispersion equation $\Omega_0 = c_t q \xi_0$, depends only on the ratio c_t/c_l, and lies, as is well known, in the range 0.874–0.955 for all substances[5]. We note that the frequency shift $\Delta\Omega$ for a solid, (10), does not depend on the length of the Rayleigh wave.

All the foregoing arguments remain in force if the two beams are incident on the surface obliquely from opposite sides, but in such a way that the projections of their wave vectors on the surface $z = 0$ are equal: $\mathbf{k}_{t1} = \mathbf{k}_{t2}$; $|\mathbf{k}_{t1,2}| = k_{0x}$; it is necessary only to replace β everywhere by the quantity

$$\beta_1 = \frac{4(k_{2z} - k_{0z})k_{2z}k_{0z}}{k_0^3(n_1n_2)^{1/2}}\sin(\varphi_1 - \varphi_2);$$
$$k_{2z} = (\varepsilon_2 k_0^2 - k_{0x}^2)^{1/2}, \ k_{0z} = (\varepsilon_1 k_0^2 - k_{0x}^2)^{1/2};$$

and stipulate satisfaction of the conditions $|q| \ll k_{0,2z}^2/2k_{0x}$, $k_{0,2z}$.

We present a few numerical estimates[3)]. For a solid transparent dielectric (for example, quartz or glass) with parameters $\rho \approx 2.5$ g/cm^3, n = 1.7, $c_t = 3 \times 10^5$ cm/sec, $c_l = 4.75 \times 10^5$ cm/sec, and $\zeta_0 = 0.91$, a phase shift $\Delta\Omega = 3 \times 10^2$ sec^{-1} is ensured at $(I_1I_2)^{1/2} = 10^7$ W/cm^2. Such a frequency shift is comparable with the damping γ_S of sound in glass and is larger by one order of magnitude than γ_S in quartz at a frequency $\Omega_0 = 2.7 \times 10^7$ sec^{-1}, i.e., at $q = 10^2$ cm^{-1}.

In the case of a liquid (for example, ethyl ether) with parameters $\alpha = 17$ dyn/cm, $\rho = 0.714$ g/cm^3, and n = 1.35 at $q = 10^3$ cm^{-1} and accordingly $\Omega_0 = 1.54 \times 10^5$ sec^{-1}, to increase the frequency by a factor $\sqrt{2}$ or to interrupt the oscillatory regime and to obtain instability (i.e., to satisfy the condition (6)) it is necessary to have $(I_1I_2)^{1/2} = 3.1 \times 10^5$ W/cm^2. On the other hand, for a frequency shift $\Delta\Omega \approx \pm 2\nu q^2$, which determines, for example, the shift at half the maximum of the scattered-light line on the surface of this liquid in the direction

$$\mathbf{k}_1 = \{q_x, q_y, -(\varepsilon_1 k_0^2 - q_x^2 - q_y^2)^{1/2}\},$$

we have $(I_1I_2)^{1/2} = 2.7 \times 10^4$ W/cm^2.

It is of interest to compare the effects of the radiation and of the gravitational field on the spectrum of the oscillations of the liquid surface. The latter can be easily taken into account by adding to the effective surface tension one more term

$$\alpha_{\text{eff}} = \alpha - (I_1I_2)^{1/2}k_0\beta / cq^2 + \rho g / q^2,$$

where g is the acceleration in free fall. It is easy to see that the frequency shifts introduced by either factor are equal in absolute magnitude at $(I_1I_2)^{1/2} = c\rho g/k_0|\beta|$. For example, for ether this amounts to $(I_1I_2)^{1/2} \approx 13$ W/cm^2. It must be recognized, however, that the effect of the gravitational field is significant in itself only for liquid-surface oscillations of sufficiently long wave, with $q \lesssim a^{-1}$, where $a = (\alpha/\rho g)^{1/2}$ is the capillary constant (usually $a \sim 0.1$ cm).

We note in conclusion that the considered effect of optical action on the "surface elasticity" of media, which is connected with the onset of ponderomotive forces, has obviously low inertia (the electronic relaxation times are $\sim 10^{-13}$ sec) and can therefore be used for frequency modulation of surface waves. We note also that a similar action on the surface waves should occur when acoustic waves interfere in two media separated by a surface.

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1) The pressure jump f is due to the difference between the densities of the electromagnetic energy on both sides of the interface and to the surface density of the polarization charges, and exists both in an electrostatic field and in a radiation field [1]. Thus, the effect considered below is possible, in principle, also in strongly inhomogeneous electrostatic fields.

2) The picture is much more complicated in the interference of two plane waves incident at arbitrary angles. In this case, resonant excitation of surface waves at the beat frequency $\Omega = |\omega_1 - \omega_2|$ and with a wave number $q = |k_{x1} - k_{x2}|$ is possible in addition to the frequency shift (see [2]. A surface-wave instability is also possible, and should be manifest also in the field of one obliquely-incident plane wave [3]. This effect, however, calls for a much higher radiation intensity and much longer evolution time than the effect discussed below.

3) In all the numerical estimates we assume $k_0 = 10^5$ cm^{-1}.

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Translated by J. G. Adashko
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