ABSORPTION OF RAYLEIGH SOUND WAVES IN METALS

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A theory of Rayleigh wave absorption by conduction electrons in metals with an isotropic dispersion law is developed. The absorption coefficient is calculated in the absence of a magnetic field and in a magnetic field perpendicular to the interface for an arbitrary electron reflection coefficient from the surface. For H = 0, the absorption of the Rayleigh wave is of the same order of magnitude as that for bulk sound waves. In a strong magnetic field, the absorption increases logarithmically. In the region of weak magnetic fields, oscillations of the geometric and acoustic cyclotron resonance are predicted which differ essentially from the resonance oscillations of bulk sound absorption.

1. INTRODUCTION

As is known, sound waves can be propagated along the plane boundary of an elastic half-space the amplitude of which falls off rapidly as one goes away from the surface. Such elastic excitations are called Rayleigh waves.[1] These waves represent the solution of the equations of elasticity theory

$$\rho_L \ddot{u}_i = \partial\sigma_{ik} / \partial x_k \qquad (1.1)$$

in the half-space $x \geq 0$. Here ρ_L is the density of the crystal, $u_i(\mathbf{r}, t)$ the vector displacement, and σ_{ik} the elastic stress tensor. On the free surface $x = 0$ the Rayleigh waves satisfy the boundary conditions

$$\sigma_{ik} n_k |_{x=0} = 0, \qquad (1.2)$$

which express the continuity of pressure at the boundary. The x axis is chosen along the inward normal $\mathbf{n}$.

For an isotropic elastic medium, the Rayleigh waves are a superposition of longitudinal and transverse oscillations:

$$\mathbf{u}(\mathbf{r}, t) = \sum_\alpha \mathbf{u}^\alpha(0) \exp[-\varkappa_\alpha x + i(\mathbf{kr} - \omega t)]. \qquad (1.3)$$

The index α takes on the values l and t, and corresponds to longitudinal and transverse types of oscillations propagating in an infinite medium; ω is the frequency, $\mathbf{k}$ is a plane wave vector with components k_y and k_z, the quantity

$$\varkappa_\alpha = (k^2 - \omega^2 / s_\alpha^2)^{1/2} \qquad (1.4)$$

represents the damping decrement of the longitudinal or transverse mode with respective phase velocity s_l or s_t. The boundary conditions (1.2) connect the longitudinal and transverse components in such fashion that the normal pressure vanishes at the interface. The spectrum of the surface wave is determined from the condition (1.2) in the form

$$\omega = s_t k \xi, \quad \xi = \xi(s_t / s_l), \qquad (1.5)$$

where $\xi < 1$ and depends on the ratio of the sound velocities.[2]

When averaged over the period of vibration, the energy density in the Rayleigh wave is

$$W = |u_x^l(0)|^2 A \rho_L \omega^2 k^{-1},$$

$$A(\xi) = \frac{\xi^4}{(1-\xi^2)^{1/2}(2-\xi^2)^6}(8 - 16\xi^2 + 11\xi^4 - 2\xi^6). \qquad (1.6)$$

The quantity W is normalized to unit area of the interface.

2. The absorption and velocity of sound in metals are essentially determined by the conduction electrons. The interaction of the electrons with the sound vibrations in the bulk have been studied in a large number of experimental and theoretical researches. At the same time, the effect of the conduction electrons on the properties of Rayleigh waves in conducting solids has been virtually unstudied.

At low temperatures, the absorption and dispersion of the sound velocity in pure metals are due mainly to the conduction electrons, inasmuch as the number of thermal phonons decreases with decrease in temperature. Measurement of the electronic absorption of sound in metals is a very convenient method for studying the volume properties of electrons (Fermi surfaces, free path lengths, deformation potentials, and so on). It can be hoped that the study of Rayleigh waves will make it possible to determine the electron characteristics near the interface, in particular, to make clear the character of the reflection of the electrons from the boundary. For example, it has been shown in[3] that Rayleigh waves can interact strongly with specific waves of the type of surface electronic sound, which accompany the transitions of electrons between magnetic surface levels in weak magnetic fields. Rayleigh waves are in a definite way analogous to electromagnetic waves in metals: both are localized near the surface. In particular, in the case of Rayleigh waves, there should also exist various effects of the type of the anomalous penetration of the electromagnetic wave into the interior of the metal. One of these possibilities has been pointed out in[4]. Therefore, the experimental study of surface ultrasonic and hypersonic waves in metals is of great interest.

In the present work, the absorption of Rayleigh waves is calculated both for the absence and the presence of a magnetic field perpendicular to the boundary surface. The metal is assumed to be isotropic acoustically, and the Fermi surface to be spherical. The reflection of the electrons from the metal-vacuum interface is described by the coefficient of specular reflection $\rho \leq 1$, which does not depend on the angle of incidence of the electron on the interface.

2. GENERAL FORMULAS FOR THE ABSORPTION COEFFICIENT

The electronic absorption coefficient is proportional to the sound energy Q absorbed by the electrons per unit time. Inasmuch as the quantity Q is expressed in the form of an integral over the volume of the sample, it is generally impossible in the case of Rayleigh waves to neglect the resultant surface integrals. It is therefore necessary to derive anew an expression for the absorption coefficient Γ. Thanks to the small value of the sound velocity in comparison with the velocity of the electrons, we shall calculate the coefficient Γ by perturbation theory, assuming the sound field $\mathbf{u}(\mathbf{r}, t)$ to be given in accord with the relation (1.3).

1. The interaction of elastic vibrations with the conduction electrons leads to the appearance in the equations of elasticity theory (1.1) of the additional component $\mathcal{F}_i$, which represents the volume force density exerted by the electrons on the lattice. This force can be written down in the following form:[5,6]

$$\mathcal{F}_i = \frac{\partial}{\partial x_k}\int d\tau_p \Lambda_{ik} F + \frac{1}{c}[\mathbf{jH}]_i + \frac{m_0}{e}\dot{j}_i. \qquad (2.1)^*$$

Here $\Lambda_{ik}(\mathbf{p})$ are the components of the deformation potential tensor, which satisfy the condition

$$\langle \Lambda_{ik} \rangle \equiv \int d\tau_p \delta(\varepsilon - \varepsilon_F)\Lambda_{ik} = 0,$$

F denotes the total distribution function of the electrons, $\mathbf{j}$ the total electric current, H the magnetic field, $-e$ the charge, m_0 the mass of the free electron, ϵ the energy, ϵ_F the chemical potential of the conduction electrons, and $d\tau_p = 2(2\pi\hbar)^{-3}d\mathbf{p}$ the element of momentum space; the dot denotes partial differentiation with respect to time, repeated indices are to be summed from 1 to 3.

The first component in (2.1) is the so-called "deformation" force, due to direct interaction of the electrons with the sound. The energy of this interaction in the deformed lattice is described by the interaction Hamiltonian $\Lambda_{ik}u_{ik}$. The second term in (2.1) is the induction force, exerted by the electrons on the lattice, due to the fact that a conductor moving under the action of sound cuts magnetic lines of force. Finally, the last term in $\mathcal{F}_i$ is the inertial force which arises from the acceleration of the electrons in the deformed crystal by the field of the sound wave.

The electron distribution function F satisfies the kinetic (Boltzmann) equation and is represented in the linear approximation in the form of a sum of equilibrium and nonequilibrium parts:

$$F = f_0(\varepsilon + \Lambda_{ik}u_{ik} - \varepsilon_F) - \chi\frac{\partial f_0}{\partial \varepsilon}, \qquad (2.2)$$

where f_0 is the Fermi distribution function, $\partial f_0/\partial\epsilon = -\delta(\epsilon - \epsilon_F)$. The presence of the term $\Lambda_{ik}u_{ik}$ in the argument of the first component takes into consideration the fact that in the deformed lattice it is just this function which cancels the collision integral and corresponds to the condition of local equilibrium. In the second component, the contribution of $\Lambda_{ik}u_{ik}$ to $\gamma f_0/\partial\epsilon$ can be neglected in the linear approximation in the amplitude of the sound wave.

Substituting (2.2) in (2.1), we represent the elasticity equation (1.1) in the form

$$\rho_L \ddot{u}_i = \frac{\partial}{\partial x_k}(\sigma_{ik} + \langle \Lambda_{ik}\chi \rangle) + \frac{1}{c}[\mathbf{jH}]_i + \frac{m_0}{e}\dot{j}_i, \qquad (2.3)$$

where

$$\sigma_{ik} = \sigma_{ik}^0 - \langle \Lambda_{ik}\Lambda_{lm} \rangle u_{lm} \qquad (2.4)$$

is the renormalized elastic strain tensor in the metal and σ_{ik}^0 the strain tensor without account of the conduction electrons. The electronic renormalization of the elastic moduli, equal to $-\langle \Lambda_{ik}\Lambda_{lm} \rangle$, is connected with the fact that the instantaneous distribution function depends on the total energy of the conduction electron in the deformed crystal. Formally, it is obtained by expanding f_0 in terms of the interaction energy $\Lambda_{lm}u_{lm}$.

The total current in the metal

$$\mathbf{j} = -e\langle \mathbf{v}\chi \rangle, \quad \mathbf{v} = \partial\varepsilon/\partial\mathbf{p}, \qquad (2.5)$$

is expressed only in terms of the anisotropic part of the distribution function χ.

All the equations actually contain the variable electromagnetic field, which accompanies the sound wave in the metal. This field should be found from the Maxwell equations

$$\mathrm{rot}\,\mathbf{E} = -\frac{1}{c}\dot{\mathbf{H}}, \quad \mathrm{rot}\,\mathbf{H} = \frac{4\pi}{c}\mathbf{j}, \quad \mathrm{div}\,\mathbf{H} = 0, \qquad (2.6)$$

where $\mathbf{E}$ is the electric field vector.

2. For the derivation of the absorption coefficient, it is convenient to express the induction force $c^{-1}\mathbf{j}\times\mathbf{H}$ in terms of the Maxwell equations (2.6). Then

$$\frac{1}{c}[\mathbf{jH}]_i = \frac{\partial T_{ik}}{\partial x_k} \equiv \frac{\partial}{\partial x_k}\frac{1}{4\pi}\left(H_i H_k - \frac{H^2}{2}\delta_{ik}\right), \qquad (2.7)$$

where T_{ik} is the Maxwell stress tensor of the total magnetic field in the metal. T_{ik} does not contain small terms due to the alternating electric field, since it is always small in metals in comparison with the alternating magnetic field. Formally this is connected with the fact that the displacement current in Maxwell's equations (2.6) is negligible in comparison with the conduction current.

We substitute (2.7) in (2.3) and write it down in the form of a continuity equation for the total momentum density of the crystal:

$$\frac{\partial}{\partial t}\left(\rho_L \dot{u}_i - \frac{m_0}{e}j_i\right) = \frac{\partial}{\partial x_k}(\sigma_{ik} + \langle \Lambda_{ik}\chi \rangle + T_{ik}). \qquad (2.8)$$

The left hand side of Eq. (2.8) contains the change in the total momentum density of the "ionic" residue and of the electrons. It is quite evident that the electron momentum density $m_0\langle \mathbf{v}\chi \rangle$ is much smaller than $\rho_L\dot{\mathbf{u}}$ to the extent that the electron mass density Nm_0 is small in comparison with ρ_L (this can be established by direct calculation). Therefore, in what follows, we shall neglect components proportional to m_0 and due to the Stuart-Tolman effect.

We now write down the law of total energy conservation. With this aim, we multiply Eq. (2.8) by $\dot{u}_i$ and integrate it over the entire volume, including the region $x < 0$. To the kinetic energy of the crystal we add the

*$[\mathbf{jH}] \equiv \mathbf{j}\times\mathbf{H}$.

energy of interaction of the field $\mathscr{H}_{em}$ which accompanies the sound vibrations. As a result we obtain

$$\frac{\partial}{\partial t}\int d\mathbf{r}\left(\frac{\rho_L \mathbf{u}^2}{2}+\mathscr{H}_{em}\right) \tag{2.9}$$

$$=\int d\mathbf{r}\left[\dot{u}_i\frac{\partial}{\partial x_k}(\sigma_{ik}+\langle\Lambda_{ik}\chi\rangle)-\frac{1}{c}[\dot{\mathbf{u}}\mathbf{H}]\mathbf{j}-\mathbf{jE}-\operatorname{div}\mathbf{S}\right],$$

where the last two components are connected with the change in the energy of the electromagnetic field because of the Joule losses $\mathbf{j}\cdot\mathbf{E}$ and the radiation, and $\mathbf{S}=(c/4\pi)\mathbf{E}\times\mathbf{H}$ is the Poynting vector. After identity transformations, it is not difficult to represent (2.9) in the following form:

$$\frac{\partial}{\partial t}\int d\mathbf{r}\left(\frac{\rho_L \mathbf{u}^2}{2}+\frac{\sigma_{ik}u_{ik}}{2}+\mathscr{H}_{em}\right) \tag{2.10}$$

$$=-\int d\mathbf{r}\left\{\mathbf{j}\left(\mathbf{E}+\frac{1}{c}[\dot{\mathbf{u}}\mathbf{H}]\right)+\langle\Lambda_{ik}\chi\rangle\dot{u}_{ik}+\frac{\partial}{\partial x_k}[S_k-\dot{u}_i(\sigma_{ik}+\langle\Lambda_{ik}\chi\rangle)]\right\}$$

On the left side of (2.10) there is the change in the energy of the sound and electromagnetic fields, and on the right the absorption of the energy by the conduction electrons. It is easy to see that the last components in (2.10) under the sign of differentiation with respect to the coordinate vanish. Actually, the component with the Poynting vector vanishes because of the continuity of the vector $\mathbf{S}$ on the interface and the possibility of transforming this term to an integral over an infinitely distant surface. The term containing $\sigma_{ik}+\langle\Lambda_{ik}\chi\rangle$ vanishes on the surface of the metal because of the boundary conditions for Eq. (2.8). In fact, the condition of mechanical equilibrium of the free surface of separation can be found if we integrate Eq. (2.8) over an infinitesimally small volume near the metal surface. As a result we obtain the relation

$$(\sigma_{ik}+\langle\Lambda_{ik}\chi\rangle+T_{ik})n_k|_{x=0}=T_{ik}^{i}n_k|_{x=0}. \tag{2.11}$$

Here T_{ik}^{i} represents the stress tensor of the electromagnetic field in a vacuum. Because of the continuity of the total magnetic field, the magnetic part of the strain tensor T_{ik}^{i} cancels with T_{ik} on the left side of (2.11). The electrical components of T_{ik}^{i} are bilinear in E and can be neglected in the linear approximation. Therefore the boundary conditions for the Eqs. (2.3) take the form

$$(\sigma_{ik}+\langle\Lambda_{ik}\chi\rangle)n_k|_{x=0}=0, \tag{2.12}$$

and the last component in (2.12) vanishes. Finally, the rate of change of the total energy of the sound and electromagnetic oscillations is represented by the following formula:

$$-\frac{\partial}{\partial t}\int d\mathbf{r}\left(\frac{\rho_L\dot{\mathbf{u}}^2}{2}+\frac{\sigma_{ik}u_{ik}}{2}+\mathscr{H}_{em}\right)=Q\equiv\int d\mathbf{r}(\mathbf{j}\mathbf{E}'+\langle\Lambda_{ik}\chi\rangle\dot{u}_{ik}), \tag{2.13}$$

where

$$\mathbf{E}'=\mathbf{E}+\frac{1}{c}[\mathbf{u}\mathbf{H}]$$

is the electric field in the system of coordinates moving with the lattice with velocity $\mathbf{u}$. The first component in Q is connected with Joule losses and the second term represents the power of deformation absorption.

The absorption coefficient can now be found by averaging the relation (2.13) over a period of the sound oscillation. We have

$$\Gamma=\frac{\bar{Q}}{2\mathscr{E}}=\frac{1}{4\mathscr{E}}\operatorname{Re}\int d\mathbf{r}(\mathbf{j}\mathbf{E}'^{*}+\langle\Lambda_{ik}\chi\rangle\dot{u}_{ik}^{*}), \tag{2.14}$$

where the asterisk denotes the complex conjugate and the prime denotes averaging over the period,

$$\mathscr{E}=\int d\mathbf{r}\left(\frac{\rho_L\dot{\mathbf{u}}^2}{2}+\frac{\sigma_{ik}u_{ik}}{2}+\mathscr{H}_{em}\right). \tag{2.15}$$

It is not difficult to establish the fact that the quantity $\mathscr{H}_{em}$ is comparable with the sound energy density only in the case of resonance coupling of a weakly attenuated electromagnetic wave with a sound wave.[6] In all remaining cases, $\mathscr{H}_{em}$ can be neglected in (2.15). Therefore, in the calculation of the absorption coefficient Γ below we shall mean by $\mathscr{E}$ the energy of the sound field. Recognizing that the field amplitudes in the half-space depend only on the x coordinate, we represent Γ in the form

$$\Gamma=\frac{1}{4W}\operatorname{Re}\int_0^\infty dx(\mathbf{j}\mathbf{E}'^{*}+\langle\Lambda_{ik}\chi\rangle\dot{u}_{ik}^{*}), \tag{2.16}$$

where $W=\mathscr{E}/\mathscr{S}$ is the mean sound energy per unit area of the surface $\mathscr{S}$.

The derivation of the foregoing expression for Γ is necessitated by the fact that one often uses another formula,

$$\Gamma=\frac{1}{2W}\int d\mathbf{r}\overline{\langle\chi\hat{\nu}\chi\rangle}, \tag{2.17}$$

in the case of sound in the bulk; here $\hat{\nu}$ is the collision integral of the electrons with the scatterers. Equations (2.17) and (2.16) are virtually identical in unbounded media, inasmuch as all the surface integrals vanish. In the case of Rayleigh waves, Eq. (2.17) gives an incorrect result that differs from (2.16) by the additional surface component

$$\frac{1}{4W}\langle v_x\overline{\chi^2(0)}\rangle,$$

which has no clear physical significance and turns out to be of the same order as the other terms in Γ.

3. In concluding this section, we give the solution of the kinetic equation for the case in which the magnetic field H is perpendicular to the surface of the metal. This equation is of the form

$$\left[i(\mathbf{kv}-\omega)+\nu+\Omega\frac{\partial}{\partial\varphi}+v_x\frac{\partial}{\partial x}\right]\chi(x,\varphi)$$
$$=g(x,\varphi)\equiv-e\mathbf{E}'(x)\mathbf{v}(\varphi)+\Lambda_{ik}(\varphi)\dot{u}_{ik}(x). \tag{2.18}$$

Here $\Omega=eH/mc$ is the cyclotron frequency, m the effective mass, and φ the angle variable describing the rotation of the electron in the magnetic field. The boundary conditions for Eq. (2.18) are the periodicity of the function χ in φ with period 2π and the condition of reflection of the electrons from the surface of the metal

$$\chi^{\downarrow}(0,\varphi)=\rho\chi^{\uparrow}(0,\varphi), \tag{2.19}$$

where the arrows ↓ and ↑ denote electrons moving from the surface ($v_x>0$) or toward it, and ρ is the coefficient of specular reflection. The solution of Eq. (2.18) is

$$\chi^{\uparrow}(x,\varphi)=\frac{1}{\Omega}\int_{-\infty}^{\varphi}d\varphi_1\exp\left[\frac{i}{\Omega}\int_{\varphi}^{\varphi_1}(\mathbf{kv}-\omega-i\nu)d\varphi'\right]g\left(x+|v_x|\frac{\varphi-\varphi_1}{\Omega},\varphi_1\right)$$

$$\chi^{\downarrow}(x,\varphi)=\frac{1}{\Omega}\int_{\Phi}^{\varphi}d\varphi_1\exp\left[\frac{i}{\Omega}\int_{\varphi}^{\varphi_1}(\mathbf{kv}-\omega-i\nu)d\varphi'\right]g\left(x-|v_x|\frac{\varphi-\varphi_1}{\Omega}\ \varphi_1\right)$$

$$+\rho\frac{1}{\Omega}\int_{-\infty}^{\Phi}d\varphi_1\exp\left[\frac{i}{\Omega}\int_{\varphi}^{\varphi_1}(\mathbf{kv}-\omega-i\nu)\,d\varphi'\right]g\left(-x+|v_x|\frac{\varphi-\varphi_1}{\Omega},\varphi_1\right),$$

$$\Phi\equiv\varphi-x\Omega/|v_x|. \tag{2.20}$$

It is not difficult to find $\chi^{\downarrow\uparrow}$ in the absence of a magnetic field, by calculating the asymptotic form of (2.20) as $\Omega\to 0$. As a result, we find

$$\chi^{\uparrow}=\frac{1}{|v_x|}\int_0^{\infty}dx'\exp\left[x'\frac{-\nu+i(\omega-\mathbf{kv})}{|v_x|}\right]g(x+x'), \tag{2.21}$$

$$\chi^{\downarrow}=\frac{1}{|v_x|}\int_0^{x}dx'\exp\left[x'\frac{-\nu+i(\omega-\mathbf{kv})}{|v_x|}\right]g(x-x')$$

$$+\rho\frac{1}{|v_x|}\int_x^{\infty}dx'\exp\left[x'\frac{-\nu+i(\omega-\mathbf{kv})}{|v_x|}\right]g(x'-x).$$

Formulas (2.16), (2.20) and (2.21) allow us to determine the absorption coefficient of the Rayleigh waves.

3. ABSORPTION IN THE ABSENCE OF A MAGNETIC FIELD

In what follows, we limit ourselves to the calculation of pure deformation absorption and neglect Joule losses from electric fields. In this case, we leave only the last terms in Eqs. (2.16) and on the right side of (2.18). Then, if we substitute Eq. (2.21) in (2.16) and carry out integration over x, x′ and ϵ, we obtain the following formulas:

$$\Gamma^{\uparrow}=\mathscr{T}\,\mathrm{Re}\sum_{\alpha,\beta}\frac{k}{\varkappa_\alpha+\varkappa_\beta}\int_{v_x<0}\frac{do}{4\pi}\frac{kvB_\alpha B_\beta^*}{\varkappa_\alpha|v_x|+\nu-i(\omega-\mathbf{kv})}, \tag{3.1}$$

$$\Gamma^{\downarrow}=\mathscr{T}\,\mathrm{Re}\sum_{\alpha,\beta}\frac{k}{\varkappa_\alpha+\varkappa_\beta}\int_{v_x>0}\frac{do}{4\pi}B_\alpha B_\beta^*\left[\frac{kv}{\varkappa_\alpha|v_x|+\nu+i(\omega-\mathbf{kv})}\right.$$

$$\left.+2\rho\frac{\varkappa_\alpha+\varkappa_\beta}{\varkappa_\beta-\varkappa_\alpha}\frac{kv(\varkappa_\alpha|v_x|+\nu)}{(\varkappa_\alpha|v_x|+\nu)^2+(\omega-\mathbf{kv})^2}\right],$$

where the arrows at Γ denote the contribution to the absorption from electrons moving toward the surface of the metal or away from it. Naturally, the dependence on ρ exists only for $\Gamma^{\downarrow}$. In Eq. (3.1),

$$\mathscr{T}=\frac{3}{8}\zeta\frac{N\varepsilon_F k}{\rho_L v} \tag{3.2}$$

represents a quantity of the order of the coefficient of collisionless absorption of bulk sound, N the concentration of the electrons, $\zeta=(\Lambda/\epsilon_F)^2$ the dimensionless parameter of the electron-phonon interaction of the order of unity, Λ the characteristic value of the deformation potential, v the velocity of the electrons on the Fermi surface, do the element of solid angle, and

$$B_\alpha=\frac{\Lambda_{ik}u_{ik}^{\alpha}(0)}{ku_x^{i}(0)\,\varepsilon_F(\zeta A)^{1/2}}.$$

In contrast with the bulk absorption, in the case of Rayleigh waves the effective frequency of collisions $\nu_{\mathrm{eff}}=\nu+\kappa|v_x|$ enters into the denominators of the integrand functions. The frequency $\kappa|v_x|$ represents the reciprocal "lifetime" of an electron with velocity v_x in the acoustic skin layer of thickness κ^{-1}. For this reason, the resonance denominators generally do not reduce to delta functions with the energy conservation law, and all the angles contribute to the integrals. If we assume for simplicity that the tensor Λ_{ik} does not depend on the angles, then direct calculation of the integrals gives the following result:

$$\Gamma=\mathscr{T}\sum_{\alpha,\beta}\mathrm{Re}(B_\alpha B_\beta^*)\,\phi(q_\alpha)\left(\frac{k}{\varkappa_\alpha+\varkappa_\beta}+\rho\frac{k}{\varkappa_\beta-\varkappa_\alpha}\right)$$

$$\phi(q_\alpha)=\frac{q_\alpha}{(q_\alpha^2-1)^{1/2}}\,\mathrm{Re\,arc\,tg}\left\{(q_\alpha^2-1)^{1/2}\right.$$

$$\left.\times\frac{1+\varkappa_\alpha\tilde{l}+[1+(k\tilde{l})^2]^{1/2}[(q_\alpha^2-1)\varkappa_\alpha\tilde{l}-1]}{(q_\alpha^2-1)(1+\varkappa_\alpha\tilde{l})[1+(k\tilde{l})^2]^{1/2}-(q_\alpha^2-1)\varkappa_\alpha\tilde{l}+1}\right\}. \tag{3.3}$$

Here $q_\alpha=k/\kappa_\alpha>1$ is the dimensionless wave number and $\tilde{l}=v/(\nu-i\omega)$ is the effective free path length.

In the case of low frequencies ($|k\tilde{l}|\ll 1$) it follows from (3.3) that

$$\Gamma=\mathscr{T}\,\mathrm{Re}\,k\tilde{l}\sum_{\alpha,\beta}k\frac{B_\alpha B_\beta^*}{\varkappa_\alpha+\varkappa_\beta}\left[1-\frac{1-\rho}{4}(\varkappa_\alpha+\varkappa_\beta)\tilde{l}\right]. \tag{3.4}$$

In this case the absorption coefficient is identical in order of magnitude with the coefficient of volume absorption $\Gamma_{\mathrm{vol}}\sim\omega^2/\nu$. The reflection of the electrons from the surface of the metal affects only the value of the correction components in the next approximation in $k\tilde{l}$. This is associated with the fact that the relative number of electrons colliding with the surface is of the order of $|k\tilde{l}|$ and is small.

In the opposite limiting case of strong spatial dispersion ($|k\tilde{l}|\gg 1$) the absorption is determined by (3.3), in which, in place of the function $\phi(q_\alpha)$, we have the asymptotic expression

$$\Psi(q_\alpha)=\frac{q_\alpha}{(q_\alpha^2-1)^{1/2}}\,\mathrm{arc\,tg}(q_\alpha^2-1)^{1/2}. \tag{3.5}$$

Even in this case the absorption coefficient of the Rayleigh waves is of the order of $\Gamma_{\mathrm{vol}}\sim\omega s/v$. However, in both cases, the dependence of Γ on the elastic moduli and the numerical factors are different than in the volume absorption. It should be noted that for $|k\tilde{l}|\gg 1$, in the range of frequencies where the wavelength of the sound is large in comparison with the thickness of the electromagnetic skin layer at the frequency ω, the vortex fields can give an absorption comparable with pure deformation.[7] Here, however, we shall not be concerned with calculation of the absorption from vortex fields.

4. ABSORPTION IN A STRONG MAGNETIC FIELD

In this section we give the results of a calculation of the coefficient of deformation absorption of Rayleigh waves in a magnetic field perpendicular to the boundary of the metal. If we substitute (2.20) in (2.16) and carry out integration over x, φ_1 and φ, then the expression for Γ takes the following form:

$$\Gamma^{\uparrow}=\frac{1}{2}\mathscr{T}\,\mathrm{Re}\sum_{\alpha,\beta}k\frac{B_\alpha B_\beta^*}{\varkappa_\alpha+\varkappa_\beta} \tag{4.1}$$

$$\times\sum_{n=-\infty}^{\infty}\int_{\pi/2}^{\pi}d\theta\sin\theta J_n^2(kR\sin\theta)\frac{kv}{\varkappa_\alpha|v_x|+\nu-i(\omega-n\Omega)},$$

$$\Gamma^{\downarrow}=\frac{1}{2}\mathscr{T}\,\mathrm{Re}\sum_{\alpha,\beta}k\frac{B_\alpha B_\beta^*}{\varkappa_\alpha+\varkappa_\beta}\sum_{n=-\infty}^{\infty}\int_0^{\pi/2}d\theta\sin\theta J_n^2(kR\sin\theta)$$

$$\times\left[\frac{kv}{\varkappa_\alpha|v_x|+\nu+i(\omega-n\Omega)}+2\rho\frac{\varkappa_\alpha+\varkappa_\beta}{\varkappa_\beta-\varkappa_\alpha}\frac{kv(\varkappa_\alpha|v_x|+\nu)}{(\varkappa_\alpha|v_x|+\nu)^2+(\omega-n\Omega)^2}\right],$$

where $v_x=v\cos\theta$, $J_n(z)$ is a Bessel function of order n, and $R=v/\Omega$. In the derivation of (4.1) it was assumed that B_α does not depend on φ and θ.

In a strong magnetic field, when

$$kR \ll 1, \quad \varkappa_\alpha R \ll 1, \tag{4.2}$$

one should keep only components with n = 0 in sums over n. In the region of weak spatial dispersion

$$|k\tilde{l}| \ll 1 \tag{4.3}$$

we have for the absorption coefficient

$$\Gamma(H) = \Gamma(0)[1 - (kR)^2/2], \tag{4.4}$$

where $\Gamma(0)$ is given by Eq. (3.4). Thus, in this region, the coefficient reaches saturation, the value of which is identical with the absorption for H = 0.

In the opposite limiting case

$$|\varkappa_\alpha \tilde{l}|, \ |k\tilde{l}| \gg 1 \tag{4.5}$$

we obtain

$$\Gamma = \frac{1}{2}\mathcal{T}\,\mathrm{Re}\sum_{\alpha,\beta} k\frac{B_\alpha B_\beta^*}{\varkappa_\alpha + \varkappa_\beta}(q_\alpha \ln \varkappa_\alpha \tilde{l} + q_\beta \ln \varkappa_\beta \tilde{l}) + \frac{\rho}{2}\mathcal{T}\,\mathrm{Re}\sum_{\alpha,\beta} k\frac{B_\alpha B_\beta^*}{\varkappa_\beta - \varkappa_\alpha}(q_\alpha \ln|\varkappa_\alpha \tilde{l}| - q_\beta \ln|\varkappa_\beta \tilde{l}|). \tag{4.6}$$

The absorption coefficient here, too, does not depend on the magnetic field, but we get a logarithmic dependence on the free path length l, which does not exist for H = 0. The value of the coefficient $\Gamma(H)$ is approximately $\ln|k\tilde{l}|$ times greater than $\Gamma(0)$. The appearance of a large logarithm in the absorption coefficient of the surface wave can be understood from the following qualitative considerations. The effective time of free flight of the electron with a given velocity projection v_x is evidently

$$\tau_{\mathrm{eff}} = \int_0^\infty dt \exp\left(-\frac{t}{\tau} + i\omega t - \varkappa|v_x|t\right) = \frac{1}{\varkappa|v_x| + \nu - i\omega} \tag{4.7}$$

The average over all electrons, i.e., over all values of $|v_x|$ from zero to v, will be equal to

$$\langle\tau_{\mathrm{eff}}\rangle = \frac{1}{\varkappa v}\ln \varkappa\tilde{l}, \qquad \tilde{l} = v/(\nu - i\omega).$$

If we take it into account that $\Gamma \sim \omega^2\langle\tau_{\mathrm{eff}}\rangle$, then a correct order-of-magnitude estimate of formula (4.6) is obtained directly. In the case of bulk sound, $\kappa = ik$, $\langle\tau_{\mathrm{eff}}\rangle = \pi/kv$, i.e., we obtain the well known estimate for $\Gamma_{\mathrm{vol}} \sim \omega s/v$ by this same method.

Thus, the logarithmic dependence of Γ on the free path length in a strong magnetic field is due to the surface character of the electron absorption in the field of an inhomogeneous sound wave.

5. ABSORPTION COEFFICIENT IN A WEAK MAGNETIC FIELD

This region of magnetic fields corresponds to the inequality

$$kR \gg 1. \tag{5.1}$$

A large number of components with $|n| \lesssim kR\sin\theta$ turn out to be important in the sums over n in (4.1). Terms with large |n| are exponentially damped, and their contribution to the absorption turns out to be small. Summation over n in (4.1) is conveniently carried out in explicit fashion, using the identity

$$\sum_{n=-\infty}^{\infty}\frac{J_n^2(z)}{i\gamma + n} = \frac{\pi}{i\,\mathrm{sh}\,\pi\gamma}J_{i\gamma}(z)J_{-i\gamma}(z).$$

Then the coefficient Γ can be represented in the form

$$\Gamma = \pi\mathcal{T}k^2R\,\mathrm{Re}\sum_{\alpha,\beta}\mathrm{Re}\,(B_\alpha B_\beta^*)\left(\frac{1}{\varkappa_\alpha + \varkappa_\beta} - \frac{\rho}{\varkappa_\beta - \varkappa_\alpha}\right) \times \int_0^{\pi/2} d\theta \sin\theta\,\frac{J_{i\gamma_\beta}(kR\sin\theta)\,J_{-i\gamma_\beta}(kR\sin\theta)}{\mathrm{sh}\,\pi\gamma^\beta}, \tag{5.2}$$

where

$$\gamma_\beta = \xi + \varkappa_\beta R\cos\theta, \quad \xi = (\nu - i\omega)/\Omega.$$

In the calculation of the integral over θ in (5.2), one should use the asymptotic expression for the Bessel function at large arguments and indices. The character of the asymptotic form of these functions depends on the relation between the argument and the index. Their absolute values turn out to be identical for $\theta = \theta_0$, where θ_0 satisfies the equation

$$q_\alpha\,\mathrm{tg}\,\theta_0 = \left|1 + \frac{1}{\varkappa_\alpha\tilde{l}\cos\theta_0}\right|. \tag{5.3}$$

In the region of strong spatial dispersion (4.5) of interest to us, the second component on the right can be neglected, and $\theta_0 = \arctan q_\alpha^{-1}$. Therefore, the integral in (5.2) should be divided into the sum of two, one over the region $\theta_0 \le \theta \le \pi/2$ and the other over $\theta \le \theta_0$. In the first region, the argument of the Bessel function $J_{\pm i\gamma\beta}(kR\sin\theta)$ is larger than the modulus of the index and the asymptotic form is

$$J_{i\gamma}(z)J_{-i\gamma}(z) = \frac{1}{\pi(z^2+\gamma^2)^{1/2}}\left\{\mathrm{sh}\,\pi\gamma + e^{-\pi\gamma} + \sin 2\left[(z^2+\gamma^2)^{1/2} + i\gamma\,\mathrm{arctg}\frac{i\gamma}{(z^2+\gamma^2)^{1/2}}\right]\right\}. \tag{5.4}$$

In the second region, where the argument is smaller than the modulus of the index, the asymptote is the following:

$$J_{i\gamma}(z)J_{-i\gamma}(z) = \frac{\mathrm{sh}\,\pi\gamma}{\pi(z^2+\gamma^2)^{1/2}}. \tag{5.5}$$

If we substitute (5.4) and (5.5) in (5.2), then the coefficient Γ can be represented as the sum of three different components:

$$\Gamma(H) = \Gamma(0) + \Gamma_1 + \Gamma_2. \tag{5.6}$$

The first component represents the absorption coefficient for H = 0 and is given by the formula (3.3). It is determined by the first term of the asymptote (5.4), which is proportional to $\sinh\pi\gamma$, and also by (5.5). The component Γ_1 is described by that term of the asymptotic expression (5.4) which is proportional to $e^{-\pi\gamma}$. Finally, the last component in (5.6) is associated with the oscillating term in (5.4).

The integral over θ in Γ_1 can be represented in the form

$$I_1 = \frac{2}{\pi kR}\sum_{n=1}^{\infty}e^{-2\pi\xi n}\int_{\theta_0}^{\pi/2}d\vartheta\exp\{-2\pi n\varkappa_\beta R\cos\theta\} \times\left[1 + \left(\frac{1}{k\tilde{l}\sin\theta} + \frac{\mathrm{ctg}\,\theta}{q_\beta}\right)^2\right]^{-1/2}.$$

Thanks to the condition (5.1), the main contribution to the integral over θ is made by the immediate vicinity of the point $\theta = \pi/2$ with a width of the order of $(2\pi n\kappa_\beta R)^{-1}$. Substituting for the exponential factor its value at $\theta = \pi/2$ and taking (4.5) into account, we obtain

$$I_1 = \frac{1}{\pi^2 k\varkappa_\beta R^2}\sum_{n=1}^{\infty}\frac{e^{-2\pi\xi n}}{n} = -\frac{\ln(1 - e^{-2\pi\xi})}{\pi^2 k\varkappa_\beta R^2}. \tag{5.7}$$

Correspondingly, we have for Γ_1

$$\Gamma_1 = -\mathcal{T}(1+\rho)\frac{\ln|1-e^{-2\pi\xi}|}{2\pi kR}\sum_{\alpha,\beta} q_\alpha q_\beta B_\alpha B_\beta^*. \qquad (5.8)$$

The conditions of applicability of this asymptotic formula are given by the inequalities (4.5) and (5.1).

Similarly, one can obtain an expression for Γ_2. In this term, the integral over θ is simplified if the argument of the sine in (5.4) is expanded in powers of γ^2 (such an expansion is permissible, since the main contribution is made by values of $\pi/2 - \theta \sim (\kappa_\beta R)^{-1} \ll 1$). Then

$$I_2 = \frac{2}{\pi kR}\sum_{n=1}^{\infty}\exp\left\{-2\pi\xi\left(n-\frac{1}{2}\right)\right\}\int_{\theta_0}^{\pi/2} d\theta \exp\left\{-2\pi\left(n-\frac{1}{2}\right)\varkappa_\beta R\cos\theta\right\}$$
$$\times \sin\left[2kR\sin\theta - \frac{(\xi+\varkappa_\beta R\cos\theta)^2}{kR\sin\theta}\right]. \qquad (5.9)$$

In the integration near $\theta = \pi/2$, the sine can be taken outside the integral sign at this point. As a result, we have for Γ_2

$$\Gamma_2 = \mathcal{T}(1+\rho)\operatorname{Re}\frac{\ln[\operatorname{cth}(\pi\xi/2)]}{2\pi kR}\sin\left(2kR-\frac{\xi^2}{kR}\right)\sum_{\alpha,\beta} q_\alpha q_\beta B_\alpha B_\beta^*. \qquad (5.10)$$

The logarithmic factors in Γ_1 and Γ_2 describe acoustic cyclotron resonance in the absorption of Rayleigh waves. This resonance takes place at multiples of the frequencies $\omega = n\Omega$ and, in contrast with bulk sound, the line does not have a Lorentz shape. Acoustic cyclotron resonance ought to appear in the form of modulation of the amplitude of the geometric resonance, which is described by the factor $\sin(2kR - \xi^2/kR)$ in Γ_2. Here there are about v/s periods of geometric oscillations between two maxima of the cyclotron resonance. Of course, satisfaction of the inequality $\omega\tau \gg 1$ is necessary for the cyclotron resonance. This condition can be realized, for example, in pure gallium at frequencies of the order of 100 MHz, at liquid helium temperatures. If $\omega\tau < 1$, then the geometric resonance will be preserved and the cyclotron resonance will be lacking.

[1] J. W. Rayleigh, Proc. London Math. Soc. **17**, 4 (1885).

[2] L. D. Landau and E. M. Lifshitz, Teoriya uprugosti (Theory of Elasticity) Nauka, 1965, Sec. 24.

[3] A. M. Grishin, É. A. Kaner, O. I. Lyubimov and N. M. Makarov, Zh. Eksp. Teor. Fiz. **59**, 629 (1970) [Sov. Phys.-JETP **32**, 342 (1970)].

[4] A. M. Gishin and O. Lyubimov, ZhETF Pis. Red. **12**, 489 (1970) [JETP Lett. **12**, 342 (1970)].

[5] V. M. Kontorovich, Zh. Eksp. Teor. Fiz. **45**, 1638 (1963) [Sov. Phys.-JETP **18**, 1125 (1964)].

[6] V. G. Skobov and É. A. Kaner, Zh. Eksp. Teor. Fiz. **46**, 273 (1964) [Sov. Phys.-JETP **19**, 189 (1964)].

[7] É. A. Kaner, Zh. Eksp. Teor. Fiz. **43**, 216 (1962) [Sov. Phys.-JETP **16**, 154 (1963)].

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