# ATTENUATION OF THE CRITICAL VIBRATIONS IN DISPLACIVE-TYPE FERROELECTRICS CONTAINING DISLOCATIONS

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The damping $\Gamma_D$ of critical phonons due to their scattering on static dislocations for $T > T_0$ ($T_0$ denotes the transition temperature) is investigated for displacive ferroelectrics. An explicit expression is given for the "strictional" contribution of the dislocation scattering to the attenuation. The dependence of the attenuation on the concentration of dislocations and on the temperature is investigated. The "strictional" dislocation attenuation of sound, due to the interaction of acoustic and soft optical phonons, is also calculated.

## 1. INTRODUCTION

An appreciable number of articles have been devoted to experimental and theoretical investigations of the damping of the critical vibrations and the dielectric losses in displacive ferroelectrics. For example, for the case of $SrTiO_3$ the temperature dependence of tan δ was studied in[1], and the width of the critical vibrations can be estimated from experiments involving the inelastic scattering of neutrons.[2] The attenuation in barium titanate was investigated by optical methods (see, for example,[3]) and also by neutron scattering.[4] Apparently the attenuation was first considered theoretically by Silverman,[5] who investigated the temperature dependence of tan δ associated with the scattering on phonons and on impurity atoms. In this connection the investigation was carried out within the framework of a one-dimensional model. The low-temperature attenuation ($T \lesssim \omega_c$) was investigated in[6]. Attempts were made in[7,8] to estimate the attenuation of the critical vibrations, and also to estimate the attenuation of sound and the thermal conductivity, on the basis of Silverman's model. The attenuation of the critical vibrations and the attenuation of sound for $\omega \ll \omega_c$ is calculated in[9] on the basis of a three-dimensional model; however, the most important quartic anharmonicities were not taken into consideration. Using a more realistic model of a ferroelectric proposed by Vaks,[11] the attenuation of the critical vibrations in an ideal crystal is calculated in article[10]. It was shown that the strictional interaction of the critical phonons with the acoustic phonons and the mutual interaction of the critical phonons with one another introduce the major contribution. The same model is used in[12] in order to calculate the attenuation of high-frequency sound.

It is interesting to compare the "self" damping of the critical vibrations, which is investigated in[10], and the damping due to the presence of a different type of defect in the crystal. The attenuation of the critical vibrations in a ferroelectric crystal containing linear defects (dislocations) is calculated below. For simplicity a diatomic crystal is considered in the cubic phase, $T > T_0$. The cited estimates are obviously only qualitatively valid in the immediate vicinity of $T_0$, when correlation effects are essential.[13]

## 2. THE ELASTIC AND THE ELECTRIC FIELDS CREATED BY A DISLOCATION IN A FERROELECTRIC CRYSTAL

As a consequence of the electrostriction and the piezoelectric effect, a dislocation not only generates an elastic field in a crystal, but it also creates an electric field. The simultaneous solution of the system of equations for the theory of elasticity and electrostatics is extremely complicated for the case of an anisotropic medium; however, in the case when homogeneity exists along a certain direction, the problem reduces to a planar problem, which allows us to use the method of functions of a complex variable.[14] This method was applied to the case of a linear dislocation in[15], and the generalization of[15] to the case of a piezoelectric is given in[16]. The influence of the piezoelectric effect on the attenuation will be considered in another article, but now we consider a cubic crystal having a center of inversion, so that it is sufficient to take only the striction into consideration. Then the density of the crystal's free energy can be written in the form

$$F = \frac{1}{2}\lambda_{\alpha\beta\gamma\delta}u_{\alpha\beta}u_{\gamma\delta} - \frac{1}{8\pi}\epsilon_{\alpha\beta}E_\alpha E_\beta + c_{\alpha\beta\gamma\delta}\frac{\partial E_\alpha}{\partial x_\beta}u_{\gamma\delta} + d_{\alpha\beta\gamma\delta}u_{\alpha\beta}E_\gamma E_\delta, \quad (1)$$

where $u_{\alpha\beta}(\mathbf{r})$ and $\mathbf{E}(\mathbf{r})$ are the deformation tensor and the electric field generated by a dislocation in the medium, $\lambda_{\alpha\beta\gamma\delta}$ is the tensor of the elastic moduli, $\epsilon_{\alpha\beta}$ is the dielectric tensor, and $c_{\alpha\beta\gamma\delta}$ and $d_{\alpha\beta\gamma\delta}$ are the electrostriction constants, where the term associated with $c_{\alpha\beta\gamma\delta}$ corresponds to the harmonic approximation and the term with $d_{\alpha\beta\gamma\delta}$ corresponds to the anharmonicities.

In the absence of external charges and volume forces, the deformation tensor $u_{ik}$ and the electric field vector $\mathbf{E}$ are described by the following system of equations:

$$\operatorname{div}\mathbf{D} = 0, \quad \operatorname{rot}\mathbf{E} = 0, \quad \partial\sigma_{ik}/\partial x_k = 0. \quad (2)$$

Here the stress tensor $\sigma_{ik}$ and the electric displacement vector $\mathbf{D}$ are expressed in terms of $u_{ik}$ and $\mathbf{E}$ by the usual equations of state.

Introducing the displacement vector $\mathbf{u}$ and the electrostatic potential $\varphi$, we obtain a system of equations for $\mathbf{u}$ and $\varphi$. In the presence of dislocations, the condi-

tion of the lack of uniqueness of the vector u, associated with a complete circuit around a closed contour enclosing the dislocation line, is added to this system. Neglecting the last term on the right-hand side of Eq. (1) and writing the tensors $\lambda_{\alpha\beta\gamma\delta}$ and $c_{\alpha\beta\gamma\delta}$ in "isotropic" form

$$\lambda_{\alpha\beta\gamma\delta}=\lambda_1\delta_{\alpha\beta}\delta_{\gamma\delta}+{}^1/_2\lambda_2(\delta_{\alpha\gamma}\delta_{\beta\delta}+\delta_{\alpha\delta}\delta_{\beta\gamma}), \tag{3}$$

having omitted the intermediate calculations we write the solution of the system (2) in the k-representation in the following form:

$$u_{ik}(\mathbf{k})=-\frac{i}{Vk^2}\oint_D dl\, e^{-i\mathbf{kl}}\Big\{k_i[\tau\mathbf{b}]_k+k_k[\tau\mathbf{b}]_i+[\mathbf{k}\tau]_i b_k+[\mathbf{k}\tau]_k b_i$$
$$-4\frac{k_ik_k}{k^2}\frac{[\eta+\mu+(e_2-e_1)k^2]}{[\eta+2\mu-e_2k^2]}(\mathbf{k}[\tau\mathbf{b}])\Big\}, \tag{4}^*$$

$$\varphi(\mathbf{k})=\frac{4\pi}{\varepsilon}\left[c_1u_{il}+c_2\frac{k_ik_l}{k^2}(u_{il}+u_{li})\right]. \tag{5}$$

Here **b** is the Burgers vector; $\eta$ and $\mu$ are the Lamé coefficients; $\tau$ is the unit vector tangent to the axis of the dislocation, the contour of the integral is taken along the dislocation line D;

$$e_1=\frac{4\pi}{\varepsilon}(c_1+2c_2)^2,\qquad e_2=\frac{4\pi c_2}{\varepsilon}(c_1+2c_2);$$

$\epsilon$ is the dielectric constant, and V is the volume of the crystal.

## 3. HAMILTONIAN OF A FERROELECTRIC CRYSTAL CONTAINING A DISLOCATION AND THE ATTENUATION OF THE CRITICAL VIBRATIONS

We shall start from the Vaks model,[11] and for the sake of simplicity we confine our investigation to a diatomic crystal. For small **k** the two transverse vibrations, described by the "optical" coordinate $\mathbf{y}_k$, are critical. According to[11], for **k** in the (100) direction (and for all directions of **k** when the anisotropy is small) the spectra of the transverse critical ($\omega_c$) and acoustic ($\omega_a$) vibrations has the following form:

$$\omega^2_{c,a}(\mathbf{k})={}^1/_2\{\omega_c^2+k^2(s_t+a_t)\pm[(\omega_c^2+k^2s_t-k^2a_t)^2+4k^4v_t^2]^{1/2}\}, \tag{6}$$

where $\omega_c^2=\lambda C^{-1}(T-T_0)$, C is the Curie-Weiss constant, $\lambda=4\pi z_c^2/v_cM$, $z_c$ is the effective charge of the critical branch, $v_c$ and M denote the volume and the mass of the elementary cell, and $s_t$, $a_t$, and $v_t$ are the constants introduced in[11]. In particular, $v_t$ characterizes the repulsion of the critical and acoustic branches.

In what follows we shall confine our attention to the isotropic approximation (therefore we set the constants $v_a$ and $a_a$ equal to zero in formulas (9) and (11) of[11]). Then one can find an explicit expression for the polarization vectors of the phonons with the repulsion of the critical and acoustic branches taken into consideration.

In writing down the Hamiltonian which describes the interaction of the phonons with the dislocation, it is convenient, just as in[11], to introduce "optical" and "acoustic" coordinates (Jacobi coordinates) according to the formulas

$$\mathbf{y}_r=\mathbf{u}_r^{(1)}-\mathbf{u}_r^{(2)},\qquad \mathbf{u}_r=\mu_1\mathbf{u}_r^{(1)}+\mu_2\mathbf{u}_r^{(2)}, \tag{7}$$

where $\mathbf{u}_r^{(i)}$ denotes the displacement of the i-th atom from its equilibrium position in the r-th cell, $\mu_i=m_iM^{-1}$, and $m_i$ denotes the mass of the i-th atom.

We shall start from the cubic anharmonicity, which we write in the form

$$H_3=\frac{1}{3!}\sum_{1,2,3}V^{(3)}_{i_1i_2i_3}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\xi_{\mathbf{k}_1}^{i_1}\xi_{\mathbf{k}_2}^{i_2}\xi_{\mathbf{k}_3}^{i_3}, \tag{8}$$

where $\xi_{\mathbf{k}}^{i}$ denotes either $\mathbf{u}_k$ or else $\mathbf{y}_k$ (the Fourier components of $\mathbf{u}_r$ and $\mathbf{y}_r$). Let us represent the quantities $\xi$ in the form $\xi=\xi_D+\xi$, that is, we shall measure the displacement from the new equilibrium position, $\xi_D$, associated with the presence of the dislocation, and moreover $\mathbf{y}_D=v_c\mathbf{P}_D/z_c$, where $\mathbf{P}_D$ is the polarization vector generated by the dislocation in the medium. Confining our attention to terms of first-order smallness in $\xi_D$, we obtain the following expression for the Hamiltonian describing the interaction of the phonons with the dislocation in terms of the coordinates $\mathbf{u}_k$ and $\mathbf{y}_k$:

$$H_{int}=\sum_{\mathbf{k}_1,\mathbf{k}_2}[K_{\alpha\beta}(\mathbf{k}_1,\mathbf{k}_2)u^{\alpha}_{-\mathbf{k}_1}u_{\mathbf{k}_2}^{\beta}+L_{\alpha\beta}(\mathbf{k}_1,\mathbf{k}_2)y^{\alpha}_{-\mathbf{k}_1}y_{\mathbf{k}_2}^{\beta}+M_{\alpha\beta}(\mathbf{k}_1,\mathbf{k}_2)u^{\alpha}_{-\mathbf{k}_1}y_{\mathbf{k}_2}^{\beta}], \tag{9}$$

where we shall not take umklapp processes into consideration. The functions K, L, and M have the following form

$$K_{\alpha\beta}(\mathbf{k}_1,\mathbf{k}_2)=\gamma_{\alpha\delta,\beta\rho,\gamma\lambda}k_1^{\delta}k_2^{\rho}u_{\gamma\lambda}(\mathbf{k}_1-\mathbf{k}_2), \tag{10}$$

$$L_{\alpha\beta}(\mathbf{k}_1,\mathbf{k}_2)=-\frac{\lambda}{4\pi}q_{\alpha\beta\gamma\delta}u_{\gamma\delta}(\mathbf{k}_1-\mathbf{k}_2), \tag{11}$$

$$M_{\alpha\beta}(\mathbf{k}_1,\mathbf{k}_2)=2\left[i\frac{\lambda v_c}{4\pi z_c}k_1^{\delta}q_{\alpha\delta\beta\gamma}P_D^{\gamma}(\mathbf{k}_1-\mathbf{k}_2)+k_1^{\delta}k_2^{\rho}e_{\alpha\delta,\beta\rho,\gamma\lambda}u_{\gamma\lambda}(\mathbf{k}_1-\mathbf{k}_2)\right], \tag{12}$$

where $u_{\alpha\beta}(\mathbf{k})$ denotes the tensor field of the deformation created by the dislocation in the medium, and the following notation has been introduced:

$$\gamma_{\alpha\delta,\beta\rho,\gamma\lambda}=\frac{i}{2}\frac{\partial^3V^{\alpha\beta\gamma}_{aaa}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)}{\partial k_1^{\delta}\partial k_2^{\rho}\partial k_3^{\lambda}}\Big|_{k=0}$$
$$q_{\alpha\delta\beta\gamma}=\frac{i}{2}\frac{v_c}{z_c^2}\frac{\partial}{\partial k_1^{\delta}}V^{\alpha\beta\gamma}_{a00}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\Big|_{k=0},$$
$$e_{\alpha\delta,\beta\rho,\gamma\lambda}=\frac{i}{2}\frac{\partial^3V^{\alpha\beta\gamma}_{00a}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)}{\partial k_1^{\delta}\partial k_2^{\rho}\partial k_3^{\lambda}}\Big|_{k=0} \tag{13}$$

(the subscript a on the potential $V^{\alpha\beta\gamma}$ corresponds to the acoustic branch, the subscript 0 corresponds to the optical branch).

In determining the functions K, L, and M we have taken into consideration that[17]

$$\lim_{\mathbf{k}\to0}u_{\mathbf{k}}^{\nu}V^{\nu\cdots}(\mathbf{k},\ldots)=-iu_{\alpha\beta}(\mathbf{k})\frac{\partial}{\partial k_\beta}V^{\alpha\cdots}(\mathbf{k},\ldots)\Big|_{k=0}. \tag{14}$$

Although the problem of determining the cross section for the scattering of long wavelength phonons on defects is essentially a classical problem, it is however convenient to carry out this calculation with the aid of quantum-mechanical perturbation theory. Therefore, we change to the operators which create and annihilate phonons:

$$u^{\alpha}(\mathbf{k})=\left(\frac{\hbar}{2MN}\right)^{1/2}\sum_{j=1}^{6}\frac{\bar e_{j\mathbf{k}}^{\alpha}}{\sqrt{\omega_{j\mathbf{k}}}}(c_{j\mathbf{k}}-c^{+}_{j,-\mathbf{k}}),$$

$$y^{\alpha}(\mathbf{k})=\left(\frac{\hbar}{2MN}\right)^{1/2}\sum_{j=1}^{6}\frac{\bar e_{j\mathbf{k}}^{\alpha}}{\sqrt{\omega_{j\mathbf{k}}}}(c_{j\mathbf{k}}-c^{+}_{j,-\mathbf{k}}), \tag{15}$$

where $\omega_j$ denotes the frequency of the j-th branch, N is the total number of cells in the crystal, and

*$[\tau\mathbf{b}]\equiv\tau\times\mathbf{b}$.

$$\tilde{e}_{jk} = \sqrt{\mu_1}\, e_{jk}^{(1)} + \sqrt{\mu_2}\, e_{jk}^{(2)}, \quad \bar{e}_{jk} = \frac{1}{\sqrt{\mu_1}} e_{jk}^{(1)} - \frac{1}{\sqrt{\mu_2}} e_{jk}^{(2)}, \qquad (16)$$

where the $\mathbf{e}_{jk}^{(m)}$ are the usual phonon polarization vectors (m is the number of the atom). One can easily determine explicit expressions for $\tilde{\mathbf{e}}_{jk}$ and $\bar{\mathbf{e}}_{jk}$ for the model of a diatomic crystal under consideration here; however, in view of their cumbersome nature we shall not present these expressions here.

In terms of the operators $c_{jk}^{+}$ and $c_{jk}$ the Hamiltonian (9) takes the form

$$H_{int} = \sum_{kk'} \sum_{jj'} \Phi_{jj'}^{kk'} (c_{jk}^{+} c_{j'k'} - c_{jk} c_{j',-k'}) + \text{h.c.}, \qquad (17)$$

where the amplitude for the scattering of phonons on the dislocations is given by

$$\Phi_{jj'}^{kk'} = \frac{\hbar}{M} \frac{1}{\sqrt{\omega_{jk}\omega_{j'k'}}} [\tilde{e}_{jk}^{\alpha} \bar{e}_{j'k'}^{\beta} K_{\alpha\beta}(\mathbf{k}, \mathbf{k}') + \bar{e}_{jk}^{\alpha} \tilde{e}_{j'k'}^{\beta} L_{\alpha\beta}(\mathbf{k}, \mathbf{k}') + \tilde{e}_{jk}^{\alpha} \tilde{e}_{j'k'}^{\beta} M_{\alpha\beta}(\mathbf{k}, \mathbf{k}')]. \qquad (18)$$

As is well-known, in $BaTiO_3$ the magnitude of the repulsion between the low-lying optical and acoustic branches is small,[18] and therefore the "critical" scattering amplitudes with j, j' = c give the major contribution to the attenuation of the critical vibrations:

$$\Phi_{c,1}^{kk'} = -\frac{\hbar\lambda}{4\pi} \frac{1}{(\omega_{ck}\omega_{ck'})^{1/2}} [q_{12}(\mathbf{e}_{2k}\mathbf{e}_{2k'})\delta_{\alpha\beta} + q_{44} e_{2k}^{\alpha} e_{2k'}^{\beta}] u_{\alpha\beta}(\mathbf{k}-\mathbf{k}'),$$

$$\Phi_{c,2}^{kk'} = -\frac{\hbar\lambda}{4\pi} \frac{1}{(\omega_{ck}\omega_{ck'})^{1/2}} [q_{12}(\mathbf{e}_{2k}, \mathbf{e}_{3k'})\delta_{\alpha\beta} + q_{44} e_{2k}^{\alpha} e_{3k'}^{\beta}] u_{\alpha\beta}(\mathbf{k}-\mathbf{k}'). \qquad (19)$$

In formulas (19) we have confined our attention to only the interaction of the "strictional" type, which is proportional to $q_{\alpha\beta\gamma\delta}$; the striction constants $q_{12}$ and $q_{44}$ are determined according to Eq. (13). The polarization vectors $\mathbf{e}_i$ are determined by the following equations:

$$\mathbf{e}_{1k} = \mathbf{k}/k, \quad \mathbf{e}_{2k} = [\mathbf{n}\mathbf{e}_{1k}], \quad \mathbf{e}_{3k} = [\mathbf{e}_{1k}\mathbf{e}_{2k}], \qquad (20)$$

where $\mathbf{n}$ is an arbitrary unit vector, non-collinear with the vector $\mathbf{k}$. Neglecting the interaction between the low-lying optical branch and the transverse acoustic phonons, expression (5) for the deformation field takes the well-known form

$$u_{\alpha\beta}(\mathbf{q}) = -\frac{i}{qV} \varphi_{\alpha\beta\gamma\delta}(\mathbf{q}_0) \sum_m b_m^{\gamma} T_m^{\delta}(\mathbf{q}) e^{-i\mathbf{q}\mathbf{r}_m},$$

$$\varphi_{\alpha\beta\gamma\delta}(\mathbf{q}_0) = -\frac{2(\eta+\mu)}{(\eta+2\mu)} q_0^{\alpha} q_0^{\beta} q_0^{\rho} e_{\gamma\delta\rho} + \frac{1}{2}(e_{\alpha\delta\gamma} q_0^{\beta} + e_{\beta\delta\gamma} q_0^{\alpha} + e_{\alpha\rho\delta}\delta_{\beta\gamma} q_0^{\rho} + e_{\beta\rho\delta}\delta_{\alpha\gamma} q_0^{\rho}),$$

$$T_m(\mathbf{q}) = \oint_{D_m} d\mathbf{l}\, \tau e^{-i\mathbf{q}\mathbf{r}_m}, \quad \mathbf{q}_0 = \frac{\mathbf{q}}{q}, \qquad (21)$$

where $\mathbf{b}_m$ and $\mathbf{r}_m$ denote, respectively, the Burgers vector and the radius vector of the m-th dislocation, and $e_{\alpha\beta\gamma}$ is the completely antisymmetric tensor of the third rank.

Now let us calculate the width of the critical phonons, associated with the scattering on dislocations. Here it is necessary to allow for the fact that the utilization of ordinary perturbation-theory formulas in the "$\delta$-function" approximation may turn out to be incorrect,[1)] since at finite temperatures the intrinsic anharmonic width of a phonon is given by $\Gamma_c/\omega_c \sim 0.01$ to 0.1.[10] However, the effective form factor associated with the scattering on dislocations has a sharp maximum in q-space (the q-momentum of the scattering), falling off rapidly at distances $q \sim R^{-1}$, where R is the characteristic size of the dislocation; in the temperature region under consideration the width of this maximum is smaller than the width $\Gamma_c$. Qualitatively one can take the finiteness of $\Gamma_c$ into account by "smearing out" the corresponding $\delta$-function.[19] Then we have the following result for the dislocation damping $\Gamma_D$:

$$\Gamma_D(\omega, \mathbf{k}) = \frac{4}{\hbar^2} \sum_{\mathbf{k}'} (\overline{|\Phi_{c,1}^{kk'}|^2} + \overline{|\Phi_{c,2}^{kk'}|^2}) \Gamma_c(\omega, \mathbf{k}') \times \left[\frac{1}{(\omega-\omega_{ck'})^2 + \Gamma_c^2(\omega, \mathbf{k}')} - \frac{1}{(\omega+\omega_{ck'})^2 + \Gamma_c^2(\omega, \mathbf{k}')}\right], \qquad (22)$$

where the bar indicates averaging over the dislocation distribution in the crystal. The averaging in Eq. (22) amounts to finding the average of the expression $T_i^{\alpha}(\mathbf{q}) T_i^{*\beta}(\mathbf{q})$, where the asterisk denotes complex conjugation. According to[20] we have the following result for dislocations of circular form:

$$T_i^{\alpha}(\mathbf{q}) T_i^{*\beta}(\mathbf{q}) = 2\pi R_i^2 (\delta_{\alpha\beta} - q_0^{\alpha} q_0^{\beta}) \int_0^1 J_2(2qR_i x)\, dx. \qquad (23)$$

Here $R_i$ is the radius of the i-th dislocation loop and $J_2$ is the Bessel function of second order.

Let us present the formulas for the dislocation damping which are obtained from expressions (19)–(23) for $\mathbf{k} = 0$. If $\sqrt{s_t}/R\omega_c \ll 1$ (this is always true for the temperature interval under consideration and typical values of R), then

a) for $\omega \ll \omega_c$ we find:

$$\Gamma_D(\omega) = 4\left(\frac{\lambda}{4\pi\omega_c^2}\right)^2 q_{str}^2 \frac{b^2 R \Gamma_c(\omega)}{v\omega_c} \omega \ln\left(\frac{\omega_c R}{\sqrt{s_t}}\right), \qquad (24)$$

b) for $\omega - \omega_c \gg \Gamma_c(\omega)$ we find:

$$\Gamma_D(\omega) = \pi\left(\frac{\lambda}{4\pi\omega_c^2}\right)^2 q_{str}^2 \frac{b^2 R}{v} \frac{\omega_c^3}{(\omega^2 - \omega_c^2)}. \qquad (25)$$

We obtain the following result for dislocation damping of the critical vibrations ($\omega = \omega_c$) under the conditions $\sqrt{s_t}/R \ll \Gamma_c(\omega) \ll \omega_c$:

$$\Gamma_D(\omega_c) = \left(\frac{\lambda}{4\pi\omega_c^2}\right)^2 q_{str}^2 \frac{b^2 R \omega_c^2}{v\Gamma_c(\omega_c)} \ln\left(\frac{\omega_c R}{\sqrt{s_t}}\right). \qquad (26)$$

Here $\Gamma_c(\omega)$ is the average anharmonic width at the frequency $\omega$, $q_{str}^2 = \alpha_1 q_{44}^2 + \alpha_2 q_{12}^2 + \alpha_3 q_{12} q_{44}$, the dimensionless constants $\alpha_i \sim 1$, v is the volume per dislocation ($v = Vn^{-1}$, where n is the total number of dislocations in the solid). It is clear from formula (24) that, since the anharmonic damping $\Gamma_c(\omega) \sim \omega$,[10] the dislocation damping is proportional to the square of the frequency of the external field.

The case of most interest is when $\xi > R^{-2}$, where $\xi$ is the dislocation concentration. This corresponds to the presence of rather extended dislocations in the crystal. Then the quantity $Rv^{-1}$ appearing in formulas (24)–(26) is equal to $\xi$. For this case let us present numerical estimates for $BaTiO_3$ at T = 500°K.[21] For $\omega \ll \omega_c$, $\Gamma_D(\omega)/\omega_c \approx 10^{-11} (\omega/\omega_c)^2 \xi$, and the critical dislocation damping $\Gamma_D(\omega_c)/\omega_c \approx 10^{-9}\xi$, where $\xi$ has the dimensions of $cm^{-2}$. Thus, the quantity $\Gamma_D(\omega_c)/\omega_c$

[1)] V. G. Vaks and R. O. Zaitsev called our attention to this point.

may amount to tens of percent for $\xi \sim 10^6$. In the case $\xi < R^{-2}$ (dislocation loops of small dimensions) the contribution due to the scattering on dislocations turns out to be small.

Let us also present a formula for the dislocation contribution $\gamma_D(\omega)$ to the attenuation of transverse sound, resulting from the strictional interaction of the transverse acoustic and the soft phonons. In the first nonvanishing approximation in powers of the parameter $v_t/a_t$ characterizing the repulsion of the branches, we obtain the following result for frequencies $\omega \ll \omega_c$:

$$\gamma_D(\omega) = 4\left(\frac{\lambda}{4\pi\omega_c^2}\right)^2 q_{str}^2 b^2 \xi \left(\frac{v_t}{a_t}\right)^2 \left(\frac{\omega}{\omega_c}\right)^4 \Gamma_c(\omega) \ln\left(\frac{\omega_c R}{\sqrt{s_t}}\right). \quad (27)$$

In contrast to the striction damping of sound in a perfect crystal,[12] where $\gamma(\omega) \sim v_t^4$ in the isotropic approximation, the dislocation part of the damping is given by $\gamma_D \sim v_t^2$.

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[1] G. Rupprecht and R. O. Bell, Phys. Rev. **125**, 1915 (1962).
[2] R. A. Cowley, Phys. Rev. **134**, A981 (1964).
[3] M. DiDomenico, Jr., S. P. S. Porto, and S. H. Wemple, Phys. Rev. Letters **19**, 855 (1967).
[4] G. Shirane, B. C. Frazer, V. J. Minkiewicz, J. A. Leake, and A. Linz, Phys. Rev. Letters **19**, 234 (1967).
[5] B. D. Silverman, Phys. Rev. **125**, 1921 (1962).
[6] V. Dvořák, Czech. J. Phys. **B17**, 726 (1967).
[7] Kensuke Tani, J. Phys. Soc. Japan **26**, 93 (1969).
[8] Kensuke Tani and Naoyuki Tsuda, J. Phys. Soc. Japan **26**, 113 (1969).
[9] E. Pytte, Phys. Rev. **B1**, 924 (1970).
[10] B. Ya. Balagurov, V. G. Vaks, and B. I. Shklovskiĭ, Fiz. Tverd. Tela **12**, 89 (1970) [Sov. Phys.-Solid State **12**, 7-70 (1970)]; Preprint IAÉ-1815 (1969).
[11] V. G. Vaks, Zh. Eksp. Teor. Fiz. **54**, 910 (1968) [Sov. Phys.-JETP **27**, 486 (1968)].
[12] B. Ya. Balagurov, Zh. Eksp. Teor. Fiz. **61**, 1627 (1971) [Sov. Phys.-JETP **34**, 868 (1972)].
[13] V. G. Vaks, Zh. Eksp. Teor. Fiz. **58**, 296 (1970) [Sov. Phys.-JETP **31**, 161 (1970)].
[14] J. D. Eshelby, W. T. Read and W. Shockley, Acta Met. **1**, 251 (1953).
[15] A. N. Stroh, Phil. Mag. **3**, 625 (1958).
[16] A. M. Kosevich, L. A. Pastur, and É. P. Fel'dman, Kristallografiya **12**, 916 (1967) [Sov. Phys.-Crystallography **12**, 797 (1968)].
[17] P. C. Kwok and P. B. Miller, Phys. Rev. **151**, 387 (1966).
[18] Y. Yamada, G. Shirane and A. Linz, Phys. Rev. **177**, 848 (1969).
[19] L. É. Gurevich and B. I. Shklovskiĭ, Fiz. Tverd. Tela **9**, 526 (1967) [Sov. Phys.-Solid State **9**, 401 (1967)].
[20] V. G. Bar'yakhtar, M. A. Savchenko, and V. V. Tarasenko, Zh. Eksp. Teor. Fiz. **51**, 936 (1966) [Sov. Phys.-JETP **24**, 623 (1967)].
[21] J. Harada, J. D. Axe, and G. Shirane, Phys. Rev. **B4**, 155 (1971).

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246