A DUAL MODEL FOR FERMIONS WITHOUT PARITY DEGENERACY ON THE PRINCIPAL TRAJECTORY

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A dual amplitude for interaction between N − 2 mesons and two fermions in a dual configuration is set up in such a way that parity degeneracy is absent on the principal fermion trajectory. The structure of the j-plane (Regge poles, fixed poles and cuts) is analyzed within the framework of the model. The degree of degeneracy of states with a given mass is calculated.

1. INTRODUCTION

ONE of the most remarkable experimental facts that has recently won universal confidence is the linearity of the Regge trajectories. It is distinctly seen, e.g., in baryon trajectories, such as Λ or Σ, each having three resonances, and specially the Δ_δ trajectory, which now is a surprisingly straight line passing through five resonances (with masses 1236, 1950, 2420, 2850, 3230)[1]. The linearity of the baryon trajectories, however, both causes surprise and leads to difficulties. The point is that the linear baryon trajectories can occur, as a rule, only if parity degeneracy is observed, (i.e., there should exist two resonances of different parity but with the same mass and spin)[2]. Such parity degeneracy has not been observed experimentally, at least for the known resonances.

The simplest way out would be to assume that such degeneracy sets in for the heavier resonances (which have not yet been observed), while for some reason the residues for the presently-observed particles vanish at the corresponding masses. However, it is not clear, first, why such a vanishing of the residues occurs for a large number of resonances. Second, it seems to us that such an approach leads to additional difficulties within the framework of the dual resonance models (DRM).

In the dual model for fermions[3,4], in which the baryon with the smallest mass has a definite parity (there is no parity twin), the amplitude of the meson-baryon scattering is given by

$$F=\bar{u}_1\gamma_5(1+\hat{q}/m)\gamma_5 u_2 V(s,\ t), \tag{1}$$

where $s = q^2$, $\alpha_s = \alpha_0 + q^2$, $\alpha(m^2) = 1/2$, and contains at the pole $\alpha_s = n + 1/2$ two terms corresponding to states with opposite parity

$$F=\frac{R_n(t)}{n+{}^1/_2-\alpha_s}\left\{\bar{u}_1\left(1-\frac{\hat{q}}{M}\right)u_2\left(1+\frac{M}{m}\right)-\bar{u}_1\left(1+\frac{\hat{q}}{M}\right)u_2\left(\frac{M}{m}-1\right)\right\}; \tag{2}$$

here $M = V\sqrt{s}$ and $R_n(t)$ is a polynomial of n-th degree in t.

It seems more natural to us to assume that there is no parity degeneracy on the entire baryon trajectory. At the present time, there are essentially only two ideas as to how this can occur.

First, it can be assumed that the singularities of the scattering amplitude in the complex angular-momentum plane are only moving poles, but the contribution of the reggeon corresponding to one of the trajectories $\alpha(s)$ with definite parity vanishes at $\alpha(s) = n + 1/2$, i.e., at values of $\sqrt{s}$ equal to the masses of the resonances. However, attempts to construct a model of such a mechanism[5] have led (at least for the time being) to a function that increases exponentially with s in the complex plane, and therefore these expressions are not suitable even for the Born term in the DRM.

The second idea, within the framework of which one can avoid parity degeneracy for linear trajectories, is to assume that in addition to moving poles there are present in the complex angular-momentum plane also standing cuts[6] at $j = \alpha_f(0)$ (where $\alpha_f(s)$ is the fermion trajectory, $\alpha_f(s) = \alpha_f(0) + \alpha_f'(s)$). We shall discuss this mechanism in greater detail in Sec. 2, and wish to note here only that the standing singularities in the j-plane appear frequently in perturbation theory, and therefore the presence of singularities of this type in the DRM, which give only expressions for the Born terms and do not satisfy the unitarity condition, seem quite natural.

Thus, the purpose of the present paper is to construct DRM for the meson-baryon scattering, which would satisfy the following requirements (we are basing ourselves here on the mechanism of Carlitz and Kislinger[6]:

1) The amplitude singularities are only simple poles with respect to all the invariant variables (e.g., s, t, u, for the four-line diagram).

2) The asymptotic growth does not exceed a power-law rate in any region of variation of the variables; this enables us to write down dispersion relations with respect to any variable (with the others fixed) and permits us to regard the proposed expressions as the first approximation of the real amplitudes (the Born term).

3) The principal baryon trajectories are not parity-degenerate in all the channels of the reactions (e.g., for the reaction M + B → M + B in the s and u channels). We require here that there be no parity degeneracy for the daughter trajectories. We note, however, that at the present time there is no firmly established daughter trajectory (especially a baryon one) and the question of parity degeneracy is all the more unclear

at this level. In addition, in all the known DRM parity degeneracy sets in for the daughter trajectory, starting with the second.

4) The residues of all the states in the 4-point terminal should be positive. This requirement, of course, is quite weak, since it is necessary to require that the residues be positive in all the 4-point diagrams (this is a complicated problem). It should be noted, however, that even such a "weak" requirements leads to rather strong limitations (see Appendix A).

Attempts to construct dual models satisfying requirements (1)–(3) were already made. Bardakci and Halpern[7] proposed expressions for the 4-line diagram, but only for its (st) and (ut) terms (see Fig. 1). Venturi[8] obtained a generalization to many-particle processes, corresponding to the simplest dual diagrams of Fig. 2a, and considered factorization in the baryon channels (the meson channels were not investigated in this case).

In the present paper we obtained the following results:

1. A prescription for writing down the dual amplitude for the simplest process M + B → M + B, which is somewhat different from that in[7] and in our opinion physically clearer; we consider different variants of this prescription and the limitations imposed by the condition that the residues be positive (see Sec. 2 and Appendix A).

2. A generalization is constructed for many-particle amplitudes, and not only for the diagrams of Fig. 2a, but also for the more general case of Fig. 2b (see Sec. 3).

3. It is shown that the proposed expression for the many-line diagrams can be factorized both in the baryon and in the meson channels, and the number of states in each of them is calculated (see Appendices B and C).

4. In conclusion we discussed those features of the proposed model, which have in our opinion a rather general character and should be present in all the dual models for meson-baryon scattering.

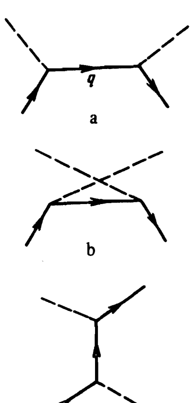

FIG. 1. a–(st) term in the scattering amplitude. b–(ut) term. c–(su) term. The dashed and solid lines correspond to mesons and baryons, respectively.

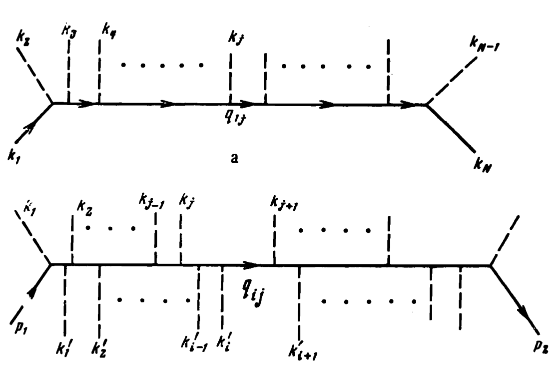

FIG. 2

2. DUAL MODEL FOR THE PROCESS M + B → M + B

If it is assumed that the singularities of the partial wave in the angular-momentum plane, besides the moving pole at $l = \alpha(s)$, is also a standing cut at $l = \alpha(0)$, then we can easily find that baryon resonances with definite parity will contribute to the elastic scattering M + B → M + B (to the 4-line diagram)[6]. To this end it suffices to write down the Sommerfeld-Watson transformation for the amplitude F of this process (see Fig. 1a)

$$F=\bar{u}_1\left\{\int_{C_1} dl\,\frac{(2l+1)}{\sin\pi l}\left(1+\frac{\hat{q}}{[l-\tilde{\alpha}(0)]^{1/2}}\right)\frac{r_l p_l(z)}{l-\tilde{\alpha}(s)}\right\}u_2. \quad (3)$$

Just as in[9], $l = j - 1/2$, where j is the angular momentum of the given reaction and $\tilde{\alpha}(s) = \alpha_f(s) - 1/2$; $\alpha_f(s)$ is the fermion trajectory. The contour C_1 and the singularities of the integrand (the pole $l = \tilde{\alpha}(s)$, the cut from $l = \tilde{\alpha}(0)$, and the poles corresponding to $\sin \pi l = 0$) are shown in Fig. 3a. It is seen from Fig. 3a that: a) the only singularities of F with respect to s are poles resulting from the fact that the contour C_1 is compressed by the moving pole $l = \tilde{\alpha}(s)$ and by the standing poles $l = 1, 2, \ldots, n$ (there is no singularity at the point $\tilde{\alpha}(s)$, i.e., the cut and the pole do not compress the contour); (b) at $s = M^2(\tilde{\alpha}(M^2) = n)$ the resonances have a definite parity, since at points at which $\alpha(s) = n$ we get

$$1+\hat{q}/(l-\tilde{\alpha}_0)^{1/2} = 1+\hat{q}/M$$

(we recall that $1 + \hat{q}/M$ is a projector on a state with definite parity, since it is equal to $\gamma_4 + 1$ in the rest system of the resonance). Inasmuch as

$$B(\tilde{\alpha}(s),\alpha(t))=\frac{1}{2\pi i}\int_C dl\int_0^1 dx\,x^{-l-1}(1-x)^{-\alpha(t)-1}\frac{1}{l-\tilde{\alpha}(s)} \quad (4)$$

(the contour C is shown in Fig. 3b), a natural generalization of (3) for the dual amplitude is (see Fig. 3b)

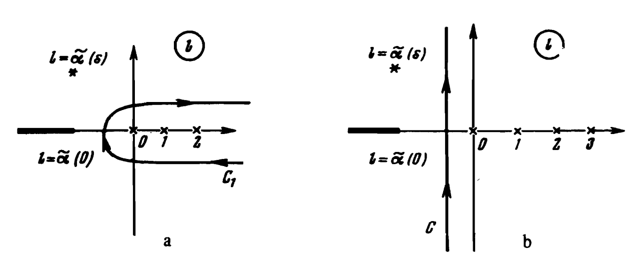

FIG. 3

$$F=\bar{u}_1\left\{\frac{1}{2\pi i}\int_C dl\int_0^1 dx\,x^{-l-1}(1-x)^{-\alpha(t)-1}\frac{1}{l-\tilde{\alpha}(s)}\right.$$
$$\left.\times\left(1+\frac{\hat{q}\varphi_1(x)}{[l\varphi_2(x)-\tilde{\alpha}(0)\varphi_3(x)]^{1/2}}\right)\right\}u_2. \quad (5)$$

A few remarks can be made with respect to the properties of the functions φ_1, φ_2, and φ_3:

1. In order not to have parity degeneracy for the principal trajectory, we must have $\varphi_1(0) = \varphi_2(0) = \varphi_3(0) = 1$.

2. The singularities of $\varphi_1(x)$ at x different from unity (and also the essential singularity with respect to x as $x \to 1$, e.g., $\varphi_1 = \exp[-bx/(1-x)]$), lead either to new singularities that are not poles with respect to the variables s and t, or to an exponential asymptotic growth with energy ($\alpha_t \to \infty$).

3. If we confine ourselves to the relation $\varphi_1 = (1-x)^{p_1}$, where p_1 is an integer (a non-integer power leads to a new meson trajectory that is shifted relative to $\alpha(t)$), then the residues in the 4-line diagram will be positive only when $p_1 \leq 1$ (see Appendix A). We consider here a meson trajectory $\alpha(t)$ such that $\alpha(0) < 0$, so as not to deal with the tachyon problem.

It is interesting to note that the condition that the residue of the 4-line diagram M + B → M + B must be positive at $p_1 = 1$ (this is the only case of interest, see the next item) imposes limitations on $\alpha(0)$ for the fermion and boson trajectories ($\tilde{\alpha}(0) < \alpha(0) - 1/2$). It is clear that inclusion of real physical trajectories can in this case be a more difficult problem than in the pure meson amplitude.

4. The condition $p_1(p_2) = 0$ leads to states with a spin larger than n, at $\alpha(t) = n$ in the t channel ("ancestors") and must therefore also be discarded.

5. It is easily seen that the relation $\varphi_2/\varphi_3 = (1-x)^m$, where $m \geq 2$ (or $\varphi_2/\varphi_3 = \exp[bx/(1-6)]$) leads to a faster than power-law growth at $\tilde{\alpha}(s) \to \infty$, with $\alpha(t)$ fixed.

6. The relation $\varphi_2/\varphi_3 = 1 - x$ leads to negative residues in the 4-line diagram (see Appendix A).

Thus, the chosen criteria are not violated by the relations

$$\varphi_1=(1-x),\quad \varphi_2=(1-x),\quad \hat{\varphi}_3=(1-x)^{p_3};\quad p_3=1,2,3,\ldots \quad (6)$$

To simplify the formulas and the arguments, we shall henceforth discuss the simplest variant of (6), namely $\varphi_1(x) = \varphi_2(x) = \varphi_3(x) = 1 - x$, which corresponds to the prescription proposed in[8].

Expression (5), which has a very lucid meaning, can unfortunately not be used at $\alpha(t) > 0$, since the corresponding integral with respect to l diverges on the large circle. To continue the amplitude into this region of t, it is convenient to integrate first with respect to l in (5), and then

$$F(s,t)=\bar{u}_1\left\{\int_0^1 dx\,x^{-\tilde{\alpha}(s)-1}(1-x)^{-\alpha(t)-1}\left[1+\hat{q}\varphi_1(x)\sqrt{\frac{-\ln x}{\varphi_2(x)}}\cdot\right.\right. \quad (7)$$
$$\left.\left.\cdot\,\Phi\left(\frac{1}{2},\ \frac{3}{2};\ \left(\tilde{\alpha}(s)-\tilde{\alpha}(0)\frac{\varphi_3(x)}{\varphi_2(x)}\right)\ln x\right)\right]\right\}u_2;$$

$\Phi(\alpha, \beta; z)$ is a confluent hypergeometric function.

From (7) we easily obtain the limitations on φ_1, φ_2, and φ_3, which have led us to (6). In addition, it is seen from (7) that $\alpha(t)$ is fixed for $|\tilde{\alpha}(s)| \to \infty$; in addition to the Regge behavior $(\tilde{\alpha}(s))^{\alpha(t)}$ there are contributions proportional to $[\tilde{\alpha}(s)]^{-n}$, n = 1, 2, ..., which correspond to fixed poles in the l-plane of the t-channel at $l = -n$. As $|\alpha(t)| \to \infty$ and at fixed $\alpha(s)$, the asymptotic expression is a sum of contributions, each of which behaves like $[\alpha(t)]^{\alpha_f(s)}$ (reggeon contribution), $(\alpha(t)/\sqrt{-\ln\alpha(t)})^{\alpha_f(0)}$ (contribution of the standing cut in the j plane), and $[\alpha(t)]^{-n/2}$, where n = 1, 3, 5... (contribution of the standing poles at j = −n/2).

3. MANY-PARTICLE PROCESSES

In this section we generalize the prescription (5) to many-particle amplitudes. For the diagram of Fig. 2a, it is quite easy to write down the amplitude in the dual model, for in this case there are no fermion channels that are dual to each other:

$$F_N=\bar{u}_1\left\{\prod_j\int_C\frac{dl_{1j}}{2\pi i}\int dx_{1j}\prod_{kl}dx_{kl}\,x_{1j}^{-l_{1j}-1}x_{kl}^{-\alpha_{kl}-1}\frac{\hat{P}_{1j}}{l_{1j}-\tilde{\alpha}_{1j}}\,\delta\left(x_{ij}+\prod x_{mn}-1\right)\right\}u_2 \quad (8)$$

where

$$\hat{P}_{1j}=1+\hat{Q}_{1j}=1+\frac{\hat{q}_{1j}[1-x_{1j}]^{1/2}}{[l_{1j}-\tilde{\alpha}(0)]^{1/2}}, \quad (9)$$

and (see Fig. 2a)

$$q_{1j}=\sum_{i=1}^{j}k_i,\qquad \alpha_{kl}=\alpha(q_{kl}^2),\qquad q_{kl}=\sum_k^l k_i.$$

Expression (8) is the amplitude for the scattering of N − 2 mesons by one fermion. For simplicity, we consider the case $\varphi_1 = \varphi_2 = \varphi_3 = 1 - x$ and $\alpha_{kl}(0) < 0$. The last condition is essential to eliminate tachyons. The energy continuation of the meson channels, just as in the case of the 4-point diagram, is realized with the aid of expressions (7).

Expression (8) can be easily rewritten in operator form, and differs in the usual VD formalism[10] from the case of meson-meson scattering in that D takes the form

$$D(s)=\int_C\frac{dl}{2\pi i}\int_0^1 dz\,z^{H-l-1}(1-z)^{\alpha(0)-1}\frac{P(l,z)}{l-\tilde{\alpha}(s)}, \quad (10)$$

where

$$\hat{P}(l,z)=1+\frac{\hat{q}[1-z]^{1/2}}{[l-\tilde{\alpha}(0)]^{1/2}}. \quad (10')$$

It should be noted here, however, that the chain VDVD... must be aligned along the fermion line. With (10) so expressed, the factorization of the amplitude F_N in the fermion channel (see (8)) becomes obvious and can be expressed in the language of the previous oscillator operators $a_\mu^{(n)}$ (as was indeed done in[8]). The meson channels must be considered separately, since the unusual form of (10) (particularly the dependence of $\hat{P}$ on z) does not make the factorization so obvious for them. The meson channels of the amplitude (8) are considered in detail in Appendix B, where it is shown that there is factorization in them, too, and the number of states increases like e^{cM} (M is the resonance mass) with increasing M, just as in[10].

For an arbitrary diagram (Fig. 2b), in which there are already fermion channels that are dual to each other, the prescription (8) must be modified somehow. Indeed, if we write the expression for the amplitude in the earlier form and write down the product $\prod_{ij} P_{ij}$ corresponding to some configuration of trees, the principal trajectory will already be parity-degenerate for the fermion channel that is dual to the separated P_{ij}. With the 4-line diagram as an example, this means that if we write for the (su) term (Fig. 1c) only the propagator $\hat{P}(l, x)$ (10′), where $q^2 = s$, then parity degeneracy will obtain for the principal trajectory in the u channel. Therefore the general prescription for a many-line diagram should consist of avoidance of the product of any propagators in (8), but for Fig. 2a this prescription should go over into the old one. We propose to generalize the prescription in the following manner: in place of $\prod_{ij} \hat{P}_{ij}(l_{ij}, x_{ij})$ we write in (8)

$$S = 1 + \sum \prod_{ij}^{ij} \hat{Q}_{ij} = 1 + \sum \hat{Q}_{ij} + \sum \hat{Q}_{i_1 j_1} \hat{Q}_{i_2 j_2} + \sum \hat{Q}_{i_1 j_1} \hat{Q}_{i_2 j_2} \hat{Q}_{i_3 j_3} + \ldots + \prod_{ij} \hat{Q}_{ij} = \hat{Q}_{i_1 j_1} \hat{Q}_{i_2 j_2} \ldots \hat{Q}_{i_m j_m}, \tag{11}$$

with either $i_k < i_{k+1}$ and $j_k \leq j_{k+1}$ or $i_k \leq i_{k+1}$ and $j_k < j_{k+1}$, and

$$\hat{Q}_{ij} = \frac{\hat{q}_{ij}(1 - x_{ij})^{1/2}}{[l_{ij} - \tilde{\alpha}(0)]^{1/2}} \qquad \hat{q}_{ij} = \hat{p}_1 + \sum^{i} \hat{k}_m + \sum^{j} \hat{k}_m{}'.$$

Thus, each term of the sum S contains the product of $\hat{Q}_{ij}$ corresponding to fermion channels that are not dual to each other, and the sum is taken over all the possible such products. It is clear that for Fig. 2a, S reduces to the product $\prod(1 + \hat{Q}_{ij})$, i.e., to a product of propagators, but this is not the case for any other diagram.

If we are interested in a certain state on the principal fermion trajectory in some particular channel, then all the duals to the given a_{ij} vanish (see (9)) under the condition $x_{ij} \to 1$. We are left only with those products which contain non-duals to the channel $i_q j_q$, i.e., of $\hat{Q}_{ij}$ of the left-hand and right-hand sides of the diagram of Fig. 4a:

$$Q^{l}_{i_m j_m} \quad (m \leqslant q) \quad \text{и} \quad Q^{r}_{i_k j_k} \quad (q < k \leqslant l),$$

therefore

$$S = \sum \{\hat{Q}_{ij}{}^{l}\}\{\hat{Q}_{ij}{}^{r}\} + \sum \{\hat{Q}_{ij}{}^{l}\}\hat{Q}_{i_q j_q}\{\hat{Q}_{ij}{}^{r}\} = \sum \{\hat{Q}_{ij}{}^{l}\}(1 + \hat{Q}_{i_q j_q})\{\hat{Q}_{ij}{}^{r}\}, \tag{12}$$

with $\{\hat{Q}_{ij}^{l,r}\} = S^{l,r} = 1$ + products of $\hat{Q}_{ij}$ pertaining to the left (right) part of the diagram of Fig. 4a.

It is seen from (12) that only states of one parity remain on the principal trajectory. The factorization in the meson and baryon channels for the most general dual diagram (Fig. 2b) is considered in Appendix C, where the number of resonances with equal mass (degeneracy) is calculated. It is curious that it turned out to be exponentially increasing with increasing square of the mass (4^{M^2} for the meson channel, 6^{M^2} for the baryon channel), and this tremendous degeneracy is connected with the matrix structure of the amplitude (with the location of $\hat{Q}_{ij}$ in S (see (11)).

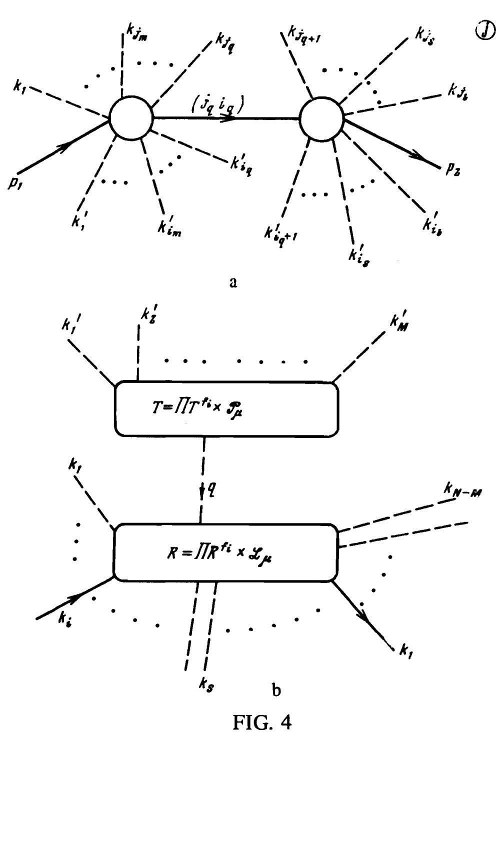

FIG. 4

4. CONCLUSIONS

Thus, we have constructed in the paper a dual model that satisfies all the requirements stipulated in the Introduction.

In the proposed model, unlike the DRM for pure mesonic processes, the number of states with definite mass is much larger than the corresponding number of states for the mesonic amplitudes (4^{M^2} for mesonic channels and 6^{M^2} for baryonic ones in place of l^{cM}; M is the resonance mass). This increase in the number of states is connected not with the chosen form of the fermion propagator, and consequently not with the requirement that there be no parity degeneracy (see Appendix B concerning the influence of the form of the propagator on the number of states), but with the presence of fermion spin, with the matrix structure of the amplitude, and with out prescription for writing down diagrams of Fig. 2b. We therefore believe that this result remains valid also in the case when it is stipulated that there be no parity degeneracy for only the first few resonances with small masses. For example, if it is assumed that there is only one resonance with mass m and spin ½ (the lightest resonance for the given baryon trajectory), then the prescription for writing the general dual diagram of Fig. 2b will be the same as in (8), (9), and (11), except that $\hat{Q}_{ik}$ will be equal to $\hat{q}_{ik}(1 - x_{ik})/m$. Naturally, the number of states is in this case the same as in the proposed model. It should be noted that the presently existing dual models for fermions[3,4] can be similarly generalized to the general form of the dual diagrams (Fig. 2b). No such generalization could be obtained so far in the usual operator formalism[4], and these failures are possibly due precisely to the large number of states, which hardly can correspond to the usual oscillator set of operators.

Naturally, in the proposed formalism it is easy to obtain a DRM in which there is no parity degeneracy for several resonances on the baryon trajectory (with masses $m_1, m_2, \ldots, m_k$). To this end it is necessary to write down the propagator corresponding to the fermion channel in the form

$$\hat{P}_{ij}=\prod_{r=1}^{r=k}\left(1+\frac{\hat{q}_{ij}(1-x_{ij})}{m_r}\right)=A(x_{ij})+B(x_{ij})\hat{q}_{ij}.$$

Thus, it seems to us that the proposed generalization to the diagram of Fig. 2b can find a wider application and can be useful even in the case when parity degenerity is observed for the heavy resonances.

In spite of the fact that we succeeded in constructing a DRM for fermions, corresponding to the realistically observable baryon spectrum, the properties of the obtained amplitude (the increase of the number of states in comparison with the DRM for pure mesonic processes) suggests that unitarization of these amplitudes can give rise to more serious difficulties in comparison with the pure mesonic processes.

We note that the proposed formalism (see (5)) can also be useful for the analysis of meson-meson scattering, if it is assumed, e.g., that the sum of the resonances in one channel is dual to the sum of the contributions of the Regge poles and the branch cuts in the other[11]. It is possible that this new duality principle agrees better with an experiment that shows the contribution of the branch cuts to be quite large[11].

The main shortcoming of the proposed scheme is the relatively large leeway in the choice of the dual models (see (6)). It is possible that the requirement that the residues in the many-line diagrams be positive limits this leeway still further, but it persists to one degree or another and most likely because our assumption that there exists a standing cut in the j plane has little physical content, since it is not represented in a very concrete form.

In conclusion, we thank I. T. Dyatlov for constant interest in the work and for exceedingly useful discussions during its course, and Ya. I. Azimov, V. M. Shekhter, and V. A. Franke for a discussion of the results.

APPENDIX A

In this Appendix we stop to discuss the limitations on the value of the exponent p_1 in the function $\varphi_1 = (1-x)^{p_1}$, which follow from the requirements that the residues for the baryon poles in meson-baryon scattering be positive, and the proof of violation of this requirement in the case when $\varphi_2 = 1 - x$ and $\varphi_3 = 1$.

We consider the amplitude of meson-baryon scattering (5) with $\varphi_1 = (1-x)^{p_1}$ and $\varphi_2 = \varphi_3 = 1 - x$, namely the (s, t) term. Its pole part ($l = \tilde{\alpha}(s)$) is the sum

$$F(s,t)=\frac{\hat{q}}{[\tilde{\alpha}(s)-\tilde{\alpha}(0)]^{1/2}}B\left(-\tilde{\alpha}(s),p_1-\alpha_t-\frac{1}{2}\right)+B(-\tilde{\alpha}(s),-\alpha_t). \quad \text{(A.1)}$$

At the pole $\tilde{\alpha}(s) = n$ this expression takes the form

$$F(s,t)\to\frac{R_n(z_s)}{n-\tilde{\alpha}(s)}=\frac{B\hat{q}+A}{n-\tilde{\alpha}(s)},\quad \tilde{\alpha}(s)\to n; \quad \text{(A.2)}$$

$$R_n(z_s)=\frac{\hat{q}}{[n-\tilde{\alpha}(0)]^{1/2}}\left(\alpha_t-p_1+\frac{3}{2}\right)\left(\alpha_t-p_1+\frac{5}{2}\right)\ldots$$
$$\ldots\left(\alpha_t-p_1+n+\frac{1}{2}\right)+(\alpha_t+1)(\alpha_t+2)\ldots(\alpha_t+n).$$

The function $\alpha_t = \alpha_0 + t$ depends linearly on the first-line of the scattering angle in the s-channel z_s:

$$\alpha_t=\alpha_0+n'(z_s-1)/2,\quad n'=n+2\alpha_0+\tilde{\alpha}(0)+O(1/n), \quad \text{(A.3)}$$

so that

$$R_n(1)=\frac{(\alpha_0-p_1+3/2,n)}{n!}\frac{\hat{q}}{[n-\tilde{\alpha}(0)]^{1/2}}+\frac{(\alpha_0+1,n)}{n!}, \quad \text{(A.4)}$$

$$R_n(-1)=\frac{(\alpha_0+\tilde{\alpha}_0+p_1-1/2,n)(-1)^n}{n!}\frac{\hat{q}}{[n-\tilde{\alpha}(0)]^{1/2}}+\frac{(-1)^n(\alpha_0+\tilde{\alpha}(0),n)}{n!}, \quad \text{(A.5)}$$

where

$$(\alpha,n)=\Gamma(\alpha+n)/\Gamma(\alpha).$$

Since we seek here the residue of a baryon with spin ½ and mass $\tilde{\alpha}(s) = n$ (with $n \gg 1$), the integral of $R_n(z_s)$

$$A^{\pm}_{1/2}=\int_{-1}^{+1}(A\pm MB)dz_s,\quad M=[n-\tilde{\alpha}(0)]^{1/2}, \quad \text{(A.6)}$$

it follows that it is asymptotically ($n \to \infty$) determined by the regions near $z_s = +1$ and $z_s = -1$, where expressions (A.4) and (A.5) can be used. The requirement that the residue be positive for a baryon with mass $\tilde{\alpha}(s) = n$ of parity $A^{+}_{1/2}$ as well as $A^{-}_{1/2}$, then means that

$$(\alpha_0+\tilde{\alpha}_0+p_1-1/2,n),\quad (\alpha_0-p_1+3/2,n),\quad (\alpha_0+\tilde{\alpha}_0,n)\ll(\alpha_0+1,n). \quad \text{(A.7)}$$

Since $p_1 \ge 1$ and $\tilde{\alpha}(0) < 0$ (absence of tachyon), this is equivalent to

$$p_1+\tilde{\alpha}_0<3/2. \quad \text{(A.8)}$$

If we take into account both the (s, t) and (s, u) terms in the amplitude, then a similar analysis of the contributions to $A^{\pm}_{1/2}$ ($n \to \infty$) leads to the following conditions in place of (A.8):

$$\left.\begin{array}{l}(\alpha_0+\tilde{\alpha}_0+p_1-1/2,n)\\ (2\alpha_0+p_1,n-p_1)\dfrac{n!}{(n-p_1)!}\\ (\alpha_0+3/2,n-p_1)\dfrac{n!}{(n-p_1)!}\end{array}\right\}\ll(\alpha_0+1,n)(\tilde{\alpha}_0+1,n), \quad \text{(A.9)}$$

i.e.,

$$\tilde{\alpha}_0<\alpha_0-1/2,\quad p_1<3/2-\tilde{\alpha}_0,\quad p_1<1-\alpha_0. \quad \text{(A.10)}$$

In addition to the natural condition that there be no tachyons, $\alpha_0 < 0$ and $\tilde{\alpha}_0 < 0$, we stipulate the absence of "ghosts" on the meson trajectories, then $\alpha_0 > -1$. The requirements (A.9) and (A.10) can be replaced by

$$p<2,\quad \text{i.e.}\quad p=1, \quad \text{(A.11)}$$

and

$$\tilde{\alpha}_0<\alpha_0-1/2. \quad \text{(A.12)}$$

2. In the case $\varphi_1 = \varphi_2 = (1 - x)$ and $\varphi_3 = 1$, the pole part ($l = \tilde{\alpha}_s$) of the amplitude F(s, t) is represented as follows:

$$F(s,t)=\int_0^1 dx\,x^{-\tilde{\alpha}(s)-1}(1-x)^{-\alpha(t)-1}\left(\frac{\hat{q}(1-x)}{[\tilde{\alpha}(s)(1-x)-\tilde{\alpha}(0)]^{1/2}}+1\right). \quad \text{(A.13)}$$

We find the residue of the pole $\tilde{\alpha}(s) = n$ for the first term:

$$V_1=\frac{\hat{q}}{[n-\tilde{\alpha}(s)]^{1/2}}\frac{1}{[n-\tilde{\alpha}(0)]^{1/2}}B_n(z_s),$$

$$B_n(z_s) = \Gamma(1-\alpha(t)) \sum_{k=0}^{n} \frac{(^1/_2, k)}{k!(n-k)!} \frac{1}{\Gamma(k-n-\alpha(t)+1)} \frac{n^k}{(n-\bar{\alpha}_0)^k} \qquad (A.14)$$

The decisive region for the residue of the baryon with spin $\frac{1}{2}$ ($A^{\pm}_{1/2}(n \to \infty)$) turns out to be the region near $z_s = -1$ in the integral (A.6) for $A^{\pm}_{1/2}$

$$B_n(-1) = \frac{(-1)^n (b,n)}{n!} \sum_{k=0}^{n} (-1)^k F_k, \qquad (A.15)$$

$$b = \alpha_0 + \bar{\alpha}_0 + 1, \quad F_k = \frac{n^k}{(n-\bar{\alpha}_0)^k} \frac{(^1/_2, k)}{(b,k)} C_n^k. \qquad (A.16)$$

The principal region in the sum (A.15) is $k \sim n/2$, where $C_n^k = n!/k!(n-k)! \sim 2^n$. To estimate the contribution of this region, we group the alternating-sign terms into a sum of terms over one sign:

$$(-1)^{n/2} \left\{ \left[F\left(\frac{n}{2}\right) - F\left(\frac{n}{2}-1\right) \right] \right.$$

$$\left. + \sum_{\substack{k=2,4,6,\ldots \\ k<n}} \left[F\left(\frac{n}{2}-k\right) - F\left(\frac{n}{2}-k-1\right) + F\left(\frac{n}{2}+k\right) - F\left(\frac{n}{2}+k-1\right) \right] \right\}$$

where $n/2$ is an integer. Thus, we obtain $B_n(-1) \sim (-1)^{n/2} 2^n$. Near $z_s = 1$, the function $R_n(z_s)$ is of the order of a certain power of n. Consequently, the region $z_s = 1$ can not cancel the contribution from the region $z_s = -1$, which is of alternating sign with respect to n, and for sufficiently large n we find baryons with spin $\frac{1}{2}$ having a negative residue. Consideration of the terms with the (s, u) terms does not change this conclusion.

APPENDIX B

We discuss here the factorization of the amplitude represented by formula (11). For the diagram of Fig. 2a, as already mentioned in Sec. 3, the factorization in the fermion channel is obvious and can be carried out with the aid of the usual operator formalism. In the meson channel of the amplitude this is no longer obvious. The general expression for the amplitude (11) (diagram of Fig. 4b) contains a sum of the products

$$\prod_{ij} \hat{Q}_{ij} = \hat{Q}_{i_1 j_1} \hat{Q}_{i_2 j_2} \ldots \hat{Q}_{i_m j_m}, \qquad (B.1)$$

$$i_k \leqslant i_{k+1},\ j_k < j_{k+1}, \quad \text{or} \quad i_k < i_{k+1},\ j_k \leqslant j_{k+1}; \qquad (B.2)$$
$$\hat{Q}_{ij} = \Phi_{ij}(\hat{p}_i + \hat{q}_j),$$

where

$$p_i = \sum^{i} k_i', \quad q_j = \sum^{j} k_l, \qquad (B.3)$$

$$\Phi_{ij} = [(-\ln x_{ij})(1-x_{ij})]^{1/2} \Phi(^1/_2, {}^3/_2; (\bar{\alpha}_{ij} - \bar{\alpha}(0)) \ln x_{ij})$$

The value of x_{ij} for the fermion channel that is dual to the given meson channel is obtained in the usual manner[10]:

$$x_{ij} = \frac{(1-\rho_i z \sigma_j)(1-\rho_{i-1} z \sigma_{j-1})}{(1-\rho_i z \sigma_{j-1})(1-\rho_{i-1} z \sigma_j)}, \qquad (B.4)$$

$$1 - x_{ij} = \frac{(\rho_i - \rho_{i-1})(\sigma_j - \sigma_{j-1})}{(1-\rho_i z \sigma_{j-1})(1-\rho_{i-1} z \sigma_j)}. \qquad (B.5)$$

This means that

$$\Phi_{ij} = \sum (p_i^2)^r (\rho_i - \rho_{i-1})^n \rho_i^l \rho_{i-1}^m (p_i q_j)^p \cdot \qquad (B.6)$$
$$\cdot z^{n+l+m} (q_j^2)^s (\sigma_j - \sigma_{j-1})^n \sigma_{j-1}^l \sigma_j^m f_{pnlmrs};$$

The summation here is over $pnlmrs$ under the conditions $p + r + s \le n$; $n \ge 1$; $p, l, m, r, s \ge 0$.

The product $\Pi(\hat{p}_i + \hat{g}_j)$ can be expanded in a sum of factorized products of the tensors $\mathscr{P}^{\{i'h\}}_{\{\mu\}}$ and $\mathscr{L}^{\{j'j''h\}}_{q\{\mu\}}$. Here $\{\mu\}$ denotes the sequence of indices $\mu_1\mu_2\ldots\mu_s$; $\{i', h\}$ is a symbol for a certain sequence of indices $i_1', i_2' \ldots i_s'$ from the sequence $i_1 i_2 \ldots i_n$ and

$$\mathscr{P}^{\{i',h\}}_{\{\mu\}} = p_{i_1'\mu_1},\ p_{i_2'\mu_2} \ldots p_{i_s'\mu_s}; \qquad (B.7)$$

$\{j'j''h\}$ denotes two sequences of indices $j_1' \ldots j_{s_1}'$ and $j_1'' \ldots j_{s_2}''$ from the general sequence $j_1 j_2 \ldots j_n$, and accordingly

$$\mathscr{L}^{\{j'j''h\}}_{q\{\mu\}} = \hat{q}_{j_1'} \ldots q_{j_m''\mu_l} \ldots \gamma_{\mu_r} \ldots \hat{q}_{j_i'} \ldots \qquad (B.8)$$

Type h of the given sampling of indices i', j', and j'' does not depend, of course, on the concrete number of these indices.

Thus, together with the usual expansion of the factors $\Pi x_{ij}^{-\alpha_{ij}-1}$ into factorized terms, the expression for our amplitude will contain one more sum that is factorized in each of its terms, namely

$$A = \int dx_i \varphi(x_i) \int dy_j \psi(y_j) \exp\left\{\sum \frac{z^n}{n} P^{(n)} Q^{(n)}\right\} \cdot \qquad (B.9)$$
$$\cdot \frac{(1-z)^{\alpha_0 - 1}}{z^{\alpha_{q^2}+1}} \sum_N z^N \sum_f \sum_h T^{\{f,N,h\}}_{\{\mu\}} R^{\{f,N,h\}}_{\{\mu\}};$$

Here

$$T^{\{f,N,h\}}_{\{\mu\}} = \sum{}' T_{i_1}^{f_1} T_{i_2}^{f_2} \ldots T_{i_n}^{f_n} \mathscr{P}^{\{i'h\}}_{\{\mu\}}, \qquad (B.10)$$

where the prime at the summation sign denotes summation with respect to $i_1, i_2, \ldots, i_n$ under the condition

$$\sum l_k + m_k + n_k = N,\ \{f\} = \{f_1, f_2, \ldots, f_n\}, \qquad (B.11)$$
$$T_{i_1}^{f_1} = (p_{i_1}^2)^{r_1} (\rho_{i_1} - \rho_{i_1-1})^{n_1} \rho_{i_1-1}^{l_1} \rho_{i_1}^{m_1} \sqrt{f_{p_1 n_1 l_1 m_1 r_1 s_1}},$$

and

$$R^{\{fNh\}}_{\{\mu\}} = \sum{}' R_{j_1}^{f_1} R_{j_2}^{f_2} \ldots R_{j_n}^{f_n} \mathscr{L}^{\{j'j''h\}}_{q\{\mu\}}, \qquad (B.12)$$

where

$$R_{j_1}^{f_1} = (q_{j_1}^2)^{s_1} (\sigma_{j_1} - \sigma_{j_1-1})^{n_1} \sigma_{j_1-1}^{m_1} \sigma_{j_1}^{l_1} \sqrt{f_{p_1 n_1 l_1 m_1 r_1 s_1}},$$
$$f_i = (n_i l_i m_i r_i s_i p_i). \qquad (B.13)$$

It must be emphasized that the number of vertices $T^{\{fNh\}}_{\{\mu\}}$ and $R^{\{fNh\}}_{\{\mu\}}$ does not depend on the number of the external mesonic lines and is determined by the number of samplings $\{f, N, h\}$ for the given N (see Appendix C). For the diagram of Fig. 2a, the factors $\hat{Q}_{ij}$ of the dual fermion channels have a fixed index $j = 1$, and accordingly q_1, σ_1, and q_1. Therefore the number of different vertices $R^{\{f,N,h\}}_{\{\mu\}}$ at a given N and under the condition $l_1 + m_1 + n_1 = N$, made up with the aid of one vector q_1, one matrix $\hat{q}_1$, and one variable σ_1, depends on N only in power-law fashion. Consequently, the degeneracy (i.e., the number of states with given mass) in the meson channel for the diagram of Fig. 2a remains the same as in the usual dual model, namely e^{cM}, where M is the mass of the state ($M^2 \gg 1$).

APPENDIX C

For the diagram of Fig. 4b, neither the symbols i nor the symbols j in the fermion factors $\hat{Q}_{ij}$ that are dual to the given meson channel are fixed, unlike the amplitude of Fig. 2a (see Appendix B), where the symbol j was fixed. We obtain the degeneracy in the meson channel for this case.

The number of vertices $T^{\{fNh\}}_{\{\mu\}}$ $(R^{\{fNh\}}_{\{\mu\}})$ (see Appendix B) is determined for a given N, first, by the number of sets of aggregates $\{f\}$, i.e., $\{l_k m_k n_k r_k s_k p_k\}$, such that

$$\sum l_k + m_k + n_k = N; \quad n_k \geq 1, \quad l_k, m_k, p_k, r_k, s_k \geq 0,$$
$$p_k + r_k + s_k \leq n_k,$$

i.e., by the number d^f_N, and second, by the number of tensors $\mathscr{P}^{\{i',h\}}_{\{\mu\}}$ $(\mathscr{L}^{\{j'j''h\}}_{q\{\mu\}})$. The number d^f_N can be obtained with the aid of a function whose Taylor-series coefficients are d^f_N. This device was used in[12].

We consider the next function,

$$f(x,y) = \prod_{\substack{n\geq 1,\\ p,r,s,l,m\geq 0,\\ p+r+s\leq n}} (1 - x^{n+l+m} y^{p+r+s})^{-1}. \tag{C.1}$$

It is easy to see that

$$\varphi(x) = f(x,1) = \sum_{N=0}^{\infty} d_N{}^f x^N. \tag{C.2}$$

For large N $(N \gg 1)$, the d^f_N can be obtained by estimating the Cauchy contour integral for d^f_N by the saddle-point method

$$\ln\varphi(x) \sim \frac{c_1}{(1-x)^6}, \quad x \to 1, \tag{C.3}$$

and hence

$$l_N{}^f \sim \exp c_1 N^{6/7}. \tag{C.4}$$

The vertex $T^{\{f,N,h\}}_{\{\mu\}}$ can be rewritten in the form

$$T^{\{fNh\}}_{\{\mu\}} = \sum_{i_1 i_2 \ldots i_n} (R_{i_1}{}^{f_1} p_{i_1\mu_1}) R_{i_2} (R_{i_3} p_{i_3\mu_3}) \ldots$$

It is easily seen that the number $d^{\mathscr{P}s}_N$ of the tensors $\mathscr{P}^{\{i'h\}}_{\{\mu\}}$, determined by the number of sub-sequences $i'_1 \ldots i'_s$ from the general sequence of the indices $i_1 \ldots i_n$, is the number of all possible distributions of s identical spheres over n − s cells, $d^{\mathscr{P}s}_n = n!/(n-s)!s!$. The maximum value of this number for $n \gg 1$ is $\sim 2^n$ $(s \sim n/2)$.

We can determine in similar fashion the number $d^{\mathscr{L}s_1 s_2}_n$ of the tensors $\mathscr{L}^{\{j'j''h\}}_{q'\{\mu\}}$. In this case we are dealing with the choice of two sub-sequences $j'_1 \ldots j'_{s_1}$ and $j''_1 \ldots j''_{s_2}$ from the sequence $j_1 \ldots j_n$ or, accordingly, the number of all-possible distributions S_1 of identical spheres of one type and s_2 identical spheres of another type with respect to $n - s_1 - s_2$ cells:

$$d^{\mathscr{L}s_1 s_2}_n = \frac{n!}{(n-s_1-s_2)!\, s_1!\, s_2!} \sim 3^n \quad (n \gg 1,\ s_1 \sim s_2 \sim n/3).$$

According to the prescription formulated in the article, identical factors $\hat{Q}_{ij}$ are not encountered in the product $\Pi\hat{Q}_{ij}$. Therefore the agreement of the indices $i_k = i_{k+1}$, on the other hand, means that $j_k < j_{k+1}$, and on the other hand, it follows from $j_k = j_{k+1}$ that $i_k < i_{k+1}$. This means that the possibilities of the equality of the indices from one of the sides must be taken into account separately. We set each sequence of indices $i_1 i_2 \ldots i_n$ in correspondence with a sequence of zeroes and unities, unity corresponding to equality of the given number i_k with the preceding i_{k-1}, and zero corresponding to the absence of such equality. In the analogous sequence of zeroes and unities, the unities for the set of indices $j_1 \ldots j_n$ will correspond to the zeroes of the sequence made up of the indices i, and vice versa. The number of such sequences is the number of combinations $C^{n^1}_n$ (n_1 is the number of coincidences); $c^{n^1}_n \sim 2^n$ for $n \gg 1$, $n_1 \sim n/2$. Thus, the matrix structure of the tensors $\mathscr{L}_q$ and $\mathscr{P}$ turns out to be the source of the strongest degeneracy. The usual degeneracy, which is connected with the expansion

$$\exp\left\{\sum \frac{z^n}{n} P^{(n)} Q^{(n)}\right\},$$

which increases for large masses M like exp(tM), and the degeneracy of $d^f_N \sim \exp(c_1 M^{12/7})$ (see (C.4)), turn out to be asymptotically $(M^2 \gg 1)$ negligible in comparison with the degeneracy $\exp(bM^2)$ that is obtained from counting the number of possible tensors $\mathscr{L}$ and $\mathscr{P}$, and with the number $C^{n^1}_n$, which takes into account the correlations from the equalities of the indices i or j.

In the case of the meson channel, the degeneracy is asymptotically determined by $d^{\mathscr{P}}_n C^{n_1}_n \sim 4^n$, i.e., the number of meson states with given mass M increases like $\exp(M^2 \ln 4)$. For the fermion channel, both vertices contain the tensors $L^{\{j'j''h\}}_{a\{\mu\}}$, and we obtain the degeneracy $d^{\mathscr{L}}_n C^{n_1}_n \sim 3^n 2^n$, i.e., the number of fermion states with given mass M increases like $\exp(M^2 \ln 6)$.

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