OSCILLATION EFFECTS OF THE "FLUX QUANTIZATION" TYPE IN NORMAL METALS

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The magnetic properties of a solid normal cylinder in a longitudinal magnetic field are considered. The surface-state spectrum is obtained for the case of weak magnetic fields. It is shown that the thermodynamic quantities oscillate as a function of the magnetic field flux with a period equal to the "quantum" $\Phi_0 = hc/e$. The effect is due to electrons located near the surface of the cylinder in a layer with a thickness of the order of the de Broglie wavelength. The possibility of studying the phenomenon experimentally is discussed.

As is well known, the phenomenon of flux quantization in a doubly-connected superconducting region, proposed by F. London[1], consists in the fact that the magnetic flux trapped by it can take on only discrete values $\Phi = n\bar{\Phi}_0$ ($\Phi_0 = hc/2e$ is the flux quantum in the superconductor). Various manifestations of this purely quantum effect are possible. Thus, after the first direct experiments on the observation of flux quantization,[2,3] oscillations of the critical transition temperature of hollow, thin-walled superconducting cylinders were discovered in the experiments of Parks and Little[4] upon variation of the applied magnetic field. They are due to the formation of a circulating current in a ring, the value of which oscillates with the change in the (unquantized) flux Φ. Quantization of the magnetic flux in singly-connected superconducting cylinders close to T_c, brought about by currents of surface superconductivity, were observed by Shablo and Dmitrenko.[5]

In all the cases enumerated, the flux quantization is due to the coherent motion of the condensate particles, i.e., it is connected with the presence of the nondiagonal long-range order inherent in superconductors. On the other hand, as Kulik has shown,[6] the flux quantization can also appear at temperatures above critical (when account is taken of the fluctuation pairing) and even in a perfectly normal metal.[7] In the latter case, the formation of an oscillating magnetic moment is brought about by quantization of the motion of an electron in a doubly-connected region, and by the sensitivity of the quantum states to the field of the vector potential **A** (the Aharonov-Bohm effect)[8]. The presence of a magnetic moment that differs from zero is equivalent to the existence of a diamagnetic current; however, in contrast with superconductors, macroscopic long-range order is absent here. Nevertheless, collisions do not lead to a damping of the current in time, since the current state corresponds to a minimum in the free energy.[7]

In the present work, the thermodynamic properties of a solid normal cylinder in a longitudinal magnetic field is studied. It is shown that in a field which satisfies the condition $r_H > R$ (r_H is the cyclotron radius, R the radius of the cylinder), the thermodynamic quantities oscillate upon change in the flux of the magnetic field $\Phi = \pi R^2 H$ with a period equal to the "flux quantum" of the normal metal $\Phi_0 = hc/e$. Skipping ahead, we note that the essential contribution to the oscillations is made by states with large values of the magnetic quantum number. They correspond to electrons located in a narrow layer near the surface of the cylinder, the spectrum of which differs from the spectrum of magnetic surface levels on a plane boundary.[9-12] The effective separation of the surface layer is equivalent to the presence of a doubly-connected region, the electrons moving in which create a surface diamagnetic current.

1. SPECTRUM OF SURFACE STATES IN A CYLINDER

Proceeding to the solution of the stated problem, we write down the Schrödinger equation in cylindrical coordinates. Assuming the solution to be of the form $\psi = e^{\pm im\theta}\mathscr{R}(r)\exp(ip_z z/\hbar)$, we obtain the equation for the radial wave function:

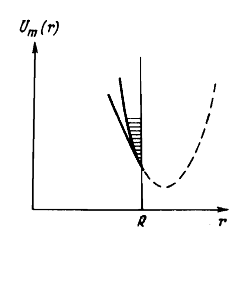
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\mathscr{R}}{\partial r}\right)+\left(\alpha^2-\frac{m^2}{r^2}-\beta^2r^2\right)\mathscr{R}=0; \qquad (1)$$

$$\alpha^2=\frac{2m^*}{\hbar^2}\left(E-\frac{p_z^2}{2m^*}\right)\mp\frac{eH}{\hbar c}m, \qquad \beta=\frac{eH}{2\hbar c},$$

where m is the absolute value of the magnetic quantum number, and m^* is the mass of the electron. The qualitative behavior of the potential energy of the electron U_m with large values of the magnetic quantum number m is shown in the drawing. It is easy to obtain the corresponding spectrum of the surface states. By means of the substitution $\mathscr{R} = r^{-1/2}\chi$, Eq. (1) can be put in the form

$$\chi''(r)+\left[\alpha^2-\frac{m^2-{}^1/_4}{r^2}-\beta^2r^2\right]\chi(r)=0. \qquad (2)$$

Expanding the expression in square brackets in (2) in powers of r = R and keeping only the linear term, which corresponds to the replacement of the potential



energy $U_m(r)$ by a straight line—the tangent at the point $r = R$ (see the drawing), we reduce (2) to the equation for the Airy functions. Introducing the quantity

$$\xi(r) = [r - R + (\alpha_1/\beta_1)^2]\beta_1^{2/3}, \quad \alpha_1^2 = \alpha^2 - \beta^2R^2 - (m^2 - 1/4)/R^2,$$
$$\beta_1^2 = 2(m^2 - 1/4)/R^3 - 2\beta^2 R,$$

we write down the solution of the latter equation in the form (C is a normalization constant)

$$\chi(\xi) = C\mathrm{Ai}(-\xi). \tag{3}$$

The asymptotic form of the Airy function at $\xi \gg 1$ is

$$\mathrm{Ai}(-\xi) \sim C\xi^{-1/4}\sin[2/3\,\xi^{3/2} + \pi/4].$$

From the requirement that the wave function vanish on the boundary of the cylinder, we obtain the spectrum of the magnetic surface levels:

$$E_{mn}(p_z) = \frac{\hbar^2}{2m^*R^2}\left\{(m \pm \eta)^2 + m^{4/3}\left[3\pi\left(n + \frac{3}{4}\right)\right]^{2/3}\right\} + \frac{p_z^2}{2m^*}, \tag{4}$$
$$\eta = \Phi/\Phi_0.$$

The obtained spectrum is valid in the region of weak magnetic fields. Actually, keeping the quadratic term in the expansion of the potential energy,

$$V(r) = \left[\frac{m^2 - 1/4}{R^4} - \beta^2\right](r - R)^2,$$

we calculate the first-order correction to the found energy levels. From the requirement of smallness of this correction in comparison with the spacing between the quantized levels (4) for a fixed magnetic quantum number, we arrive at the condition $\Phi/\Phi_0 \ll (R/\lambda)^{4/3}$, which is equivalent to $R/r_H \ll (R/\lambda)^{1/3}$ (λ is the de Broglie wavelength of the electron). Inasmuch as $R \gg \lambda$ ($\lambda \sim 10^{-8}$ cm for "typical" metals and $\lambda \sim 10^{-6}$ cm for metals of the bismuth type), the given expression for the spectrum still remains valid for $R \sim r_H$.[1)]

It will be shown below that the spectrum (4) can be obtained as a limiting case from the more general expression for the energy levels in a cylinder in a weak magnetic field. To find it, we return to Eq. (1). The solution of Eq. (1) is a confluent hypergeometric function. The desired spectrum of the electrons $E = E_{nn}(p_z)$ is found from the condition of the vanishing of this function on the boundary of the metal. To obtain the explicit analytic expression of the spectrum, we can solve Eq. (1) by perturbation theory, choosing the term $\beta^2 r^2$ to be small, which is valid for $r_H \gg R$ (weak field). This leads to the following expression:[2)]

$$E_{mn}(p_z) = \frac{\hbar^2}{2m^*R^2}\varepsilon_{mn} + \frac{p_z^2}{2m^*}, \tag{5}$$
$$\varepsilon_{mn} = \gamma_{mn}^2 \pm 2\eta m + 1/3\,\eta^2[1 + 2(m^2 - 1)/\gamma_{mn}^2],$$

here γ_{mn} are the zeroes of the Bessel function J_m.

The spectrum (5) describes the magnetic surface levels in a cylinder. The origin of these states is connected with the localization of the excitations in the two-dimensional region (the cross section of the cylinder), in which they differ significantly from the ordinary magnetic surface levels.[9-12] In the quasiclassical approximation, the electron describes a complicated trajectory inside the circular region, all portions of which touch a certain curve which serves as a caustic for it. The condition of quantization of the angular momentum leads to the result that only certain trajectories are realized, corresponding to the possible values of the magnetic quantum number. Keller and Rubinow[14] developed a multi-dimensional quasiclassical method to find the asymptotic form of the wave functions and the corresponding eigenvalues as applied to the problems of geometric optics. They showed directly that the Debye expansion of the Bessel function ($1 \ll m < z$) serves asymptotically as the accurate approximation of the eigenfunction $J_m(z)$:[15]

$$J_m(z) \sim \left(\frac{2}{\pi}\right)^{1/2}(z^2 - m^2)^{-1/4}\sin\left[(z^2 - m^2)^{1/2} - m\arccos\frac{m}{z} + \pi/4\right], \tag{6}$$

and is valid in the region between the caustic and the boundary of the cylinder. Inside the caustic ($m \gg z$), the solution decays exponentially. Using (6), we find the equation for the asymototic zeroes of the Bessel function:

$$(\gamma^2 - m^2)^{1/2} - m\arccos(m/\gamma) = \pi(n + 3/4). \tag{7}$$

The set of equations (5) and (7) determines completely the quasiclassical spectrum of the electrons in the cylinder for weak magnetic fields. In the important limiting case $m \approx \gamma$ (it corresponds to caustic dimensions of the order of the radius of the cylinder) we get from (7)

$$\gamma \approx m + 1/2\,m^{1/3}[3\pi(n + 3/4)]^{2/3} + \ldots, \tag{8}$$

which, after substitution in (5), leads to the spectrum (4) obtained previously. The "whispering gallery" modes in acoustics are the analog of the states (4).

2. OSCILLATIONS OF THERMODYNAMIC QUANTITIES

We first calculate the density of states

$$\nu(E) = \sum_{n,m,p_z}\delta[E - E_{mn}(p_z)].$$

After integration over p_z and introduction of the dimensionless variable $\mathscr{E} = 2m^*R^2E/\hbar^2$, we get

$$\nu(\mathscr{E}, H) = \frac{2Lm^*R}{\pi\hbar^2}\sum_{m,n}\frac{\theta(\mathscr{E} - \varepsilon_{mn})}{(\mathscr{E} - \varepsilon_{mn})^{1/2}}, \qquad \theta(x) = \begin{cases}1, & x > 0\\ 0, & x \leqslant 0\end{cases}, \tag{9}$$

where L is the height of the cylinder. We use the Poisson summation formula to separate the oscillating term. It consists of two different components, which result from the contribution of both the point when the phase is stationary and of the critical end points of the region of integration. Although the end points lead to oscillations of smaller amplitude, they actually determine the effect under study. The role of this component is furthermore decisive when account is taken of the diffuse reflection of electrons from the walls of the cylinder (see below). Carrying out the summation over m in (9) and introducing the integration variable $m/\mathscr{E}^{1/2} = l$, we obtain for the oscillating component

$$\nu^{\mathrm{osc}}(\mathscr{E}, H) = \frac{2Lm^*R}{\pi\hbar^2}\sum_{s=-\infty}^{\infty}I(s),$$

[1)]It was shown by Prange [13] that, in the case $r_H \ll R$, the Hamiltonian of the electrons located in a magnetic field on a cylindrical surface reduces to the Hamiltonian of a plane semi-infinite space as $R \to \infty$.

[2)]We emphasize that small parameter of perturbation theory is the ratio of the potential energy of the electron in the magnetic field to the Fermi energy, and not the quantity R/r_H.

$$I(s)=\int_{-\infty}^{\infty} dl \exp(-2\pi i s \mathscr{E}^{1/2} l) \sum_{n=0}^{n_{max}(l)} \frac{\theta(\mathscr{E}-\varepsilon_{ln})}{(1-\varepsilon_{ln}/\mathscr{E})^{1/2}}. \qquad (10)$$

We now calculate the integral I(s) asymptotically for $\mathscr{E}^{1/2} \gg 1$, recognizing that the contribution of the ends of the interval correspond to the minimum value of the quantum number n = 0. For the limiting values of the magnetic number m we have $\epsilon_{l0} = \mathscr{E}(l \pm \eta/\sqrt{\mathscr{E}})^2 + (9\pi/4)^{2/3}\mathscr{E}^{2/3} l^{4/3}$. Introducing the variable $\tilde{m} \equiv l \pm \eta/\mathscr{E}^{1/2}$, which is equivalent to the displacement of the integration boundaries, we obtain for ν_1:

$$\nu_1 \approx \frac{2Lm^*R}{\pi\hbar^2} \sum_{s=-\infty}^{\infty} e^{2\pi i s \eta} \int_{m_-}^{m_+} d\tilde{m} \exp(-2\pi i s \mathscr{E}^{1/2}\tilde{m}) f(\tilde{m}), \qquad (11)$$

$$f(\tilde{m}) = [1-\tilde{m}^2 - (\tilde{m} \mp \eta/\mathscr{E}^{1/2})^{4/3}\xi]^{-1/2}.$$

The parameter $\xi = (9\pi/4)^{2/3}\mathscr{E}^{-1/2} \ll 1$ and the limits of the integration are

$$m_\pm = \pm 1 \mp \xi\left(\frac{1}{2} \mp \frac{2}{3}\eta\mathscr{E}^{-1/2}\right) + \ldots.$$

Carrying out the integration in (11),[16] we obtain the following expression for the oscillating component ν_1^{osc}:[3)]

$$\nu_1^{osc}(E;H,R) = \sum_{s=1}^{\infty} A_s \cos\left[2\pi s \frac{\Phi}{\Phi_0}\right], \qquad (12)$$

$$A_s = \left(\frac{2}{s}\right)^{1/2} \frac{L\sin[2\pi s(E/E_0)^{1/2} + \pi/4]}{\pi R E_0^{3/4} E^{1/4}},$$

where the characteristic energy is $E_0 = \hbar^2/2m^*R^2$.

Thus the density of states of a normal solid cylinder in a weak magnetic field oscillates with change of flux Φ, having a period equal to $\Phi_0 = hc/e$. A relative small fraction of the electrons is responsible for these oscillations; these electrons are located in a narrow layer near the surface of the cylinder, with extremely large magnetic quantum numbers. The bulk of the electrons, on the other hand, produce oscillations of a different type, to find which we return to Eq. (10). Applying the Poisson formula to the sum over n, we calculate the resulting integral asymptotically, recognizing that the maximum value of n (for given m) is determined by Eq. (7). The integral over l is estimated by the stationary-phase method, which gives for ν_2^{osc} (for its first harmonic) the expression

$$\nu_2^{osc} \approx \frac{L}{\pi R E_0} \cos\left\{4\left(\frac{E}{E_0}\right)^{1/2}\left[1-\delta\left(\frac{E}{E_0}\right)\left(\frac{\Phi}{\Phi_0}\right)^2\right]\right\}, \qquad (13)$$

where the constant δ is of order unity. The component (13) depends on the magnetic field in complicated fashion and describes oscillations of the type of the de Haas-van Alphen effect under conditions of additional size quantization. Its amplitude is $(E/E_0)^{1/4} \gg 1$ times greater than the amplitude of the oscillations ν_1^{osc}.[4)]

Allowance for the diffuseness of the relfection influences the amplitudes of ν_1^{osc} and ν_2^{osc} differently. Qualitatively, this can be understood if we recall the situation with ordinary surface levels. As shown by Kaner et al.,[18] most electrons colliding with the surface undergo diffuse scattering and their damping is so large, even for weak diffuseness, that the spectrum of the surface states becomes continuous. Only for "glancing" orbits, the effective de Broglie wavelength of which is large in comparison with the characteristic surface inhomogeneities, is the reflection close to specular and the discrete character of the spectrum preserved. It is clear from what has been said that the most favorable conditions in the problem under discussion for the appearance of effects of quantization are states of the "whispering gallery" type, since the electrons described by them collide with the surface of the cylinder at small angles. For this reason, the oscillations ν_2^{osc} should disappear completely when account is taken of even a weak diffuseness (in what follows, we shall not take them into account) and all the periodic changes in the thermodynamic quantities will be determined by oscillations of the density of states ν_1^{osc}.

It is easy to estimate the width of the surface layer that contributes to the oscillations ν_1^{osc}. We make use of the conditions of quasi-classical quantization:

$$\frac{1}{2\pi}\oint p_r\,dr = (n + 3/4)\hbar, \qquad (14)$$

$$p_r = \left[2m^*\left(E - \frac{p_z^2}{2m^*}\right) \mp \frac{eH}{c}\hbar m - \left(\frac{eH}{2c}\right)^2 r^2 - \frac{\hbar^2 m^2}{r^2} - U_1(r)\right]^{1/2}.$$

The potential $U_1(r)$ corresponds to an infinitely high wall at the boundary of the cylinder. From the requirement of the vanishing of the integrand of (14), we find the equation that connects the magnetic quantum number m with the radius of the caustic a_0. Solving it relative to m, we have

$$m(H) = \mp \pi a_0^2 H/\Phi_0 \pm a_0 k, \qquad (15)$$

where the wave vector $\hbar k = (2m^*E - p_Z^2)^{1/2}$. If the magnetic field is equal to zero, we obtain the simple relation $ka_0 = m$ (see, for example,[14]).

An important change in the integrand of the expression in (10) takes place over the interval $\Delta m \sim 1$. The dimensions of the caustic for the states (4) are equal in order of magnitude to the value of the radius of the cylinder. Setting $k \approx k_F$ ($\hbar k_F$ is the Fermi momentum) and taking into account the condition $r_H > R$, we obtain the estimate $\Delta R \sim \lambda$, i.e., the oscillations in the flux function are produced by electrons concentrated in a narrow layer of the order of the de Broglie wavelength. This value can be taken to be equal to the dimensions of the layer in which the "glancing" electrons are specularly reflected from the surface. Assuming the radius of the cylinder to be much greater than the characteristic dimensions σ of the roughness of the surface, we obtain the estimate $\Delta R \sim (\rho_0/\sigma)^2\lambda$, where $\rho_0 = (\hbar c/eH)^{1/2}$. For the weak fields of interest to us, the width of the surface layer of the cylinder with specular scattering of the electrons is much greater than λ.

We now determine the relative amplitude of the oscillations. The nonoscillating part of the density of states is equal in order of magnitude to $\nu_0 \sim LE^{1/2}/RE_0^{3/2}$; therefore, we get for the relative amplitude of the oscillations

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3)In obtaining Eq. (12), we set $\xi \to 0$. Retaining the terms of order ξ leads only to a weak modulation of the amplitude of the oscillation with the field.

4)Dingle [17] obtained only an oscillating term of type (13) in the calculation of the thermodynamic quantities of a normal cylinder in a magnetic field, since terms of order $(E_0/E)^{1/4}$ were omitted from the calculation.

$$\nu^{osc}/\nu_0 \sim (E_0/E)^{3/4}. \quad (16)$$

For large electron groups[19] and at a cylinder radius $R \sim 10^{-4}$ cm, we have $E_0 \sim 10^{-7}$ eV, which gives a relative amplitude of the order of 10^{-5}–10^{-6}. However, for metals of the bismuth type, which contain small groups, we have

$$\nu^{osc}/\nu_0 \sim 10^{-2} - 10^{-3}.$$

This estimate, naturally, does not take into account the decrease in the amplitude of oscillations due to a smearing of the energy levels following surface and volume collisions.

We now proceed to the calculation of the oscillating contribution to the thermodynamic potential Ω. Carrying out similar calculations, we get

$$\Omega^{osc} \approx \Theta \frac{2}{\pi} \frac{L}{R} \left(\frac{\zeta}{E_0}\right)^{1/4} \sum_{s=1}^{\infty} \frac{\cos(2\pi s(\zeta/E_0)^{1/2} - \pi/4)}{s^{3/2}\,\mathrm{sh}[\pi^2 s(\zeta E_0)^{-1/2}\Theta]} \cos\left[2\pi s \frac{\Phi}{\Phi_0}\right], \quad (17)$$

ζ is the Fermi energy and Θ is the temperature in energy units. Using the formula $\mu = -\partial\Omega/\partial H$, we obtain from (17) the oscillating component of the magnetic moment:

$$\mu^{osc} = \sum_{s=1}^{\infty} \mu_s \sin\left[2\pi s \frac{\Phi}{\Phi_0}\right],$$

$$\mu_s = 2\Theta \frac{eRL}{\hbar c} \left(\frac{\zeta}{E_0}\right)^{1/4} \frac{\cos[2\pi s(\zeta/E_0)^{1/2} - \pi/4]}{s^{1/2}\,\mathrm{sh}[\pi^2 s(\zeta E_0)^{-1/2}\Theta]}. \quad (18)$$

The presence of a non-zero moment is equivalent to the existence of an undamped surface "diamagnetic" current (cf.[7]). In contrast with the usual dissipative current, the collisions of the electrons do not lead to a disruption of the current state, since it corresponds in the given case to a minimum in the free energy of the system.

We now touch briefly on the behavior of a solid cylinder in the field of the vector potential and of a thin-walled cylinder in a longitudinal magnetic field. We have in mind the following. The effect of "flux quantization" in[7] was obtained in a situation in which the field H inside a hollow cylinder is equal to zero, and there exists only the field of the vector potential generated by a solenoid in a cavity. In the case of a solid cylinder (more accurately, a hollow cylinder, the inner radius of which $r_1 \to 0$) the solution can be represented in the form

$$A_\theta = \Phi/2\pi r, \quad (19)$$

which is valid over the entire region of the cylinder except the point $r = 0$. The magnetic field here vanishes identically. Although it is difficult to realize this situation experimentally, it is interesting in that it demonstrates unambiguously the existence of the effect under discussion, inasmuch as in the case of the gauge (19) the spectrum of states is found in explicit form. We have

$$E_{mn}(p_z) = \frac{\hbar^2}{2m^* R^2} \gamma^2_{n,m+\eta} + \frac{p_z^2}{2m^*}, \quad (20)$$

where $\eta = \Phi/\Phi_0$, $\gamma_{n,m+\eta}$ is the n-th zero of the Bessel function of order $m + \eta$, and m takes on both positive and negative values. Again using the asymptotic form (7), and taking into account the postiveness of $m + \eta$, which follows from the requirement of the finiteness of the wave function at the origin, we get the following, after calculation of the density of states:

$$\nu^{osc} = \sum_{k=2}^{\infty} \sum_{s=1}^{k/2} A_{sk} \cos\left[2\pi s \frac{\Phi}{\Phi_0}\right], \quad (21)$$

$$A_{sk} = \frac{L}{\pi R E_0 k} \sin\left\{2k\left(\frac{E}{E_0}\right)^{1/2} \sin\left(\frac{\pi s}{k}\right) + \frac{\pi k}{2}\right\} \sin\frac{\pi s}{k},$$

(Φ is the flux of the vector **A** through the cross section of the cylinder). We note that the amplitude (21) exceeds the amplitude of the oscillations ν_1^{osc} (12) in a magnetic field. This is due to the fact that in the field of the vector potential the point of stationary phase makes a contribution to oscillations of the flux-quantization type.

Finally, for a thin-walled cylinder, the spectrum of the electrons in a longitudinal magnetic field, calculated in a fashion similar to (5) with the aid of perturbation theory, agrees exactly with the spectrum of the thin-walled cylinder in the field of the vector potential.[7] The amplitude of the produced oscillations as a function of the flux agrees with the amplitude (6) from[7].

3. DISCUSSION OF THE RESULTS

Oscillations of thermodynamic quantities of the flux-quantization type should take place in a weak magnetic field, when there are no Landau states. Inasmuch as the finiteness of the motion is connected with the collisions of the electron with the boundaries of the cylinder, all the quantum states are "surface" ones. As a consequence of the complete lifting of the degeneracy in m, there exists a unique connection beween the size of the caustic and this number. When the flux Φ through the cross section of the cylinder changes, the maximum permissible value of the magnetic quantum number also changes. One can show that the reason for the appearance of oscillations of the flux-quantization type in a magnetic field is the "creeping" of the maximum dimension of the caustic through the boundary of the cylinder, accompanied by restructuring of every state of the metal. Setting $a_0 = R$, we have, according to (15),

$$m(H) = \mp \frac{\Phi}{\Phi_0} \pm R \frac{(2m^* E - p_z^2)^{1/2}}{\hbar}. \quad (22)$$

The period of the oscillations is sought from the condition $\Delta m(H) = 1$, which gives $\Delta H = \Phi_0/\pi R^2$.

Oscillations of the flux-quantization type in a solid cylinder in the field of the vector potential are due with the periodic mutual congruence of the set of quantized levels with one another. As is seen from (20), this takes place each time that the change in the vector potential changes the index of the Bessel function by unity. The effect in the field of the vector potential is a volume effect; for just this reason, the amplitude of the oscillations is large in this case.

The calculation carried out above was performed under the assumption of an infinite length of the free path of the electron. However, it is physically evident that the effect of the oscillations should be preserved even when account is taken of collisions of the electrons with the boundaries, volume defects, and the like. Account of the scattering leads to a decrease in the amplitude of the oscillations, which is described phenomenclogically by means of the Dingle factor.[19] For the glancing electrons, we neglect the level broadening that results from surface collisions, assuming that the smearing width is entirely determined by the impurity concentration and by the temperature of the metal. From the condition of smallness of the level width $\hbar/\tau$ (τ is the time of free flight) in comparison with the spacing between the quantized levels $\delta E \sim \zeta\lambda/R$, one can obtain an estimate of the temperature required to detect the effect. However, for pure samples, a stronger limitation on the temperature arises from the temperature dependence of the amplitude of the oscillations (18): $\Theta \ll \hbar v_F/2\pi^2 R$. We note that in the experiment one usually measures the oscillation of the magnetic moment μ^{osc} or the magnetic susceptibility χ^{osc}, the relative amplitude of which is greater than the density of states. Thus, for the magnetic moment we have, according to (18),

$$\frac{\mu^{osc}}{\mu_0} \sim \frac{1}{\pi^2}\left(\frac{\rho_0}{R}\right)^2 (\zeta/E_0)^{1/12},$$

which exceeds the estimate (16). This is connected with the fact that the oscillations have a very small period.

In addition to the study of the oscillations of μ or χ, the study of oscillations of the longitudinal electrical resistance of the cylinder can be experimentally more convenient. One can become convinced of the existence of such an effect by considering it as the analog of the Shubnikov—de Haas effect (if the given oscillation effect is compared with the de Haas—van Alphen effect). A similar situation holds in superconductors for $T > T_c$. It has been discussed in the work of Kulik and Mal'chuzhenko,[20] where it was shown that with account of the fluctuation coupling, the effect of flux quantization in a hollow cylinder can appear in two equivalent phenomena—the appearance of a circulating current and oscillations of the fluctuation conductivity.

In conclusion, we point out the interesting possibility of the study of the spectrum of magnetic surface levels of the cylinder. If a normal cylinder located in a weak magnetic field is placed in a high-frequency electromagnetic field, then one should observe singularities of the surface impedance, brought about by transitions of the electrons between quantized levels. The spectrum of surface states (4) describes a system of energy levels characterized by two discrete quantum numbers. The separation between neighboring levels is equal to $E_{m+1,n} - E_{m,n} \approx \zeta\lambda/R$ for a fixed quantum number n and to $E_{m,n+1} - E_{m,n} \approx \zeta\,(\lambda/R)^{2/3}$ for a fixed magnetic quantum number. Each time that the frequency of the high frequency field is a multiple of one of the characteristic frequencies, the surface impedance of the sample should reveal singularities. This resonance mechanism of absorption of energy of the high-frequency field is similar to the oscillations of the surface impedance of a plane parallel plates, first observed by Khaĭkin.[9]

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