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RELAXATION OF FAST ELECTRONS IN GASES AND SEMICONDUCTORS

R. I. LYAGUSHCHENKO

Leningrad State University

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The energy distribution in a gas or semiconductor is analyzed for energies $\epsilon \gg \langle\epsilon\rangle$ ($\langle\epsilon\rangle$ is the mean electron energy) in the presence of sources of nonequilibrium electrons with energies $\epsilon_0 \gg \langle\epsilon\rangle$. The case when the electron source density depends on the coordinate is considered. A method for solving the problem is proposed which simultaneously takes into account electron diffusion in coordinate space and energy relaxation due to interelectron interaction, impact ionization, interaction with phonons (for semiconductors) or elastic and inelastic scattering by atoms (for gases).

THE problem of relaxation of fast non-equilibrium electrons produced in a medium is frequently encountered in gas and semiconductor electronics. By way of example, we can point to two rather extensive groups of phenomena where this problem arises:

1) Photoelectric phenomena in semiconductors. In this case fast non-equilibrium electrons are produced inside the semiconductor by the incident radiation.

2) Phenomena in the near-cathode region of a gas discharge; here the reason for the appearance of fast electrons is the acceleration of the emission electrons in the region of the cathode potential drop.

A feature of the problem is that in the general case the electron-energy distribution function depends on the coordinate, since the density of the sources of the primary electrons is not uniformly distributed in space. This problem was solved earlier for different particular cases. The distribution function was obtained in a number of papers[1-6] without allowance for the dependence on the coordinate. The dependence of the distribution on the coordinate was taken into account by others[7-9], but the solution methods used by them were suitable only for the case when all the coefficients of the kinetic equation are independent of the energy. This condition either imposes stringent limitations on the energy dependence of the collision frequencies, or yields the form of the distribution only in a narrow range of energy values $|\epsilon - \epsilon_0| \ll \epsilon_0$ near the energy ϵ_0 of the primary electrons.

We show in the present paper that the problem of the relaxation of fast non-equilibrium electrons can be posed and solved in general form by making certain realistic assumptions. The method proposed by us for the solution of the problem yields the distribution function for arbitrary energy dependences of the collision frequencies in a wide energy interval, with the exception of a narrow region $|\epsilon - \epsilon_0| \ll \epsilon_0$. In connection with the latter limitation, our method and the method employed earlier[7-9] are mutually complementary.

1. FORMULATION OF PROBLEM

Assume that the electrons are produced with a characteristic energy of the order of ϵ_0. We shall henceforth be interested in the energy distribution function $f_0(\epsilon, x)$ of fast non-equilibrium electrons under the conditions

$$\varepsilon_0, \varepsilon \gg T, T_e, \tag{1}$$

where T is the temperature of the medium, $T_e = (2/3)\langle\epsilon\rangle$ is the temperature of the slow electrons in the medium, and $\langle\epsilon\rangle$ is the average electron energy. We solve the problem in the diffusion approximation, assuming that the electron distribution depends little on the angles in momentum space[10]. This is true if the distribution of the primary electrons that appear in the medium is isotropic. If the last condition is not satisfied (as, for example, when a beam of fast electrons enters the medium), then it is meaningful to solve the problem in the diffusion approximation if the electron momentum relaxation length λ_p is noticeably smaller than the energy relaxation length Λ:

$$\lambda_p \ll \Lambda. \tag{2}$$

As usual[10]

$$\lambda_p = \left(\frac{2\varepsilon}{m}\right)^{1/2}\tau_p, \quad \frac{1}{\tau_p} = 2\pi\int_0^\pi P(\varepsilon, \vartheta)(1 - \cos\vartheta)\sin\vartheta\, d\vartheta, \tag{3}$$

where m is the electron mass, τ_p is the momentum relaxation time, and $P(\epsilon, \vartheta)$ is the probability of scattering through an angle ϑ. An expression for Λ will be derived later on.

If we neglect the action of the space-charge field in the medium on the fast electrons, then the kinetic equation for a medium occupying the half-space $x \geq 0$ can be written in the form

$$D(\varepsilon)\frac{\partial^2 f_0}{\partial x^2} + S(f_0) - \frac{f_0}{\tau(\varepsilon)} = -R(\varepsilon, x). \tag{4}$$

Here $D(\epsilon)$ is the diffusion coefficient of electrons of energy ϵ, $\tau(\epsilon)$ is the electron lifetime, $R(\epsilon, x)$ is the density of the primary-electron sources, and $S(f_0)$ is the collision operator. We assume that at $\epsilon \gg T_e$

$$f_0(\varepsilon, x) \gg f_{00}(\varepsilon, T_e),$$

where $f_0(\epsilon, T_e)$ is the equilibrium distribution function. As the boundary condition at x = 0 we assume the condition of total electron reflection:

$$\partial f_0(\varepsilon, x) / \partial x|_{x=0} = 0. \tag{5}$$

In the case of the photoeffect in semiconductors, this condition is satisfied if the electron energy is lower than the work function and if the surface-recombination coefficient is small.

In the near-cathode region, condition (5) is satisfied for electrons with energy $\epsilon < eU_c$ (U_c is the cathode potential drop). At $\epsilon > eU_c$, the condition (5) is a good approximation for the energy region $\epsilon - eU_c \ll \epsilon$, for in this case the fraction of the electrons satisfying the condition for returning to the cathode ($\frac{1}{2}mv_x^2 \geq eU_c$) is small. The energy of the electrons emitted from the cathode is usually much lower than eU_c, so that the boundary condition (5) makes it possible to describe well almost the entire distribution of the fast electrons, with the exception of the energy region $\epsilon > eU_c$, where the number of electrons is exponentially small.

We assume the condition

$$f_0(\varepsilon, x) \to 0 \quad \text{as} \quad x \to \infty. \tag{5a}$$

More accurately speaking, $f_0(\epsilon, x)$ should tend to the equilibrium distribution function as $x \to \infty$. However, in the energy region $\epsilon \gg T_e$ the number of equilibrium electrons is exponentially small and we can assume the condition (5a).

The form of the function $R(\epsilon, x)$ depends on the mechanism whereby the fast electrons are produced. In the case of the photoeffect, $R(\epsilon, x)$ can be easily expressed in terms of the spectral density of the incident radiation and the absorption coefficient. When the primary electrons are introduced in the medium in the form of a beam, then the beam relaxes over a distance $x \approx \lambda_p$. If $\lambda_p \ll \Lambda$, then a diffusion flux of electrons should exist at $x > \lambda_p$, with a density equal to the density in the primary beam (under the condition of total reflection at $x = 0$). The same result is obtained when $x > \lambda_p$, if it is assumed that the electrons are produced in the $x = 0$ plane and with an isotropic momentum distribution, so that we can write in this case

$$R(\varepsilon, x) = \frac{j_0}{g(\varepsilon)} r(\varepsilon)\, \delta(x - \sigma), \tag{6}$$

where $\delta(x - \sigma)$ is a δ function, $\sigma \to 0$ after integration with respect to x, j_0 is the emission-current density, $r(\epsilon)$ is the distribution with respect to energy in the beam, and $g(\epsilon)$ is the density of states.

2. FORM OF THE COLLISION OPERATOR

In the general case, we take into account in the collision operator the interelectron interaction, the interaction of the electrons with the phonons (for semiconductors) or with atoms (for a gas), and impact ionization. We write down the form of the collision operator for these processes.

1. Interelectron interaction. For electrons with energy $\epsilon \gg T_e$, when the number of fast electrons is much less than the total number of electrons in the medium, the interelectron interaction operator can be represented in the form[11,12]

$$S_e(f_0) = \frac{1}{\varepsilon^{1/2}} \frac{d}{d\varepsilon} \left\{ \nu_e(\varepsilon)\, \varepsilon^{3/2} \left[T_e \frac{df_0}{d\varepsilon} + f_0 \right] \right\}, \tag{7}$$

$\nu_e(\epsilon) = 2^{3/2}\pi e^4 n_e L / m^{1/2}\epsilon^{3/2}$ is the frequency of the interelectron collisions, n_e is the electron concentration, and L is the Coulomb logarithm.

2. Elastic scattering by gas atoms or scattering by acoustic phonons. In this case, the collision operator is given by[10]

$$S_a(f_0) = \frac{1}{\varepsilon^{1/2}} \frac{d}{d\varepsilon} \left\{ \delta \nu_a(\varepsilon)\, \varepsilon^{3/2} \left[T \frac{df_0}{d\varepsilon} + f_0 \right] \right\}. \tag{8}$$

Here δ is the average fraction of the energy lost in a single scattering act, and $\nu_a(\epsilon)$ is the collision frequency. For scattering by atoms we have $\delta = 2m/M$, and for scattering by phonons $\delta = 2ms^2/T$; M is the mass of the atom and s is the speed of sound.

3. Excitation of atoms or emission of optical phonons. If we neglect the small dispersion of the optical phonons, then the collision operator has the same form for the two processes[13,14]:

$$S^*_a(f_0) = \frac{1}{\varepsilon^{1/2}} \sum_n \left[(\varepsilon + \varepsilon_n)^{1/2} \nu_n^*(\varepsilon + \varepsilon_n) f_0(\varepsilon + \varepsilon_n) - \varepsilon^{1/2} \nu_n^*(\varepsilon) f_0(\varepsilon) \right]. \tag{9}$$

Here $\nu_n^*(\epsilon)$ is the frequency of excitation of the n-th level of the atom or of the emission of an optical phonon of the n-th type, and ϵ_n is respectively the excitation energy of the atom or the energy of the optical phonon. For simplicity we neglect the phonon absorption or the deactivation of the excited atoms, assuming that in the case of a semiconductor $\epsilon_n \gg T$, and in the case of a gas the relative number of excited atoms is small.

At $\epsilon \gg \epsilon_n$, the inelastic-process operator (9) can be written in differential form, expanding the first term in powers of ϵ_n:

$$S_a^*(f_0) = \frac{1}{\varepsilon^{1/2}} \frac{d}{d\varepsilon} \left[\varepsilon^{1/2} \sum_n \varepsilon_n \nu_n^*(\varepsilon) f_0(\varepsilon) \right]. \tag{10}$$

4. Impact ionization. For simplicity we confine ourselves to the case when the mass of the hole is noticeably larger than the mass of the electron. In this case we can write for a gas and for a semiconductor

$$S_i(f_0) = \frac{1}{\varepsilon^{1/2}} \left[\int_{\varepsilon+\varepsilon_i}^{\infty} w_i(\varepsilon', \varepsilon) f_0(\varepsilon') \sqrt{\varepsilon'}\, d\varepsilon' + \int_{\varepsilon+\varepsilon_i}^{\infty} w_i(\varepsilon', \varepsilon' - \varepsilon - \varepsilon_i) f_0(\varepsilon') \sqrt{\varepsilon'}\, d\varepsilon' \right] - \nu_i(\varepsilon) f_0(\varepsilon). \tag{11}$$

Here $w_i(\epsilon', \epsilon)$ is the probability of a process in which the primary electron with energy ϵ' produces a secondary electron with energy ϵ, $\nu_i(\epsilon)$ is the frequency of the ionization processes, and ϵ_i is the ionization energy. At energies $\epsilon \gg \epsilon_i$, this expression can also be written in differential form.

At high incident-electron energies, processes with small energy transfer predominates[15], so that the function $w_i(\epsilon', \epsilon)$ has a sharp maximum with respect to the argument ϵ in the energy region $\epsilon' \gg \epsilon$. Taking this into account, we can neglect the first term of (11) when $\epsilon \gg \epsilon_i$, introduce the variable $\Delta\epsilon = \epsilon' - \epsilon - \epsilon_i$ in the second integral, and expand the integrand function of ϵ' in powers of $\epsilon_i + \Delta\epsilon$, retaining the first two terms of the expansion. Then, recognizing that

$$\int_0^{\infty} w_i(\varepsilon, \Delta\varepsilon)\, d(\Delta\varepsilon) = \nu_i(\varepsilon),$$

we obtain for $\epsilon \gg \epsilon_i$

$$S_i(f_0) = \frac{1}{\varepsilon^{1/2}} \frac{d}{d\varepsilon} \left[\varepsilon^{1/2} \int_0^{\infty} w_i(\varepsilon, \Delta\varepsilon)\, (\varepsilon_i + \Delta\varepsilon)\, d\Delta\varepsilon f_0 \right] \tag{12}$$

3. SOLUTION OF KINETIC EQUATION

We make a simplification that plays an important role in what follows. The collision operators S_e and S_a defined in (7) and (8) have the form of divergences of the flux in energy space. The part of the flux proportional to f_0 is always directed toward lower energies and determines the deceleration of the fast electrons by the dynamic friction force. The part of the flux proportional to $df_0/d\epsilon$ is due to diffusion of the electrons in energy space, and its direction depends on the sign of the derivative. The relation between the diffusion and the dynamic friction depends on the form of f_0. In the case of a Maxwellian distribution, these two terms in the flux are equal and opposite.

In a problem of our type, when there is a source of fast electrons concentrated in the energy region $\epsilon \approx \epsilon_0 \gg T_e \geq T$, it is natural to expect the principal process governing the distribution in the region $\epsilon < \epsilon_0$ to be deceleration by dynamic friction, and the resultant distribution varies little over energy intervals of the order of T_e or T.

In the region $\epsilon > \epsilon_0$, where the source density $R(\epsilon, x)$ becomes small or exactly equal to zero, one cannot neglect the diffusion in energy space, since electrons are present in this energy region only because of diffusion.

The depth of penetration of the electrons in the energy region where $R(\epsilon, x) = 0$ is obviously of the order of T_e or T, and therefore the number of electrons in this region turns out to be small at $\epsilon_0 \gg T_e$ and $\epsilon_0 \gg T$. If this small fraction of the electrons is of no special interest, then the distortion of the form of the distribution in this region is immaterial. Taking the foregoing into account, we assume that the following conditions are satisfied

$$T_e\frac{df_0}{d\varepsilon} \ll f_0,\ T\frac{df_0}{d\varepsilon} \ll f_0, \tag{13}$$

and neglect subsequently the diffusion of the electrons in energy space, assuming that

$$S_e(f_0) = \frac{1}{\varepsilon^{1/2}}\frac{d}{d\varepsilon}[\nu_e(\varepsilon)\varepsilon^{3/2}f_0],\ S_a(f_0) = \frac{1}{\varepsilon^{1/2}}\frac{d}{d\varepsilon}[\delta\nu_a(\varepsilon)\varepsilon^{3/2}f_0]. \tag{14}$$

This assumption will be justified by the results.

We assume also that the density $R(\epsilon, x)$ of the sources is either exactly equal to zero at $\epsilon > \epsilon_0$, or decreases rapidly with increasing energy at $\epsilon > \epsilon_0$ in the interval $\epsilon - \epsilon_0 \ll \epsilon_0$. The form of the solution of the kinetic equation depends on the relation between the characteristic energy ϵ_0, the minimum excitation energy ϵ_1 of the atom or of the optical phonon, and the ionization energy ϵ_i. We consider in succession several possible cases:

1. The case $T_e \ll \epsilon_0 < \epsilon_1$, $T_e \ll \epsilon_0 < \epsilon_i$. It suffices to take into account in the collision operator only the interelectron interaction and the processes of interaction with acoustic phonons or elastic collisions with atoms. Using for the collision operators the expressions in (14), we can rewrite the kinetic equation in the form

$$D(\varepsilon)\frac{\partial^2 f_0}{\partial x^2} + \left(\frac{2}{m\varepsilon}\right)^{1/2}\frac{d}{d\varepsilon}[\varepsilon Q f_0] - \frac{f_0}{\tau(\varepsilon)} = -R(\varepsilon, x), \tag{15}$$

$$Q = \left(\frac{m\varepsilon}{2}\right)^{1/2}[\nu_e(\varepsilon) + \delta\nu_a(\varepsilon)].$$

The function Q has the meaning of the average energy lost by the electron per unit length, and is usually called the "decelerating ability of the medium."

We introduce a new function $\psi(\epsilon)$ and a new variable $t(\epsilon)$ defined by

$$\psi = \varepsilon Q \exp\left[\int_{\varepsilon}^{\varepsilon_0}\left(\frac{m}{2\varepsilon}\right)^{1/2}\frac{d\varepsilon}{Q\tau}\right]f_0, \tag{16}$$

$$t = \int_{\varepsilon}^{\varepsilon_0}\frac{D(\varepsilon)}{Q(\varepsilon)}\left(\frac{m}{2\varepsilon}\right)^{1/2}d\varepsilon.$$

Then Eq. (13) reduces to the heat-conduction equation with a source

$$\frac{\partial\psi}{\partial t} = \frac{\partial^2\psi}{\partial x^2} + F(x, t), \tag{17}$$

$$F(x, t) = \frac{R(x, t)\varepsilon Q}{D}\exp\left[\int_{\varepsilon}\left(\frac{m}{2\varepsilon}\right)^{1/2}\frac{d\varepsilon}{Q\tau}\right] \tag{18}$$

with conditions

$$\psi \to 0 \text{ as } x \to \infty;\quad \partial\psi/\partial x|_{x=0} = 0,\quad \psi \to 0 \text{ as } t \to -\infty.$$

Using the Green's function of the heat-conduction equation[16], we can write the required solution of (17) in the form

$$\psi = \frac{1}{2\sqrt{\pi}}\int_{-\infty}^{t}dt'\int_{-\infty}^{\infty}dx'\frac{F(|x'|, t')}{(t-t')^{1/2}}\exp\left[-\frac{(x-x')^2}{4(t-t')}\right]. \tag{19}$$

Formulas (16) and (19) yield the general solution of the problem at $\epsilon < \epsilon_1$ and $\epsilon_0 < \epsilon_i$.

2. The case $T_e \ll \epsilon_1 < \epsilon_0 \leq 2\epsilon_1$, $T_e \ll \epsilon_0 < \epsilon_i$. In this case it is necessary to take into account processes such as excitation of atoms or emission of optical phonons. We assume for simplicity that it suffices to consider one energy level or one type of phonon. Then the kinetic equation takes the form

$$D(\varepsilon)\frac{\partial^2 f_0}{\partial x^2} + \left(\frac{2}{m\varepsilon}\right)^{1/2}\frac{d}{d\varepsilon}[\varepsilon Q f_0] - \frac{f_0}{\tau(\varepsilon)} + R(\varepsilon, x)$$
$$= \begin{cases} \nu^*(\varepsilon)f_0(\varepsilon) & \text{if } \varepsilon \geqslant \varepsilon_1 \\ -(\varepsilon+\varepsilon_1)^{1/2}\nu^*(\varepsilon+\varepsilon_1)f_0(\varepsilon+\varepsilon_1)/\varepsilon^{1/2} & \text{if } \varepsilon < \varepsilon_1, \end{cases} \tag{20}$$

where $Q(\epsilon)$ is given by formula (15).

The solution in the region $\epsilon_1 \leq \epsilon \leq 2\epsilon_1$ can be obtained by using (16), (18), and (19) and replacing in them $1/\tau$ by $1/\tau + \nu^*$. We then get

$$f_0(\varepsilon) = \frac{1}{2\sqrt{\pi}\,\varepsilon Q}\exp\left[-\int_{\varepsilon}^{\varepsilon_0}\left(\frac{m}{2\varepsilon}\right)^{1/2}\left(\frac{1}{\tau} + \nu^*\right)\frac{d\varepsilon}{Q}\right]$$
$$\times\int_{-\infty}^{t}dt'\int_{-\infty}^{\infty}dx'\frac{F(|x'|, t')}{(t-t')^{1/2}}\exp\left[-\frac{(x-x')^2}{4(t-t')}\right]. \tag{21}$$

In the region $\epsilon \leq \epsilon_1$, the right-hand side of (20) is a known function if (21) is taken into account. We introduce again the variable t in accordance with (16) and make the substitution

$$\psi = \varepsilon Q \exp\left[\int_{\varepsilon}^{\varepsilon_0}\left(\frac{m}{2\varepsilon}\right)^{1/2}\left(\frac{1}{\tau} + \nu^*\right)\frac{d\varepsilon}{Q}\right]f_0. \tag{22}$$

Recognizing that $\nu^* = 0$ when $\epsilon < \epsilon_1$, we obtain for the function ψ the same equation (17) with the function

$$\tilde{F}=\left[R(x,\varepsilon)+\left(\frac{\varepsilon+\varepsilon_1}{\varepsilon}\right)^{1/2}\nu^*(\varepsilon+\varepsilon_1)f_0(\varepsilon+\varepsilon_1)\right]\frac{\varepsilon Q}{D}$$

$$\times\exp\left[\int_{\varepsilon}^{\varepsilon_0}\left(\frac{m}{2\varepsilon}\right)^{1/2}\left(\frac{1}{\tau}+\nu^*\right)\frac{d\varepsilon}{Q}\right] \tag{23}$$

in place of F.

A solution of (23) can be obtained with the aid of (19). Recognizing that $f_0(\epsilon) = 0$ at $\epsilon \geq 2\epsilon_1$, we obtain for the kinetic equation (18) at $\epsilon \leq \epsilon_1$ a solution that goes over into (21) at the point $\epsilon = \epsilon_1$ and has the same form (21) with $\tilde{F}$ from (23) in place of F from (18).

3. The case $\epsilon_0 \gg \epsilon_1 \gg T_e$, $\epsilon_0 < \epsilon_i$ or $\epsilon_0 \gg \epsilon_1 \gg T_e$, $\epsilon_0 \gg \epsilon_i \gg T_e$. We obtain the form of the distribution for high energies $\epsilon \gg \epsilon_1$ and $\epsilon \gg \epsilon_i$. In this case we can use the approximate expressions (10) and (12) for the operators of the inelastic processes (9) and (11). For electrons of energy $\epsilon \gg \epsilon_1$ we can almost always neglect recombination processes and processes of the type of elastic scattering by atoms or scattering by acoustic phonons. The kinetic equation then coincides with (15), from which we can leave out the term f_0/τ, and Q is given by

$$Q=\left(\frac{m\varepsilon}{2}\right)^{1/2}\left[\nu_e(\varepsilon)+\sum_n\varepsilon_n\nu_n^*(\varepsilon)\right] \tag{24}$$

at $\epsilon_0 < \epsilon_i$ and

$$Q=\left(\frac{m\varepsilon}{2}\right)^{1/2}\left[\nu_e(\varepsilon)+\sum_n\varepsilon_n\nu_n^*+\int_0^\infty w_i(\varepsilon,\Delta\varepsilon)(\varepsilon_i+\Delta\varepsilon)\,d\Delta\varepsilon\right] \tag{25}$$

at $\epsilon_0 \gg \epsilon_i$.

A solution of the equation can be obtained from (16), (18) and (19) by putting in these formulas $1/\tau = 0$ and substituting (24) or (25) for $Q(\epsilon)$.

4. EXAMPLES AND DISCUSSION OF RESULTS

By way of examples of the application of the general formulas, let us find the form of the electron distribution for two particular cases:

1. Distribution in the near-cathode region of the discharge at $\epsilon \gg \epsilon_i > \epsilon_1$, $\epsilon_0 = eU_c \gg \epsilon_i$. In this case we can put $r(\epsilon) = \delta(\epsilon - \epsilon_0)$ in (6). We then easily obtain from (6), (16), (18), and (19)

$$f_0(\varepsilon,x)=\frac{m^2j_0}{8\pi^{3/2}\varepsilon Qt^{1/2}}\exp\left(-\frac{x^2}{4t}\right)\quad\text{if}\quad\varepsilon\leqslant\varepsilon_0 \tag{26}$$

$f_0(\epsilon, x) = 0$ at $\epsilon > \epsilon_0$. It follows from (26) that the number of electrons with energy ϵ decreases practically to zero at a distance x on the order of several times $\sqrt{t(\epsilon)}$. In order of magnitude we have

$$t(\varepsilon)\cong\left(\frac{m\varepsilon_0}{2}\right)^{1/2}\frac{D(\varepsilon_0)}{Q(\varepsilon_0)}\left(1-\frac{\varepsilon}{\varepsilon_0}\right). \tag{27}$$

It follows from (27) that we can use for the energy relaxation length the quantity

$$\Lambda=\left[\left(\frac{m\varepsilon_0}{2}\right)^{1/2}\frac{D(\varepsilon_0)}{Q(\varepsilon_0)}\right]^{1/2}. \tag{28}$$

2. Distribution of photoelectrons in a semiconductor illuminated with monochromatic light when $T_e \ll \epsilon_0 < \epsilon_1$. In this case the source density is

$$R(x,\varepsilon)=ke^{-kx}\frac{I_0}{g(\varepsilon_0)}\delta(\varepsilon-\varepsilon_0), \tag{29}$$

where I_0 is the photon flux density and k is the absorption coefficient. From (29), (16), and (19) we obtain

$$f_0(\varepsilon,x)=\frac{m^2kI_0}{16\pi^{3/2}\varepsilon Qt^{1/2}}\exp\left[-\int_{\varepsilon}^{\varepsilon_0}\left(\frac{m}{2\varepsilon}\right)^{1/2}\frac{d\varepsilon}{Q\tau}\right]$$

$$\times\int_{-\infty}^{\infty}\exp\left[-k|x'|-\frac{(x-x')^2}{4t}\right]dx'\quad\text{if}\quad\varepsilon\leqslant\varepsilon_0, \tag{30}$$

$f_0(\epsilon, x) = 0$ at $\epsilon > \epsilon_0$.

Let us examine the form of the distribution at $\epsilon < \epsilon_0$ in different limiting cases.

A. $[\nu_e(\epsilon_0) + \delta\nu_a(\epsilon_0)]\tau(\epsilon_0) \gtrsim 1$.
In this case

$$\int_0^{\varepsilon_0}\left(\frac{m}{2\varepsilon}\right)^{1/2}\frac{d\varepsilon}{Q\tau}\lesssim 1,$$

and the characteristic value of $t(\epsilon)$ can be taken to be equal to Λ. From (30) with

$$k\Lambda\equiv k\left[\frac{D(\varepsilon_0)}{\nu_e(\varepsilon_0)+\delta\nu_a(\varepsilon_0)}\right]^{1/2}\gg 1$$

we then have

$$f_0(\varepsilon,x)\cong\frac{m^2I_0}{8\pi^{3/2}\varepsilon Qt^{1/2}}\exp\left[-\frac{x^2}{4t}-\int_{\varepsilon}^{\varepsilon_0}\left(\frac{m}{2\varepsilon}\right)^{1/2}\frac{d\varepsilon}{Q\tau}\right], \tag{31}$$

and for $k\Lambda \ll 1$ we get

$$f_0(\varepsilon,x)=\frac{m^2kI_0}{8\pi\varepsilon Q}\exp\left[-kx-\int_{\varepsilon}^{\varepsilon_0}\left(\frac{m}{2\varepsilon}\right)^{1/2}\frac{d\varepsilon}{Q\tau}\right]. \tag{32}$$

B. $[\nu_e(\epsilon_0) + \delta\nu_a(\epsilon_0)]\tau(\epsilon_0) \ll 1$.
In this case the bulk of the fast electrons is concentrated in an energy region where

$$\int_{\varepsilon}^{\varepsilon_0}\left(\frac{m}{2\varepsilon}\right)^{1/2}\frac{d\varepsilon}{Q\tau}\approx\left(\frac{m}{2\varepsilon_0}\right)^{1/2}\frac{(\varepsilon_0-\varepsilon)}{Q(\varepsilon_0)\tau(\varepsilon_0)}\lesssim 1.$$

Using (27), we obtain from this the characteristic value

$$t\cong D(\varepsilon_0)\tau(\varepsilon_0);\quad\text{if}\quad k[D(\varepsilon_0)\tau(\varepsilon_0)]^{1/2}\gg 1\ \text{we have}$$

$$f_0(\varepsilon,x)=\frac{m^2I_0}{8\pi^{3/2}\varepsilon Qt^{1/2}}\exp\left[-\int_{\varepsilon}^{\varepsilon_0}\left(\frac{m}{2\varepsilon}\right)^{1/2}\frac{d\varepsilon}{Q\tau}-\frac{x^2}{4t}\right]. \tag{33}$$

If inequality B is satisfied, then the function (33) has a sharp peak at $\epsilon \approx \epsilon_0$ and almost all the electrons have an energy close to ϵ_0. To find how the number of fast electrons varies in coordinate space, we integrate the function (33) with respect to energy, taking into account the presence of the sharp peak at $\epsilon \approx \epsilon_0$. The integration yields

$$n_c(\varepsilon_0)\equiv\int_0^{\varepsilon_0}f_0(\varepsilon)g(\varepsilon)\,d\varepsilon=\left[\frac{\tau(\varepsilon_0)}{D(\varepsilon_0)}\right]^{1/2}I_0\exp\left[-\frac{x}{(D(\varepsilon_0)\tau(\varepsilon_0))^{1/2}}\right]. \tag{34}$$

This result is obtained also directly from the kinetic equation, if we neglect the collisions.

At $k[D(\epsilon_0)\tau(\epsilon_0)]^{1/2} \ll 1$ we get from (30)

$$f_0(\varepsilon,x)=\frac{m^2kI_0}{8\pi\varepsilon Q}\exp\left\{-kx-\int_{\varepsilon}^{\varepsilon_0}\left(\frac{m}{2\varepsilon}\right)^{1/2}\frac{d\varepsilon}{Q\tau}\right\}. \tag{35}$$

In the case of weak interelectron interaction $\nu_e \ll \delta\nu_a$, our formulas (32) and (35) coincide with the result of Ladyzhenskiĭ[6]. In the particular case considered by Abakumov and Yassievich[3], our formula (31) and their formulas coincide in the region of applicability of our theory if

$$\frac{\varepsilon_0T_e}{(\varepsilon_0-\varepsilon)^2}\frac{x^2}{\Lambda^2}\ll 1.$$

Let us see how the true form of the distribution is distorted by the approximation made in the theory. It follows from our results that $f_0(\epsilon, x) = 0$ at $\epsilon > \epsilon_0$ if $R(\epsilon, x) = 0$ at $\epsilon > \epsilon_0$. The reason is that by neglecting diffusion in energy space we disregard the possible acceleration of the electrons. In fact, diffusion in energy space causes a small number of electrons to penetrate into the region $\epsilon > \epsilon_0$.

For a correct description of the distribution in this energy region, it is necessary to retain the diffusion terms of the type $T_e\, df_0/d\epsilon$ or $T\, df_0/d$ in the energy flux, and to solve the problem by previously developed methods[7-9].

In general, our results are reliable only in the range of values of ϵ and x that satisfy the conditions (13). For example, for the functions (26), (31), and (32), the conditions (13) reduce to the inequalities

$$\frac{1}{t}\frac{dt}{d\varepsilon}\frac{x^2}{4t} \ll \frac{1}{T}, \frac{1}{T_e}, \qquad (36)$$

$$T,\ T_e \ll Q(2\varepsilon/m)^{1/2}\tau. \qquad (37)$$

It follows from (36) that our results do not describe the distribution correctly at energies too close to ϵ_0 or when $x \gg \Lambda$. We note, however, that if the inequalities (36) are violated, then the distribution function is already quite small.

It should be recognized that the inequalities (36) limit the applicability of the results only if $\nu_e + \delta\nu_a \gtrsim \nu^* + 1/\tau$, when the interaction with the electrons, atoms, or acoustic phonons is appreciable, but if $\nu_e + \delta\nu_a \ll \nu^* + 1/\tau$, then these processes have little effect on the distributions, the inequalities (36) and (37) can be disregarded, and the limits $\nu_e \to 0$ and $\delta\nu_a \to 0$ can be taken in all the formulas.

Another source of distortion is the changeover to a differential form of the operators of the inelastic processes (9) and (11). For example, if the electrons are produced with a distribution of the type $R(\epsilon) \sim \delta(\epsilon - \epsilon_0)$ and relax as a result of excitation of the atoms or emission of optical phonons, then it is quite obvious that the distribution function have peaks at the points $\epsilon_0 - n\epsilon_1$ (n is an integer). But we obtain from (24) a smooth function in this case. Thus, the changeover to differential form smooths out the distribution. However, by comparing the approximate solution with the exact solution of the problem, which can be obtained at a source density R independent of x, we can verify that in an energy interval of length ϵ_1 the number of electrons at $\epsilon \gg \epsilon_1$ is close in both cases.

The energy spectrum of the electrons was assumed in this paper to be quadratic and isotropic, but all the results can be easily generalized to include the case of an arbitrary isotropic spectrum.

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