# *KADOMTSEV-NEDOSPASOV HELICAL INSTABILITY IN A STRONG PINCH EFFECT IN AN ELECTRON-HOLE PLASMA*

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The excitation criterion, oscillation frequency and helical instability increment for a strong pinch effect in semiconductors are calculated. Instability arises when the longitudinal magnetic field exceeds the maximal magnetic field of the current.

**1.** Investigations of helical instability and of the pinch effect in an electron-hole plasma of semiconductors started practically simultaneously. In 1958, Ivanov and Ryvkin[1] observed current instability in Ge samples placed in a sufficiently strong longitudinal magnetic field. As shown by Glicksman[2], this instability is due to excitation of diffusion helical waves, the theory of which as applied to the stability of the positive column of a gas discharge was developed by Kadomtsev and Nedospasov[3]. The instability sets in when the drift current of the electrons and holes in the longitudinal magnetic field and in electric fields transverse to it is directed towards the surface of the sample and exceeds the diffusion current.

In 1959 Glicksman and Steel[4] first investigated the pinch effect in the electron-hole plasma of InSb. In this phenomenon, which is well known in gas-discharge plasmas[5], the flow of a strong current causes the plasma to contract towards the axis of the sample as a result of the drift of the electrons and the holes in the electric field and in the current's own magnetic field. If a sufficiently strong longitudinal magnetic field is applied to samples in which a strong pinch effect is produced, then all the symptons of strong contraction disappear[6]. The disintegration of the pinch could not be explained within the framework of the model of the longitudinal magnetic field frozen into the plasma[7], since the Maxwellian diffusion time of the magnetic field is much shorter than the characteristic time of the process.

A hypothesis was therefore advanced[6] that the disintegration of the pinch is due to the development of a helical instability that leads to an anomalously large flux of particles to the surface of the sample. This hypothesis was confirmed by experiments[8]; it turned out then that the helical instability arises under conditions when the longitudinal magnetic field exceeds the current's own magnetic field on the surface of the sample. It is therefore obvious that when a theory is constructed for the helical instability under conditions of a strong pinch effect, it is necessary to take into account the current's own magnetic field. In the present paper such a theory is constructed for cylindrical and planar sample geometries and for a non-degenerate electron-hole plasma.

**2.** We present certain data on the stationary state of a plasma under the conditions of the pinch effect, which we shall need in the analysis of the stability. It is assumed that in the absence of the pinch effect the quasineutral electron-hole plasma fills uniformly the cross section of the sample, with a density $n_0$. In the case when the diffusion length of the carriers exceeds the transverse dimension of the sample in the contraction direction, and the rate of surface recombination is low, the equation describing the spatial distribution of the plasma density in the pinch effect takes the following form in the case of cylindrical geometry

$$\frac{dq}{d\rho}=-\frac{\alpha q}{\rho}\int_0^\rho q(\rho')\,\rho'\,d\rho', \tag{2.1}$$

where

$$q=\frac{n(\rho=0)}{n_0},\quad \rho=\frac{r}{R},\quad \alpha=\frac{2\pi e^2 b_e^2 E_z^2 n_0 R^2}{c^2 T},$$

R is the sample radius, $E_z$ is the longitudinal electric field, $b_c$ is the electron mobility, T is the carrier temperature in energy units; it was assumed that the electron and hole temperatures are equal, and $b_e \gg b_h$ (case of InSb). This equation was solved by Bennett back in 1934[5]:

$$q=\frac{q_0}{(1+\rho^2/\rho_0^2)^2}, \tag{2.2}$$

$$q_0\rho_0^2=8/\alpha. \tag{2.3}$$

An additional relation between the parameters $q_0$ and $\rho_0$ can be obtained with the aid of the boundary condition, which in the case of a sufficiently pure sample surface corresponds to the balance of the volume recombination and generation processes. Thus, in the case of linear volume recombination[9], when the number of particles is conserved upon contraction, we have

$$q_0=8/(8-\alpha),\quad \rho_0^2=(8-\alpha)/\alpha, \tag{2.4}$$

from which we see that strong contraction sets in as $\alpha\to\infty$ (the Bennett criterion). The contraction has in this case the character of a collapse: $q_0\to\infty$ and $\rho_0\to 0$ when $\alpha\to\infty$.

Naturally, in the region of the collapse it is necessary to take into account the effects of quadratic volume recombination[10] or of the strong carrier degeneracy[11]. These effects eliminate the collapse. For example, in the case of quadratic recombination[10] and strong contraction ($\alpha \gg 1$) we have

$$q_0=3\alpha/8,\quad \rho_0^2=64/3\alpha^2. \tag{2.5}$$

The total number of particles decreases with increasing electric field ($\alpha$) and the current-voltage characteristics assume an N-shaped form.

With the aid of (2.2) we can easily obtain the current's own magnetic field:

$$H_\varphi(\rho)=\frac{\beta\rho}{1+\rho^2/\rho_0^2},\quad \beta=\frac{2\pi}{c}eb_en_0q_0RE_z. \tag{2.6}$$

For stronger contraction ($\rho_0^2 \ll 1$), the maximum value of the current's own magnetic field is reached inside the sample at the point $\rho_0$; then

$$H_\varphi^{max}/H_\varphi^0=1/2\rho_0, \tag{2.7}$$

where $H_\varphi^0 = \beta\rho_0^2$ is the current's own magnetic field on the surface of the sample ($\rho = 1$).

Taking (2.3) into account, it is easy to show that in the case of a strong pinch effect

$$\bar{E}_z=\frac{8}{\bar{H}_\varphi^0},\quad \bar{E}_z=\frac{eRE_z}{T},\quad \bar{H}_\varphi^0=\frac{b_e}{c}H_\varphi^0. \tag{2.8}$$

in the case of planar geometry, the corresponding spatial distribution for the plasma density was obtained by Polovin and Tsintsadze[12]:

$$q=\frac{q_0}{\operatorname{ch}^2(\xi/\xi_0)}; \tag{2.9}$$

In this case

$$q_0\xi_0^2=2/\alpha. \tag{2.10}$$

Here $\xi = x/a$, where 2a is the thickness of the plate in the direction of the X axis; it was assumed that the sample dimensions in the Y and Z directions are much larger. The current's own magnetic field is directed along the Y axis; the compression occurs in the direction of the X axis, towards the center of the sample.

In the case of linear volume recombination, when $\alpha \gg 1$, we have

$$q_0=\alpha/2,\quad \xi_0=2/\alpha, \tag{2.11}$$

and for quadratic recombination[10]

$$q_0=3^{2/3}\cdot 2^{-1}\alpha^{1/3},\quad \xi_0=2\cdot 3^{-1/3}\alpha^{-2/3}. \tag{2.12}$$

As seen from (2.11), no collapse occurs in the case of planar geometry.

For strong contraction, the currents own magnetic field is given by

$$H_y=H_y^0\operatorname{th}\frac{\xi}{\xi_0},\quad H_y^0=\frac{4\pi}{c}eb_en_0q_0a\xi_0E_z, \tag{2.13}$$

where $H_y^0$ is the magnetic field on the surface of the sample. In the case of planar geometry, the maximum value of the current's own field is reached on the sample surface.

Taking (2.10) into account, we can show that in strong contraction

$$\bar{E}_z=4/\xi_0\bar{H}_y^0. \tag{2.14}$$

In the subsequent investigations of the stability we shall use the distributions (2.2), (2.6), (2.9), and (2.13) and the relations (2.3), (2.8), (2.10), and (2.14).

**3.** In the analysis of the stability of the pinch in a longitudinal magnetic field, we start from the equations of motion and continuity for the electrons and holes, which are linearized with respect to small quasineutral perturbations, and Maxwell's equations, assuming that $b_e \gg b_h$ and that the magnetization of the holes can be neglected. These equations are

$$\boldsymbol{\Gamma}_e=-b_en\mathbf{E}-D_e\nabla n-\frac{b_e}{c}[\boldsymbol{\Gamma}_e\mathbf{H}],\quad \boldsymbol{\Gamma}_h=b_hn\mathbf{E}-D_h\nabla n, \tag{3.1*}$$

$$\frac{\partial n}{\partial t}+\operatorname{div}\boldsymbol{\Gamma}_e=0,\quad \frac{\partial n}{\partial t}+\operatorname{div}\boldsymbol{\Gamma}_h=0, \tag{3.2}$$

$$\operatorname{rot}\mathbf{H}=-\frac{4\pi}{c}e(\boldsymbol{\Gamma}_e-\boldsymbol{\Gamma}_h), \tag{3.3'}$$

$$\operatorname{div}\mathbf{H}=0, \tag{3.3}$$

where $\Gamma_i$ are the particle fluxes and $D_i$ are the diffusion coefficients (i = e, h).

Since the oscillation frequency is not very high, we do not take into account the displacement current, and assume the electric field of the wave to be potential: $\mathbf{E}' = -\nabla\varphi$ (the primes denote perturbed quantities).

Unlike the usual theory of helical instability[2,3], in the case of a strong pinched effect it is necessary to take into account the perturbed magnetic field produced by the particle fluxes.

We present below calculations for a sample in the form of a cylinder. It is assumed that all the perturbed quantities are given by

$$A'=A_1(r)\exp(im\varphi+ikz-i\omega t),$$

where $m^2 = 1$ (helical mode). The external longitudinal magnetic field is directed along the Z axis and is assumed to be homogeneous, in view of the short diffusion time of the magnetic field. The calculations are performed in the approximation

$$\bar{H}_z^2=\left(\frac{b_e}{c}H_z\right)^2\ll 1,\quad \bar{H}_\varphi^2=\left(\frac{b_e}{c}H_\varphi\right)^2\ll 1, \tag{3.4}$$

so that the terms proportional to $\bar{H}_z^2$ and $\bar{H}_\varphi^2$ are neglected throughout in comparison with unity. We note that the strong-contraction criterion ($\alpha \gg 1$) does not contradict this approximation in a sufficiently large interval of the values of $H_\varphi$.

From the equations of motion we easily obtain expressions for the perturbed fluxes of the electrons

$$\begin{aligned}\Gamma_{er}'&=b_en\frac{d\varphi'}{dr}-D_en\frac{d}{dr}\frac{n'}{n}-\frac{im}{r}\bar{H}_z(b_en\varphi'-D_en')\\&\quad+ik\bar{H}_\varphi(b_en\varphi'-D_en')-\frac{b_e^2}{c}E_znH_\varphi',\\ \Gamma_{e\varphi}'&=\frac{im}{r}(b_en\varphi'-D_en')+\bar{H}_z\left(b_en\frac{d\varphi'}{dr}-D_en\frac{d}{dr}\frac{n'}{n}\right)+\frac{b_e^2}{c}E_znH_r',\\ \Gamma_{ez}'&=-b_eE_zn'+ik(b_en\varphi'-D_en')-\bar{H}_\varphi\left(b_en\frac{d\varphi'}{dr}-D_en\frac{d}{dr}\frac{n'}{n}\right),\end{aligned} \tag{3.5}$$

and of the holes

$$\begin{aligned}\Gamma_{hr}'&=-b_hn\frac{d\varphi'}{dr}-D_en\frac{d}{dr}\frac{n'}{n},\quad \Gamma_{h\varphi}'=-\frac{im}{r}(b_en\varphi'+D_hn'),\\ \Gamma_{hz}'&=b_hE_zn'-ik(b_hn\varphi'+D_hn').\end{aligned} \tag{3.6}$$

Substituting (3.5) and (3.6) in (3.2) we obtain a system of two equations for the perturbations of the density and of the potential:

$$\hat{L}\bar{\varphi}_1-\frac{i}{\rho q}\left(m\bar{H}_z\frac{dq}{d\rho}-\bar{k}\frac{d\rho q\bar{H}_\varphi}{d\rho}\right)\bar{\varphi}_1-\hat{L}\bar{n}_1$$

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*$[\Gamma_e\mathbf{H}] \equiv \Gamma_e \times \mathbf{H}$.

$$+i\left(\frac{m\overline{H}_z}{\rho q}\frac{dq}{d\rho}-\frac{\tilde{k}}{\rho q}\frac{d\rho q\overline{H}_\varphi}{d\rho}-\tilde{k}\overline{E}_z-\omega_e\right)\tilde{n}_1$$

$$+\frac{4\pi e b_e E_z R^2 q}{c^2}\Gamma_{ez}'-\frac{b_\iota R E_z}{c}\frac{dq}{d\rho}H_\varphi'=0, \qquad (3.7)$$

$$\hat{L}\tilde{\varphi}_1+\hat{L}\tilde{n}_1-i(\tilde{k}\overline{E}_z-\omega_h)\tilde{n}_1=0, \qquad (3.8)$$

where

$$\hat{L}=\frac{1}{\rho}\frac{d}{d\rho}\left(\rho q\frac{d}{d\rho}\frac{1}{q}\right)-\frac{1}{\rho^2}-\tilde{k}^2, \quad \tilde{\varphi}_1=q\varphi_1, \quad \tilde{n}_1=\frac{T}{e}q_1,$$

$$q_1=n_1/n_0, \quad \tilde{k}=kR, \quad \omega_i=\omega R^2/D_i \quad (i=e,\ h).$$

From Maxwell's equations (3.3) we obtain an expression for $H'_\varphi$. We note that two out of the three equations (3.3′) with respect to $H'_r$, $H'_\varphi$, and $H'_z$ are linearly independent. We can find

$$H_\varphi'=-\frac{4\pi e}{cr}\int_0^r\Gamma_{ez}'r\,dr+\frac{im}{r}\int_0^r H_r'\,dr, \qquad (3.10)$$

where the function

$$Y=\int_0^r H_r'\,dr$$

is defined by the equation

$$\frac{d^2Y}{dr^2}+\frac{1}{r}\frac{dY}{dr}-\left(\frac{1}{r^2}+k^2\right)Y=i\frac{4\pi em}{cr^2}\int_0^r\Gamma_{ez}'r\,dr-i\frac{4\pi ek}{c}\int_0^r\Gamma_{e\varphi}'\,dr=S, \qquad (3.11)$$

which has a solution that vanishes when there are no perturbed fluxes, namely

$$Y=I_1(kr)\int_0^r\frac{d\mu}{\mu I_1^2(k\mu)}\int_0^\mu\lambda I_1(k\lambda)S(\lambda)\,d\lambda, \qquad (3.12)$$

where $I_1$ is a Bessel function of imaginary argument.

Equations (3.7) and (3.8) must be supplemented by boundary conditions corresponding to the vanishing of the perturbed radial fluxes of the electrons and holes on the surface of the sample (the rate of surface recombination is assumed to be small). One could find the dispersion relation while solving the system (3.7) and (3.8), using the boundary conditions. But even in the simplest cases, when the pinch effect is weakly pronounced, such a rigorous procedure of obtaining the dispersion relation for a volume helical wave (nonzero density gradient) encounters serious difficulties when attempts are made to obtain solutions of equations such as (3.7) and (3.8).

The Galerkin variational method[13] was successfully applied to problems of this type in a number of papers[2,3]. In this case the spatial distribution of the functions $\tilde{n}_1$ and $\tilde{\varphi}_1$ are specified in the form of certain approximate profiles, which should satisfy definite conditions that follow from the symmetry of the initial equations. We were unable to find exact solutions of Eqs. (3.7) and (3.8), and likewise used the profiling method to obtain the dispersion relation.

Recognizing that the functions $\tilde{n}_1$ and $\tilde{\varphi}_1$ enter in the initial operators in symmetrical fashion, we choose the same form for the profiles of $\tilde{n}_1$ and $\tilde{\varphi}_1$[14]. The corresponding profile for the mode $|m| = 1$ near the axis tends to zero like $\sim\rho$[2,3]. Since the carrier density falls off rapidly away from the sample surface under the conditions of the strong pinch effect, we choose the profile of $\tilde{n}_1(\tilde{\varphi}_1)$ such that in the region of vanishingly small density the perturbations are also small. In the choice of the profile we must take into account the structure of the stationary solution[2,3]. After choosing the profile $n_1(\tilde{\varphi}_1) = \hat{n}_1(\hat{\varphi}_1)f(\rho)$, ($\hat{n}_1$ and $\hat{\varphi}_1$ are constants), we multiply (3.7) and (3.8) by $f(\rho)$ and integrate them over the cross section of the sample. Thus, the system of integro-differential equations (3.7) and (3.8) reduces to a system of algebraic equations with respect to $\hat{n}_1$ and $\hat{\varphi}_1$. Equating the determinant of this instant to zero, we obtain the dispersion relation.

We present here the results of an investigation for the profile

$$f(\rho)=\frac{\rho}{(1+\rho^2/\rho_0^2)^2}. \qquad (3.13)$$

With $f(\rho)$ in this form, we have obtained the smallest excitation threshold, which in our opinion agrees well with experiment.

The dispersion relation under the conditions of strong contraction ($\rho_0^2 \ll 1$) for the profile (3.13), with the relations (2.2), (2.3), (2.6), (2.8), and (3.4) taken into account, is

$$\omega_h=-\frac{2}{\rho_0^2}\left(m\overline{H}_z+\frac{1}{8}\tilde{k}\overline{H}_\varphi{}^0\right)-\frac{4i}{\rho_0^2(1+\tilde{k}^2\rho_0^2/2)}\left(1+\frac{5}{4}\tilde{k}^2\rho_0^2+\frac{1}{4}\tilde{k}^4\rho_0^4+2m\tilde{k}\rho_0^2\frac{H_z}{H_\varphi{}^0}\right). \qquad (3.14)$$

As follows from (3.14), the wave with $m = -1$ is unstable when $\tilde{k} > 0$. No helical wave is excited in the absence of the longitudinal magnetic field.

Recognizing that $d\,\mathrm{Im}\,\omega/d\tilde{k} = 0$ at the excitation threshold[3], we can easily obtain the minimal criterion for the excitation of the mode $m = -1$:

$$H_z > H_\varphi{}^0/\rho_0 \quad \text{or} \quad H_z > 2H_\varphi^{max}, \qquad (3.15)$$

the oscillation frequency at the excitation threshold:

$$\mathrm{Re}\,\omega\approx\frac{2}{R^2\rho_0^2}D_h\overline{H}_z, \qquad (3.16)$$

the instability increment

$$\mathrm{Im}\,\omega\approx 4\frac{D_h}{R^2\rho_0}\frac{H_z}{H_\varphi{}^0}, \qquad (3.17)$$

and the value of the wave vector k at which the wave excitation is minimal

$$k\approx 0.8/R\rho_0. \qquad (3.18)$$

Stronger contraction increases the longitudinal length of the helical wave, owing to the increasing role of the transverse diffusion.

It should be noted that all the other profiles $f(\rho)$ chosen by us lead qualitatively to the same relations for the parameters of the helical wave (only the numerical coefficients differ).

4. An analogous calculation was carried out for a sample in the form of a plate. The perturbations were chosen in the form

$$A'=A_1(x)\exp(ik_y y+ik_z z-i\omega t).$$

We present the results for the profile

$$f(\xi)=\frac{\mathrm{sh}(\xi/\xi_0)}{\mathrm{ch}^2(\xi/\xi_0)}, \qquad (4.1)$$

which is the analog of (3.13) in the case of planar geometry.

When the longitudinal magnetic field is sufficiently strong, waves with $k_y$, $k_z < 0$ are excited (helical wave). The instability criterion is

$$H_z > 2H_y{}^0, \qquad (4.2)$$

The oscillation frequency is

$$\operatorname{Re}\omega \approx \frac{2D_h}{a^2\xi_0^2}\overline{H}_z, \tag{4.3}$$

The increment is

$$\operatorname{Im}\omega \approx \frac{D_h}{a^2\xi_0^2}\frac{H_z}{H_y^0}, \tag{4.4}$$

and the wave vectors at which the excitation threshold is minimal are

$$k_z \approx 0.8/\xi_0 a, \quad k_y \approx 0.5/\xi_0 a. \tag{4.5}$$

The relations (3.15)—(3.18) and (4.2)—(4.5) can be made more specific with the aid of expressions (2.5), (2.11), and (2.12).

It is easy to show that in the case of a strong pinch effect the longitudinal magnetic field at which the helical instability is excited increases with increasing electric field, this being due to the increasing role of transverse diffusion with increasing contraction.

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