FERRIMAGNETIC ECHO IN THE SUBSIDIARY ABSORPTION REGION

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The theory of ferrimagnetic echo is considered for the case when the frequency of the second pulse field is equal to the doubled frequency of the first pulse. This echo has previously been observed experimentally. The Heisenberg equations are solved for the spin-wave amplitudes and an expression is found for the echo signal amplitude. The three-magnon process of homogeneous precession decay into spin wave quanta plays an important role in the formation of the response. The possibility of observing amplification of the signal with increase of the delay time is discussed.

COMSTOCK and Raymond[1] reported observation of echo signals in an yttrium iron garnet, when the frequency of the second pulse of the alternating field, equal to double the frequency of the first pulse, coincided with the resonant frequency of the homogeneous precession. The echo was observed at the frequency of the field of the first pulse, i.e., at a frequency corresponding to the region of additional absorption[2]. So far there is no calculation whatever of the indicated phenomenon, with the exception of the qualitative discussion in[1].

As is well known[3], an alternating field applied perpendicular to the constant field ensures a linear connection between the spin waves and the electromagnetic field, and this connection becomes stronger in the presence of inhomogeneity of the constant field. Accordingly, the first pulse of the exciting field, of duration Δt_1, excites linearly in the sample coherent spin waves with wave vectors $\mathbf{k}$ (k-magnons), the frequency ω_k of which is equal to the frequency ω_1 of the pulse field. During the time τ between the exciting pulses, the coherent phase of the spin system becomes dephased because of the inhomogeneity of the local field.

The action of a second pulse of duration Δt_2 and carrier frequency $\omega_2 = 2\omega_1 = \omega_0$ on the sample (ω_0 is the homogeneous-precession frequency) can be described in the following manner. We assume that the power of the pulse field exceeds the threshold for the first pulse excitation zone[2]. In the initial period $\Delta t'$ of the pulse, the magnons of the homogeneous precession (0-magnons) increase linearly to the threshold level or somewhat higher, and after the instability threshold is reached, during the time $\Delta t''$ ($\Delta t' + \Delta t'' = \Delta t_2$) parametric excitation of the k-magnons begins (the so-called first-order process). After the pulse has passed, the macroscopic coherent moment is again dephased in the local field.

The Hamiltonian of the problem is

$$\mathscr{H} = \mathscr{H}_0 + \mathscr{H}_1 + \mathscr{H}_2; \quad \mathscr{H}_0 = \sum_k \hbar\omega_k c_k^+ c_k,$$

$$\mathscr{H}_1 = \hbar \sum_k (\beta_{kj} \exp(-i\omega_j t) c_k^+ + \beta_{kj}^* \exp(i\omega_j t) c_k),$$

$$\mathscr{H}_2 = \frac{\hbar}{2} \sum_{k\neq 0} (g_k c_0 c_k^+ c_{-k}^+ + g_k^* c_0^+ c_k c_{-k}). \quad (1)$$

Here $\mathscr{H}_0$ is the Hamiltonian of the unperturbed system of magnons, $\mathscr{H}_1$ is the operator of magnon interaction with the alternating magnetic field, and $\mathscr{H}_2$ describes the parametric connection between the 0- and k-magnons. The operators c_0^+ (c_0) and c_k^+ (c_k) are respectively the creation (annihilation) operators of the 0- and k-magnons, β_{kj} is the parameter of the coupling between the alternating field and the spin waves, g_k is the parameter of the three-magnon interaction of the homogeneous precession with the spin waves, and $j = 1, 2$.

To calculate the response we solve the Heisenberg equations for the operators $c_0(t)$ and $c_k(t)$:

$$\dot{c}_{0(k)} = (i\hbar)^{-1}[c_{0(k)}, \mathscr{H}]. \quad (2)$$

Immediately before the second pulse, the operator $c_k(\tau)$ is given by

$$c_k(\tau) = (c_k(0) - i\beta_{k1}\Delta t_1)\exp\{-i\omega_k\Delta t_1 - i(\omega_k - i\eta_k)\tau\}. \quad (3)$$

The quantity η_k, introduced as an imaginary increment to the natural frequency of the spin waves, describes the irreversible damping of the k-magnons (this damping is neglected during the time of action of the pulses).

The action of the second pulse is described by the equations

$$\dot{c}_0 = -i\omega_0 c_0 - \frac{i}{2}\sum_{k\neq 0} g_k^* c_k c_{-k} - i\beta_{02}\exp(-i\omega_2 t), \quad (4a)$$

$$\dot{c}_k = -i\omega_k c_k - i g_k c_0 c_{-k}^+, \quad \dot{c}_{-k}^+ = i\omega_k c_{-k}^+ + i g_k^* c_0^+ c_k. \quad (4b)$$

In the initial period of the second pulse (during the time $\Delta t'$) there are excited mainly 0-magnons, so that the linear term of (4a) can be neglected. We then have

$$c_0(\Delta t') = [c_0(\tau) - i\beta_{02}\Delta t']\exp(-i\omega_2\Delta t'). \quad (5)$$

We assume that, after a certain value above threshold is reached, the amplitude of the homogeneous precession remains stationary during the time $\Delta t''$:

$$c_0(t) = c_0(\Delta t')\exp(-i\omega_2 t).$$

Substituting this expression in (4b) we obtain for the parametrically excited spin waves

$$c_k(\Delta t_2) = [c_k(\Delta t')\operatorname{ch}\rho_k\Delta t'' - i(g_k/\rho_k)c_0(\Delta t')c_{-k}^+(\Delta t')\operatorname{sh}\rho_k\Delta t''] \times \exp(-i\omega_k\Delta t''), \quad (6)$$

$$\rho_k = |g_k|[c_0^+(\Delta t')c_0(\Delta t')]^{1/2}. \quad (6a)$$

After the end of the second pulse, at the instant of time t (the time is reckoned from the instant of termination of the first pulse), $c_k(t)$ takes the form

$$c_k(t) = c_k(\Delta t_2)\exp\{-i(\omega_k - i\eta_k)(t-\tau)\}. \tag{7}$$

Using (3), (6), and (7) we obtain for the diagonal part of the operator $c_k(t)$, which describes the amplitude of the coherent echo signal ($\tau \gg \Delta t_{1,2}$)

$$[c_k(t)]_{\text{diag}} = -i\beta_{k1}^*\Delta t_1\beta_{02}\Delta t'(g_k/\rho_k)\,\text{sh}\,\rho_k\Delta t'' \times \exp\{-\eta_k t - i\omega_k(t-2\tau)\} \tag{8}$$

Taking into account the scatter of the local frequency, we represent ω_k in the form

$$\omega_k(r) = \omega_k^0 + \Delta\omega_k(r);$$

it follows then from (8) that the macroscopic coherent echo signal appears at the instant of time $t = 2\tau$. It is also seen from (8) that the amplitude of the echo attenuates exponentially with increasing τ. In principle, the echo signals can become amplified with increasing τ, as in the case of the (ω, ω) sequence of the pulses[4]. If the second pulse is strong enough, then the number of 0-magnons will exceed the threshold value after termination of the pulse, and the unstable growth of the k-magnons will continue during the initial period of the relaxation. Notice should be taken, however, of the following essential differences in the excitation of the echo for the (ω, ω) and $(\omega, 2\omega)$ pulse sequences: a) In the case of the (ω, ω) sequence the echo is the response of homogeneous precession, whereas in the second case the echo is the response of a spin-wave system with much weaker coupling to the electromagnetic fields. b) In the case of the (ω, ω) sequence, the most important role in the amplification of the echo is played by the two-magnon scattering by inhomogeneities of the local magnetic field, wherein 0-magnons are converted into k-magnons and vice versa. In the case of the $(\omega, 2\omega)$ sequence, this process contributes exclusively to the damping of the signal. It is therefore more likely to observe not amplification but attenuation of the signal with increasing τ.

It would be necessary to take into account in the calculation the irreversible phase relaxation due to the propagation of the spin waves over the sample. Actually, the propagation velocity is small for the long-wave k-magnons, and the diffusion can be neglected. For example, putting $v_{gr} \approx 10^3$ cm/sec and $\tau = 10^{-7}$ sec, we obtain for the distance traversed by the magnon between pulses the value $\sim 10^{-4}$ cm, which is much smaller than the sample dimensions.

In conclusion we note that, just as in the case of cyclotron echo[5], calculation analogous to the foregoing does not lead to the occurrence of an echo for the $(2\omega, \omega)$ sequence of pulses.

Note added in proof (21 August 1972). Remarks a) and b) pertain to the case of short-wave magnons and are of no importance for long-wave magnetostatic modes, so that in the latter case there is amplification of the signal, as was demonstrated experimentally by V. V. Danilov, V. I. Suchakov, and A. V. Tychinskiĭ (ZhETF Pis. Red. 15, 520 (1972) [JETP Lett. 15, 368 (1972)]).

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