INVESTIGATION OF THE MOLECULAR SCATTERING OF LIGHT IN A SAPPHIRE CRYSTAL

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An Ar^+ laser was used to investigate the fine structure of the Rayleigh line of a synthetic sapphire crystal. The velocities of the longitudinal and transverse hypersonic waves were determined and the relative intensities of the Mandelstam-Brillouin components were measured. A theoretical calculation of these quantities was carried out. A comparison of the experimental and calculated results made it possible to refine the value of the photoelastic constant p_{41}. The investigation established that sapphire satisfies the polarization selection rules for the scattering of light by acoustic waves and that the intensity distribution in a polarization spectrum is not affected if the source and the detector are interchanged. A characteristic feature of the scattering of light in a sapphire is the negligibly low intensity of the "longitudinal" Mandelstam-Brillouin components compared with the intensity of the "transverse" components obtained by illumination with light polarized at right-angles to the scattering plane. It is shown qualitatively that this feature of the scattering of light in sapphire owes its origin to the fact that the shear and compression deformations generated by an acoustic wave balance out the associated changes in the permittivity manifested in the VV spectra.

KRISHNAN[1] was the first to study the Mandel'shtam-Brillouin scattering of light in a sapphire crystal. Krishnan used the $\lambda = 2537$ Å mercury line as the excitation source. He recorded one pair of the Mandel' shtam-Brillouin components which were fairly wide. Therefore, he could not determine whether the longitudinal or the transverse hypersonic waves were responsible for the observed Mandel'shtam-Brillouin components, especially as at that time no experimental data were available on the velocity of sound in sapphire.

Kitaeva and Chistyĭ[2] investigated the scattering of light in a sapphire crystal using an Ar^+ laser. The laser technique made it possible to observe the "longitudinal" (corresponding to the scattering by a longitudinal hypersonic wave) and the "transverse" (corresponding to the scattering by a transverse hypersonic wave) Mandel'shtam-Brillouin components and to reveal the characteristic feature of the scattering of light in sapphire, which was the negligibly low intensity of the "longitudinal" Mandel'shtam-Brillouin components compared with the intensity of the "transverse" components in the case of illumination with light polarized at right-angles to the scattering plane.

The present paper describes the results of experimental investigations and calculations (based on the Motulevich formulas[3]) of the intensities of the Mandel' shtam-Brillouin components of light scattered in a synthetic sapphire single crystal. The investigation was carried out using different crystallographic directions of the propagation of the exciting light and of the observation of the scattered light, and different polarizations of the exciting light relative to the scattering plane. A qualitative explanation of the special features of the scattering of light in sapphire crystals is given.

A sapphire crystal does not have piezoelectric or electrooptic properties. The anisotropy of the refractive index of sapphire is weak (for example, for the $\lambda = 4880$ Å we have $n_o = 1.775$, $n_e = 1.767$[4]) and, therefore, the Motulevich theory[3,5] is fully applicable to this material. Sapphire crystals belong to the trigonal system. The point group is $\bar{3}m$.

DESCRIPTION OF EXPERIMENTS

A sample of synthetic sapphire was cut in the form of a six-sided prism. The optic axis of the crystal was aligned along the Z axis. The X axis made an angle of 14° with the normal to the front face of the crystal. The scattered light was observed along the Z axis. The measurements were obtained for four orientations of the crystal relative to the exciting light beam. This beam propagated as follows: 1) along the X axis; 2) along the Y axis; 3) along the −Y axis; 4) along a line in the XY plane making an angle 16° with the Y axis (the designations of the principal crystallographic axes are the same as those used in[6]). The apparatus used is shown schematically in Fig. 1.

The radiation of an Ar^+ laser of 0.5 W power

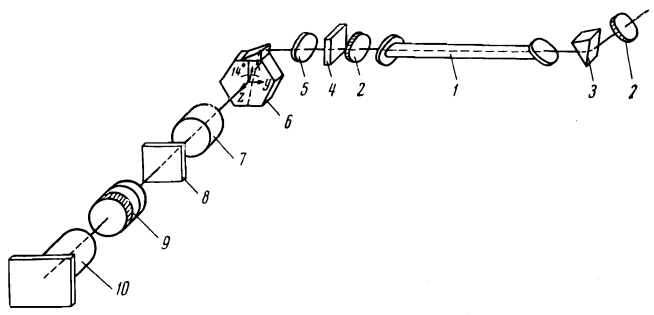

FIG. 1. Schematic diagram of the apparatus: 1–Ar^+ gas-discharge laser tube; 2–resonator mirrors; 3–Brewster prism ensuring laser emission at $\lambda = 4880$ Å; 4–crystal quartz plate for rotation of plane of polarization of exciting light by 90°; 5–lens for focusing Ar^+ laser radiation in sapphire; 6–sapphire crystal; 7–collimating lens (f = 300 mm); 8–SZS-21 filter; 9–Fabry-Perot etalon; 10–camera objective (f = 270 mm) and film casette. The XYZ coordinate system is right-handed and the X axis is parallel to the dashed line.

(λ = 4880 Å) was focused in the sapphire crystal. The Rayleigh scattering spectrum was investigated using a Fabry-Perot etalon which received a parallel beam. The scattered light was recorded photographically. The exciting light was polarized either in the scattering plane or at right-angles to this plane. The plane of polarization was rotated by 90° with the aid of a quartz plate 8.7 mm thick. An immersion liquid (glycerin) was used in order to eliminate influence of imperfections of the polished surface of the sapphire crystal and to minimize the effect of stray light. The red luminescence of sapphire was absorbed by a SZS-21 filter. Some of the spectrograms obtained are shown in Fig. 2.

An analysis of these spectrograms showed that when the exciting light was polarized at right-angles to the scattering plane, only the Mandel'shtam-Brillouin components corresponding to the scattering by a quasi-transverse hypersonic wave were observed; the components corresponding to the scattering by a quasilongitudinal acoustic wave had a negligibly low intensity (Fig. 2a).

In the second case, when the exciting light was polarized in the scattering plane, both the "transverse" and the "longitudinal" Mandel'shtam-Brillouin components were observed. The intensities of these components were similar (Figs. 2b–2d). It was established that the "transverse" components in Fig. 2a were polarized in the scattering plane whereas in Figs. 2b–2d they were polarized at right-angles to the scattering plane; the "longitudinal" components in Fig. 2b–2d were polarized in the scattering plane.

The characteristic feature of the scattering of light in sapphire crystals was the negligibly low intensity of the "longitudinal" Mandel'shtam-Brillouin components compared with the intensity of the "transverse" components in the case when the exciting light was polarized at right-angles to the scattering plane.

The analysis of the spectrograms consisted of measurements of the relative intensities I_T/I_L (T and L refer to the "transverse" and the "longitudinal" components) of the Mandel'shtam-Brillouin components by the conventional photographic photometry method and of the shift of these components ($\Delta\nu$) relative to the exciting light. The shifts were used in the calculation of the velocity of hypersound:

$$v = \frac{\Delta\nu}{\nu}\frac{c}{n\sqrt{2}}. \qquad (1)$$

The results of the analysis of the spectrograms are presented in Table I (columns 3 and 5).

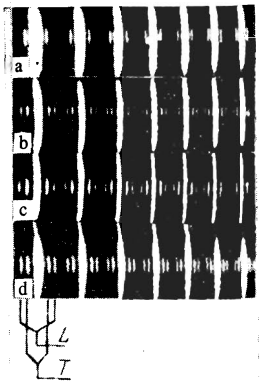

FIG. 2. Spectrograms of scattered light obtained for exciting light traveling along the Y axis (a, b), along the X axis (c), and along a direction making an angle of 16° with the Y axis (d). Polarization of the exciting light: a) at right-angles ot the scattering plane; b)-d) in the scattering plane. T represents the "transverse" and L the "longitudinal" Mandel'shtam-Brillouin components.

DISCUSSION OF RESULTS

1. Velocity of Hypersonic Waves in Sapphire

The velocities of the longitudinal and transverse hypersonic waves can be calculated if the elastic constants of a crystal are known. We calculated the velocities of hypersound in sapphire using the elastic constants determined from ultrasonic measurements for a synthetic sapphire crystal.[6] The results of this calculation are presented in Table I (column 4). It is evident from Table I that the velocity of the transverse hypersonic waves depended on the direction of propagation and was ~5700–6700 m/sec; the corresponding velocity of the longitudinal hypersonic waves ~10 400–11 100 m/sec. Krishnan[1] reported that the velocity of hypersound was 7300 m/sec, i.e., one may assume that the Mandel'shtam-Brillouin components observed by him were due to a transverse hypersonic wave. Our values of the velocities were in good agreement with the calculations. This indicated that there was no dispersion of the elastic constants at hypersonic frequencies.

It is interesting to note that the elastic constants of the natural and synthetic sapphire differ quite considerably. Naturally, the velocities of hypersonic waves in these crystals are also different. In particular, the synthetic crystals are more isotropic in respect of the velocity of the sound than the natural crystals. This is illustrated by the graphs given in[7], which represent polar diagrams of the distribution of the velocity of hypersound in principal sections of a sapphire crystal. We may assume that the isotropy is due to the presence of a very small number of impurities which are always encountered in synthetic sapphire and are responsible for its red luminescence.

2. Intensities of Scattered-Light Components. Refinement of the Constant p_{41}

Column 4 in Table I gives the results of a theoretical calculation of the ratio of the intensities of the Mandel'shtam-Brillouin components. According to Fabelinskiĭ,[5] the intensity of the Mandel'shtam-Brillouin components is given by the relationship

$$I_{\beta\alpha}{}^{st} = R_{\beta\alpha}{}^{st} I_0 V / 2r^2,$$

where I_0 is the intensity of the exciting light; V is the

Table I

Direction of incident light*	Wave**	Velocity of hypersound, m/sec		I_T/I_L		
		Experiment	Calculation	Experiment	расчет $p_{41}=0{,}01$	расчет $p_{41}=0{,}00$
Along X axis	qL	10650	10670	1.50	1.35	1.42
	qT_2	6160	6200			
Along Y axis	qL	10440	10380	1.19	1.59	1.19
	T_2	6700	6730			
Along –Y axis	qL	10950	10930	1.85	1.31	1.78
	T_2	5760	5790			
Along line making 16° with Y axis	qL	10370	10480	1.18	1.27	1.19
	qT_2	6480	6530			

*The scattered light is observed along the Z axis (the incident light is polarized in the scattering plane).

**Here, T is a transverse acoustic wave; qL and qT are quasilongitudinal and quasi-transverse waves.

scattering volume; r is the distance from the scatterer to the point of observation, which governs the solid angle in which the scattered light is observed; $R^{st}_{\alpha\beta}$ is the scattering coefficient.[3] We calculated directly the scattering coefficients from

$$R_{\beta\alpha}{}^{st} = \frac{\pi^2}{\lambda^4} kT \frac{n^8}{\rho v^2} (p_{ijlm}\, \alpha_i \gamma_j \gamma_l \varkappa_m)^2. \qquad (2)$$

Here, λ is the wavelength of the incident light; $n = (n_O + n_e)/2$ is the average refractive index at this wavelength; ρ is the density of sapphire; k is the Boltzmann constant; T is the temperature in °K; v is the velocity of an acoustic wave of polarization γ traveling along the direction $\varkappa$; s is the direction of the incident light; t is the direction of the scattered light; β and α are directions of polarization of the incident and scattered light; $p_{ij\nu m}$ is the photoelasticity tensor. The relative intensities of the Mandel'shtam-Brillouin components are equal to the ratios of the corresponding scattering coefficients. Our calculations were carried out using the isothermal photoelastic constants of synthetic rose-colored ruby:[8] $p_{11} = -0.23$, $p_{33} = -0.20$, $p_{44} = -0.10$, $p_{12} = -0.03$, $p_{13} = 0.02$, $p_{31} = -0.04$, $p_{14} = 0.00$, $p_{41} = 0.01$. We took into account only the symmetric (in respect of the acoustic indices) part of the photoelasticity tensor because the antisymmetric part,[9] governed by the anisotropy of the refractive index, is small and its inclusion in the scattering coefficients represents a correction of the order of 1%.

It is evident from Table I that the experimental values of the intensity ratios of the Mandel'shtam-Brillouin components differ from the calculated values. This difference increases with increasing contribution of the constant p_{41} (rows 2 and 3 in Table I). If its contribution is small, the discrepancy between the experimental and calculated results is less (rows 1 and 4 in Table I). Apart from the constant p_{41}, the calculations required the knowledge of the constant p_{44}. However, the precision with which the latter constant can be determined[8] is much higher (a few percent) than that of the constant p_{41} (a few tens of percent). Thus, the observed discrepancy between the theory and experiment can be explained by the low precision of the determination of the photoelastic constant p_{41} in[8]. The constant p_{41} can be determined more accurately from the ratios of the intensities of the Mandel'shtam-Brillouin components measured for several orientations of a crystal relative to the exciting light beam. Our measurements carried out using four orientations yield

$$p_{41} = 0.00 \pm 0.002.$$

The last column of Table I gives the ratios of the intensities calculated using the above value of the constant p_{41}. It follows from[8] that $p_{14} = 0.00$ and our measurements indicate that $p_{41} = 0.00 \pm 0.002$, i.e., within the limits of the experimental error the constants p_{14} and p_{41} of a sapphire crystal are equal:

$$p_{41} = p_{14} = 0.00. \qquad (3)$$

Hence, it follows that the symmetry of the tensor of the elastooptic constants of sapphire is higher than the symmetry of the point group of the crystal and is equal to the symmetry of the point group $\bar{6}m2$.[10]

SPECIAL FEATURES OF THE SCATTERING OF LIGHT IN A SAPPHIRE CRYSTAL

We used the refined values of the photoelastic constants in a calculation of the scattering coefficients for five orientations of the incident and scattered light relative to the crystallographic axes. The calculations were carried out on a computer and the results are presented in Table II (here, k is the wave vector of the incident wave, k′ is the wave vector of the scattered wave; qL is a quasilongitudinal acoustic wave with a polarization vector γL, qT_1 is a quasitransverse acoustic wave with a polarization vector γT_1, T_2 is a transverse acoustic wave with a polarization vector γT_2, qT_2 is a quasitransverse acoustic wave with a polarization vector γT_2).

A. It is evident from Table II that sapphire obeys the polarization selection rules for the scattering of light by acoustic waves,[11] according to which the longitudinal (L) phonons are active in the VV and HH spectra,[1)] the transverse (T_2) phonons polarized at right-angles to the scattering plane are active in the VH and HV spectra, and the transverse (T_1) phonons polarized in the scattering plane do not scatter light at 90° with respect to the incident beam. Mixed spectra are possible in the case of quasilongitudinal and quasitransverse acoustic waves: a quasitransverse phonon mixed in respect of polarization with a quasilongitudinal or another quasitransverse phonon may be active in the spectra associated with the phonons with which it is mixed. Thus, in the YZ scattering plane a quasitransverse phonon qT_1 is active in the VV and HH spectra because this qT_1 phonon is mixed in respect of polarization with a quasilongitudinal phonon qL. However, the qT_1 phonon is not active in the HV and VH spectra because it is not mixed in respect of its polarization with a T_2 phonon. In the XY scattering plane a quasitransverse phonon qT_1 has a large component in the direction perpendicular to the scattering plane and, therefore, it can be observed in the VH and HV spectra. However, in all cases the main contribution to the scattering of light is made by the quasilongitudinal phonons qL and the quasitransverse phonons qT_2 or the transverse phonons T_2 and the contribution is the same as if these phonons were purely longitudinal and purely transverse, respectively.

B. A specific feature of sapphire is the negligibly low value of the scattering coefficient for the quasilongitudinal (longitudinal) waves in the case of the VV spectra. This property leads to a strong depolarization of the scattered light in sapphire. The depolarization coefficient of the scattered light[3]

$$D = \sum_{L,T_1,T_2} (R_{HH} + R_{VH}) \Big/ \sum_{L,T_1,T_2} (R_{VV} + R_{HV}) \qquad (4)$$

in the cases of interest to us (Table II) is, respectively, 1.65, 1.47, 1.45, 1.54, and 1.61, and its value is considerably higher than the depolarization coefficient of the light scattered in diamond ($D \sim 1.24$) which until recently has been regarded as the highest value of this coefficient.[3]

[1)] V denotes the direction perpendicular to the scattering plane and H is a direction parallel to the scattering plane. The first index represents the polarization of the incident light and the second that of the scattered light.

C. It is evident from Table II that

$$R_{xy}^{yz} = R_{zx}^{yz}, \quad R_{zx}^{xy} = R_{yz}^{xy}, \quad R_{yx}^{xz} = R_{zy}^{xz}, \quad R_{Nr}^{rz} = R_{zN}^{rz}. \quad (5)$$

The above equalities show that in the same polarization spectrum the intensity of the light scattered in sapphire does not change when the source and the scattered-light detector are interchanged.

Let us consider the first equality. If the light is incident on a crystal along the Y axis and the scattered light is observed along the Z axis, the scattering coefficient R_{xy}^{yz} governs the intensity of the scattered light in the VH spectrum representing quasilongitudinal and quasitransverse (or transverse) phonons in the YZ scattering plane. However, if the source and the scattered-light detector are interchanged (in this case the light is incident along the −Z axis and the scattered light is observed along the −Y axis), the intensity in the VH spectrum obtained in the YZ scattering plane is governed by the scattering coefficient $R_{XZ}^{(-x)(-y)}$. According to the reciprocity theorem,[3] we have $R_{XZ}^{(-z)(-y)} = R_{ZX}^{yz}$ and it therefore follows from the first equality of Eq. (5) that

$$R_{xy}^{yz} = R_{xz}^{(-z)(-y)}. \quad (6)$$

in Similar conclusions follow from the other equalities in Eq. (5). These qualities yield relationships between the photoelastic constants for which the intensity of the same polarization spectrum does not alter when the scattering process is reversed:

$$^1/_2(p_{11} - p_{12}) = p_{44}, \quad p_{14} = p_{41} = 0. \quad (7)$$

The relationships in Eq. (7), which apply to isotropic media, are satisfied quite well by sapphire and they should also be obeyed by other trigonal centrosymmetric crystals with a weak optical anisotropy.

QUALITATIVE EXPLANATION OF THE SPECIAL FEATURES OF THE SCATTERING OF LIGHT IN A SAPPHIRE CRYSTAL

We shall now consider the reason for the very low intensity of the VV spectra representing the scattering of light by quasilongitudinal phonons in sapphire.

The scattering coefficients in the case of incident and scattered light directed along the principal axes of a crystal are given by the relationships

$$R_{ij}^{L} \sim (\Delta\varepsilon_{ij}^{L})^2, \quad R_{ij}^{T} \sim (\Delta\varepsilon_{ij}^{T})^2, \quad (8)$$

where $\Delta\epsilon_{ij}^{L}$ and $\Delta\epsilon_{ij}^{T}$ are the changes in the permittivity resulting from the interaction with longitudinal and transverse acoustic waves. Thus, the special features of the scattering of light are governed by the corresponding features of the tensor $\Delta\epsilon_{ij}$. Let us first consider an isotropic solid. According to Motulevich,[3] we have

$$\Delta\varepsilon_{ij} = n^4 p_{ijlm} u_{lm}. \quad (9)$$

Here, n is the refractive index of the medium and u_{lm} are the elastic deformations. (For standing thermal elastic waves we have

Table II. Scattering coefficients* R for YZ, XY, XZ, and rZ planes**

Wave	v, km/sec	VV	VH	HV	HH	γ
		K (0, *K*, 0), **K′** (0, 0, *K*), ϰ (0, −0,707, 0,707)				
		R_{xx}^{yz}	R_{xy}^{yz}	R_{zx}^{yz}	R_{zy}^{yz}	
qL	10,378	0.03	0	0	23,26	γ_L (0, −0,670, 0,742)
qT_1	6,957	3.30	0	0	0,14	γ_{T_1}(0, 0,742, 0,670)
T_2	6,735	0	27,70	27,70	0	γ_{T_2}(1, 0, 0)
		K (0, −*K*, 0), **K′** (0, 0, *K*), ϰ (0, 0,707, 0,707)				
		R_{xx}^{-yz}	R_{xy}^{-yz}	R_{zx}^{-yz}	R_{zy}^{-yz}	
qL	10,928	0.08	0	0	21,99	γ_L (0, 0,735, 0,678)
qT_1	6,950	3.18	0	0	0,08	γ_{T_1}(0, −0,678, 0,735)
T_2	5,792	0	37,45	37,45	0	γ_{T_2}(1, 0, 0)
		K (− *K*, 0, 0), **K′** (0, *K*, 0), ϰ (0,707, 0,707, 0)				
		R_{zz}^{xy}	R_{zx}^{xy}	R_{yz}^{xy}	R_{yx}^{xy}	
qL	11,180	3,21	0,02	0,02	20,05	γ_L (0,708, 0,7046, 0,048)
qT_1	6,649	0,01	6,87	6,87	0,04	γ_{T_1}(0,598, −0,634, 0,491)
qT_2	5,872	0,02	27,56	27,56	0,12	γ_{T_2}(−0,376, −0,320, 0,870)
		K (*K*, 0, 0), **K′** (0, 0, *K*), ϰ (−0,707, 0, 0,707)				
		R_{yy}^{xz}	R_{yx}^{xz}	R_{zy}^{xz}	R_{zx}^{xz}	
qL	10.676	0.05	0,08	0,08	21,89	γ_L (−0,705, −0,084, 0,704)
qT_1	6.999	3,05	1,05	1,05	0,02	γ_{T_1}(0,679, 0,202, 0,705)
qT_2	6,204	0,20	31,08	31,08	0,44	γ_{T_2}(−0,201, 0,976, −0,086)
		K (K_x, K_y, 0), **K′** (0, 0, *K*), ϰ (0,195, −0,680, 0,707)				
		R_{NN}^{rz}	R_{Nr}^{rz}	R_{zN}^{rz}	R_{zr}^{rz}	
qL	10,484	0,04	0,05	0,05	22,73	γ_L (0,254, −0,673, 0,721)
qT_1	6,999	2,97	1,96	1,96	0,01	γ_{T_1}(0,455, 0,585, 0,672)
qT_2	6,526	0,31	27,11	27,11	0,33	γ_{T_2}(0,853, 0,502, 0,141)

*The scattering coefficients are given in units of $\pi^2\lambda^{-4}n^8kT \times 10^{-16}$ [cm^{-1}].

**The direction r lies in the XY plane and makes an angle of 16° with the Y axis and the direction N is perpendicular to the rZ scattering plane.

$$u_{lm} = (kT/\rho v^2)^{1/2}\varkappa_l\gamma_m \sin(\Omega t - \mathbf{qr});$$

here, $q = q_K$ is the wave vector of the ultrasonic wave involved and Ω is its frequency.) Any deformation u_{lm} can be expanded into two deformations--pure shear and pure compression[12]--and represented in the form

$$u_{lm} = (u_{lm} - {}^1/_3\delta_{lm}u_{kk}) + {}^1/_3\delta_{lm}u_{kk}. \quad (10)$$

Similarly, we also obtain

$$n^{-4}\Delta\varepsilon_{ij} = K^0u_{kk}\delta_{ij} + 2M^0(u_{ij} - {}^1/_3\delta_{ij}u_{kk}). \quad (11)$$

The quantity K^0 can be interpreted as the optical bulk modulus which represents the change in ϵ as a result of pure compression and M^0 can be regarded as the optical shear modulus representing the change in ϵ as a result of pure shear. Equating Eqs. (9) and (11) and writing the tensor p_{ijlm} in the two-index form in accordance with the well-known rule[10] for an isotropic body, we obtain

$$p_{11} = K^0 + {}^4/_3M^0, \quad p_{12} = K^0 - {}^2/_3M^0. \quad (12)$$

If the incident light travels along the Y axis and the scattered light is observed along the Z axis (the case represented in Table II), the change in ϵ for longitudinal and transverse waves is given by the following formulas[3]

$$\frac{\Delta\varepsilon_{ij}^L}{N} = \begin{pmatrix} p_{12} & 0 & 0 \\ 0 & p^* & -p \\ 0 & -p & p^* \end{pmatrix}, \quad \frac{\Delta\varepsilon_{ij}^T}{N} = \frac{\sqrt{2}}{2}\begin{pmatrix} 0 & p & -p \\ p & 0 & 0 \\ -p & 0 & 0 \end{pmatrix}. \quad (13)$$

Here, $p^* = {}^1/_2(p_{11} + p_{12})$, $p = {}^1/_2(p_{11} - p_{12})$, $N = (kT/\rho v^2)^{1/2}n^4\sin(\Omega t - \mathbf{q}\cdot\mathbf{r})$. Substituting Eq. (12) into Eq. (13), we obtain

$$\frac{\Delta\varepsilon_{ij}^L}{N} = \begin{pmatrix} K^0 - {}^2/_3M^0 & 0 & 0 \\ 0 & K^0 + {}^1/_3M^0 & -M^0 \\ 0 & -M^0 & K^0 + {}^1/_3M^0 \end{pmatrix},$$

$$\frac{\Delta\varepsilon_{ij}^T}{N} = \begin{pmatrix} 0 & M^0 & -M^0 \\ M^0 & 0 & 0 \\ -M^0 & 0 & 0 \end{pmatrix} \quad (14)$$

It follows from these expressions that the scattering of light by transverse waves is solely due to the nondiagonal elements of the tensor $\Delta\epsilon_{ij}$ (VH and HV spectra), whereas the scattering by longitudinal waves is due to the diagonal elements (VV spectra) and nondiagonal elements (HH spectra). The nondiagonal elements of the tensor $\Delta\epsilon_{ij}$ are governed, in the case of longitudinal and transverse waves, solely by the optical shear modulus M^0. The diagonal element of the tensor $\Delta\epsilon_{ij}^L$, responsible for the scattering in the VV spectrum, is governed by the difference between the optical moduli K^0 and M^0. If K^0 and M^0 have the same sign and are of approximately the same magnitude, the shear deformations compensate the changes in ϵ due to pure compression. In this case $|M^0| \gg |K^0 - (2/3)M^0|$. A similar situation also occurs in sapphire. When the light is incident along the Y axis and the scattered light is observed along the Z axis, Eq. (9) yields the following relationships for sapphire:

$$\frac{\Delta\varepsilon_{ij}^L}{N} = \begin{pmatrix} 3{,}7 & 0 & 0 \\ 0 & 98{,}4 & -100 \\ 0 & -100 & 124 \end{pmatrix}\cdot 10^{-3}, \frac{\Delta\varepsilon_{ij}^{T_2}}{N}\begin{pmatrix} 0 & 100 & -100 \\ 100 & 0 & 0 \\ -100 & 0 & 0 \end{pmatrix}\cdot\frac{\sqrt{2}}{2}\cdot 10^{-3}. \quad (15)$$

A comparison of Eqs. (14) and (15) shows that the photoelastic properties of sapphire are approximately the same as those of an isotropic body with the moduli $M^0 \sim 0.100$ and $K^0 \sim 0.078$ in the YZ scattering plane. Therefore, the diagonal element $\Delta\epsilon_{XX}^L$, which determines the intensity of the VV spectrum associated with the qL phonons, is extremely small because of the m mutual compensation of the changes in ϵ resulting from pure shear and pure compression. However, the values of the elements $\Delta\epsilon_{YZ}^L$ and $\Delta\epsilon_{XY}^{T_2}$ depend only on the shear deformations and cannot be compensated by pure compression. Therefore, the intensities of the HH spectra for the qL phonons and of the HV (VH) spectra for the T phonons are relatively high and similar. In the scattering planes XY, XZ, and rZ (Table II) a sapphire crystal can again be regarded as an isotropic body in respect of its photoelastic properties but the moduli are now somewhat different.

This discussion shows that the special features of the scattering of light in a sapphire crystal arise because the acoustic-wave-generated shear and compression deformations compensate mutually the changes in the permittivity in the case of the VV spectra. A similar situation arises also in $LiNbO_3$ and $LiTaO_3$ crystals, in which light is scattered[13,14] in a manner similar to the scattering of light in sapphire. All these crystals have the same molecular structure at the unit cell level.

In conclusion the authors regard it as their pleasant duty to thank I. L. Fabelinskiĭ and G. P. Motulevich for their valuable advice and comments.

[1] R. S. Krishnan, Proc. Indian Acad. Sci. **26A**, 450 (1947).
[2] V. F. Kitaeva and I. L. Chistyĭ, ZhETF Pis. Red. 13, 611 (1971) [JETP Lett. 13, 435 (1971)].
[3] G. P. Motulevich, Tr. Fiz. Inst. Akad. Nauk SSSR 5, 9 (1950). Izv. Akad. Nauk SSSR, Ser. Fiz. 11, 390 (1947).
[4] M. A. Jeppesen, J. Opt. Soc. Amer. 48, 629 (1958).
[5] I. L. Fabelinskiĭ, Molekulyarnoe rasseyanie sveta, Nauka, M., 1965 (Molecular Scattering of Light, Plenum Press, New York, 1968).
[6] J. B. Wachtman, Jr., W. E. Tefft, D. G. Lam, Jr., and R. P. Stinchfield, J. Res. Nat. Bur. Stand., Sect. **A64**, 213 (1960).
[7] G. W. Farnell, Can. J. Phys. 39, 65 (1961).
[8] R. W. Waxler and E. N. Farabaugh, J. Res. Nat. Bur. Stand., Sect. A 74, 215 (1970).
[9] O. V. Kachalov, Zh. Eksp. Teor. Fiz. 61, 1352 (1971) [Sov. Phys.-JETP 34, 719 (1972)].
[10] J. F. Nye, Physical Properties of Crystals, Clarendon Press, Oxford, 1957 (Russ. Transl., Mir. M., 1967).
[11] R. W. Gammon, in: Proc. First Intern. Conf. on Light Scattering Spectra in Solids, New York, 1968 (ed. by G. B. Wright), publ. by Springer Verlag, New York (1969); abstract only in: Solid State Commun. 6, No. 10, xx (1968).

[12] L. D. Landau and E. M. Lifshitz, Teoriya uprugosti, Nauka, M., 1965 (Theory of Elasticity, 2nd ed., Pergamon Press, Oxford, 1970).
[13] D. F. Kiselev, T. M. Glushkova, and M. M. Firsova, Fiz. Tverd. Tela 11, 3592 (1969) [Sov. Phys.-Solid State 11, 3007 (1970)].
[14] Z. M. Khashkhozhev, V. V. Lemanov, and R. V. Pisarev, Fiz. Tverd. Tela 12, 128 (1970) [Sov. Phys.-Solid State 12, 101 (1970)]; Izv. Akad. Nauk SSSR, Ser. Fiz. 35, 987 (1971).

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161