LATERAL SPOTS OF X-RAY TRANSITION RADIATION IN CRYSTALS AND THEIR EFFECT ON THE CENTRAL SPOT

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The radiation emitted by an ultrarelativistic charged particle on passage through a crystalline plate of arbitrary thickness is investigated within the framework of microscopic theory by taking into account the size of the atoms and thermal oscillations of the latter. The problem is solved by a method similar to that of the dynamic theory of diffraction of x-rays in the two-wave approximation. It is shown that in sufficiently thick crystals the formulas for the radiation emitted strongly differ in the vicinity of Bragg frequencies from the respective formulas of the macroscopic theory in which the crystalline structure is not taken into account. At these frequencies narrow and high maxima appear in the spectrum of the central spot. Similar maxima also appear in the lateral spots.

1. INTRODUCTION

IN our earlier papers[1,2] we investigated the radiation produced by a uniformly moving ultrarelativistic charged particle in a crystal, on the basis of an integral equation that determines the Fourier components of the radiation field. This equation was solved in[1] in the first approximation of the iteration method for a thin plate, while in[2] the integral equation was solved by a method of reduction to a system of algebraic equation for a plate of arbitrary thickness, using a model in which the crystal consisted of point-like immobile atoms. The result was a radiation picture constituting a peculiar Laue pattern with a central spot coinciding with the particle motion direction, and with lateral spots whose positions are determined by the crystal symmetry. It was shown in[2] that in a sufficiently thick crystal, at frequencies satisfying the condition for Bragg reflection in an almost exact backward direction, narrow and high maxima appear both in the emission spectrum of the central spot and in the spectrum of radiation emitted backwards.

On the other hand, this problem can be solved also in another approach, based on the use of the dynamic theory of x-ray diffraction[3-5].

This approach will be used in the present paper to investigate the lateral radiation spots and their effect on the central spot.

2. SCATTERED FIELD OF ULTRARELATIVISTIC CHARGED PARTICLE IN A CRYSTAL IN THE TWO-WAVE APPROXIMATION

We determine the field of an ultrarelativistic charged particle moving with velocity v inside a crystalline plate of arbitrary thickness l. To this end, we calculate the field component $\mathbf{E}^{sc}$ that is added to the vacuum field $\mathbf{E}_{ch}$ as a result of the interaction with the crystal ("scattered" field of the charge). The equation for the Fourier components of the scattered field in the microscopic theory (Eq. (10) of[1]) takes the following form after the sums over the crystal lattice sites are replaced approximately by the corresponding δ-functions (see (11) of[1]):

$$(k^2-\omega^2/c^2-\omega^2 g(\mathbf{k},\mathbf{k})/c^2)\mathbf{E}^{sc}(\mathbf{k})-\sum_h{}'\frac{\omega^2}{c^2}g(\mathbf{k},\mathbf{k}_h)\mathbf{E}^{sc}(\mathbf{k}_h) \quad (1)$$
$$+\mathbf{k}\Big[g(\mathbf{k},\mathbf{k})(\mathbf{k}\mathbf{E}^{sc}(\mathbf{k}))+\sum_h{}'g(\mathbf{k},\mathbf{k}_h)(\mathbf{k}\mathbf{E}^{sc}(\mathbf{k}_h))\Big]$$
$$=g(\mathbf{k},\mathbf{k})[\omega^2\mathbf{E}_{ch}/c^2-\mathbf{k}(\mathbf{k}\mathbf{E}_{ch})],$$

where we have put for simplicity

$$g^{ql}(\mathbf{k},\mathbf{k}')=g(\mathbf{k},\mathbf{k}')\delta^{ql},\quad \mathbf{E}_{sc}(\mathbf{k},\omega)=\mathbf{E}^{sc}(\mathbf{k}),$$
$$\mathbf{E}_{зар}(\mathbf{k},\omega)=\mathbf{E}_{зар}=8\pi^2 ei\frac{\omega\mathbf{v}-\mathbf{k}c^2}{k^2c^2-\omega^2}\delta(\omega-\mathbf{k}\mathbf{v}),$$

and the remaining notation remains the same. The unknowns in (1) are $\mathbf{E}^{sc}(\mathbf{k})$ and $\mathbf{E}^{sc}(\mathbf{k}_h)$, where h runs through all the reciprocal-lattice sites with the exception of the site $\mathbf{K}_h = 0$ (this circumstance is indicated by the primes at the summation signs). To find these quantities it is necessary to write down analogous equations for $\mathbf{E}^{sc}(\mathbf{k}_h)$ and solve the obtained system of equations simultaneously.

Assume now that there are only two vectors near the Ewald sphere, $\mathbf{k}$ and $\mathbf{k}_h = \mathbf{k} + \mathbf{K}_h$, where $\mathbf{K}_h$ now stands for a certain perfectly defined reciprocal-lattice vector, multiplied by 2π. Then, confining ourselves only to the quantities $\mathbf{E}^{sc}(\mathbf{k})$ and $\mathbf{E}^{sc}(\mathbf{k}_h)$ and introducing the notation

$$g_{ij}=g(\mathbf{k}_i,\mathbf{k}_j),\quad \eta_i=k_i^2c^2/\omega^2-1,$$
$$a_{ij}=\mathbf{k}_i\mathbf{E}^{sc}(\mathbf{k}_j),\quad q_i=\mathbf{k}_i\mathbf{E}_{ch}$$

(the indices i, j = 0 and h correspond to $\mathbf{k}$ or $\mathbf{k}_h$), we obtain the following system of equations:

$$(\eta_0-g_{00})\mathbf{E}^{sc}(\mathbf{k})-g_{0h}\mathbf{E}^{sc}(\mathbf{k}_h)+(\mathbf{k}c^2/\omega^2)[g_{00}a_{00}+g_{0h}a_{0h}] \quad (2)$$
$$=g_{00}[\mathbf{E}_{ch}-\mathbf{k}q_0c^2/\omega^2],$$
$$-g_{h0}\mathbf{E}^{sc}(\mathbf{k})+(\eta_h-g_{hh})\mathbf{E}^{sc}(\mathbf{k}_h)$$
$$+(\mathbf{k}_hc^2/\omega^2)[g_{h0}a_{h0}+g_{hh}a_{hh}]=g_{h0}[\mathbf{E}_{ch}-\mathbf{k}_hq_hc^2/\omega^2].$$

We have retained in the right-hand sides of (1) and (2)

only the terms with $\mathbf{E}_{ch}(\mathbf{k}, \omega)$, since we take the vector $\mathbf{k}$ to be the wave vector of the central spot.

If it is assumed that the principal process is Thomson scattering of the waves by the atoms, and also take into account the absorption of the radiation as a result of the photoeffect, then $g_{00} \approx g_{hh} = g' + ig''$, where $g' = -\omega_0^2/\omega^2$ and $|g|' \gg g'' > 0$. As to $g_{oh} = g'_{oh} + ig''_{oh}$ and $g_{ho} = g'_{ho} + ig''_{ho}$, we have

$$g_{ho}' = -\frac{\omega_0^2}{Z\omega^2} f(\mathbf{K}_h) \exp\{-M(\mathbf{K}_h)\}$$

(if the unit cell contains only one atom), and g'_{oh} is determined by the same formula, but with $\mathbf{K}_h$ replaced by $-\mathbf{K}_h$. Here $f(\mathbf{K}_h)$ is the atomic form factor, $\exp\{-M(\mathbf{K}_h)\}$ determines the Debye-Waller factor, and Z is the atomic number. The quantities g''_{oh} and g''_{ho} are much smaller in absolute value than $|g'_{oh}|$ and $|g'_{ho}|$.

To solve the system (2) we obtain first the values of a_{ij}. Taking the scalar products of (2) with $\mathbf{k}$ and $\mathbf{k}_h$, we obtain four linear inhomogeneous algebraic equations with respect to a_{ij}:

$$\begin{pmatrix} 1+g_{00} & g_{0h} & 0 & 0 \\ \zeta g_{00} & \zeta g_{0h} & \eta_0 - g_{00} & -g_{0h} \\ -g_{h0} & \eta_h - g_{hh} & \zeta g_{h0} & \zeta g_{hh} \\ 0 & 0 & g_{h0} & 1+g_{hh} \end{pmatrix} \begin{pmatrix} a_{00} \\ a_{0h} \\ a_{h0} \\ a_{hh} \end{pmatrix} = \begin{pmatrix} -g_{00}q_0 \\ -g_{00}(\zeta q_0 - q_h) \\ g_{h0}(q_0 - \zeta q_h) \\ -g_{h0}q_h \end{pmatrix}, \tag{3}$$

where $\zeta = \mathbf{k}\cdot\mathbf{k}_h c^2/\omega^2$. Since $|g_{ij}| \ll 1$ and $1 - \beta^2 \ll 1$, it follows from the first and the last equations that $|a_{00}| \ll |a_{oh}|$ and $|a_{hh}| \ll |a_{ho}|$. This means physically that the nontransverse component of the scattering field is small. Then, confining ourselves only to the second and third equations of the system (3), we obtain (if $|g_{oh}|$ and $|g_{ho}|$ are not much smaller than $|g_{oo}|$ and $|g_{hh}|$):

$$\begin{aligned} a_{0h} &= [(\zeta^2 g_{00} + \eta_0 - g_{00})q_0 - \eta_0 \zeta q_h] g_{h0}/D_p, \\ a_{h0} &= [-\zeta(g_{0h}g_{h0} + g_{00}(\eta_h - g_{hh}))q_0 + (\zeta^2 g_{0h}g_{h0} \\ &\quad + g_{00}(\eta_h - g_{hh}))q_h]/D_p, \end{aligned} \tag{4}$$

where

$$D_p = (\eta_0 - g_{00})(\eta_h - g_{hh}) - \zeta^2 g_{0h}g_{h0}, \tag{5}$$

and, in addition

$$\begin{aligned} q_0 &= \tilde{q}_0 \delta(k_z - \omega/v), \quad q_h = \tilde{q}_h \delta(k_z - \omega/v), \\ \tilde{q}_0 &= -\frac{8\pi^2 ei}{v}, \quad \tilde{q}_h = \tilde{q}_0 \left[1 + \frac{K_{hz}(1-\beta^2)\omega/v + \mathbf{k}_\perp \mathbf{K}_{\perp h}}{k_\perp^2 + \omega^2/v^2 - \omega^2/c^2}\right]. \end{aligned} \tag{6}$$

Here $\mathbf{k}_\perp$ and $\mathbf{K}_{\perp h}$ are the transverse components of the vectors $\mathbf{k}$ and $\mathbf{K}_h$. We have assumed here that the charged particle moves along the z axis.

Solving (2) with respect to $\mathbf{E}^{sc}(\mathbf{k})$ and $\mathbf{E}^{sc}(\mathbf{k}_h)$ and neglecting a_{00} and a_{hh} in comparison with a_{oh} and a_{ho}, we obtain

$$\begin{aligned} \mathbf{E}^{sc}(\mathbf{k}) &= \{[g_{00}(\eta_h - g_{hh}) + g_{0h}g_{h0}]\mathbf{E}_{ch} - (\eta_h - g_{hh})(g_{00}q_0 \\ &\quad + g_{0h}a_{0h})\mathbf{k}c^2/\omega^2 - g_{0h}g_{h0}(q_h + a_{h0})\mathbf{k}_h c^2/\omega^2\}/D_n, \\ \mathbf{E}^{sc}(\mathbf{k}_h) &= g_{h0}\{\eta_0 \mathbf{E}_{ch} - (g_{00}q_0 + g_{0h}a_{0h})\mathbf{k}c^2/\omega^2 \\ &\quad - (\eta_0 - g_{00})(q_h + a_{h0})\mathbf{k}_h c^2/\omega^2\}/D_n, \end{aligned} \tag{7}$$

where

$$D_n = (\eta_0 - g_{00})(\eta_h - g_{hh}) - g_{0h}g_{h0}. \tag{8}$$

Expressions (7) when added to the Fourier component of the field of the charge in vacuum $\mathbf{E}_{ch}$ determine the Fourier components of the field of the charge inside the crystal. Far from the Bragg frequencies, when $|\eta_h| \gg |\eta_0|$, $|g_{ij}|$, we have $|\mathbf{E}^{sc}(\mathbf{k}_h)| \ll |\mathbf{E}^{sc}(\mathbf{k})|$, and

$$\mathbf{E}^{sc}(\mathbf{k}) \approx g_{00}[\mathbf{E}_{ch} - (\mathbf{k}\mathbf{E}_{ch})\mathbf{k}c^2/\omega^2]/(\eta_0 - g_{00}).$$

After simple calculations, we obtain the usual expression for the field of the charge in matter[6].

3. FIELD OF FREE RADIATION INSIDE AND OUTSIDE THE CRYSTAL

We now obtain the free-radiation field $\mathbf{E}^{fr}$ inside and outside a crystal. Outside the crystal we have vacuum and the problem is trivial, viz., the scattered field of the charge is equal to zero and the free field is the field of the free electromagnetic radiation in vacuum[7]. As to the free field in the crystal, it must be determined by starting from the homogeneous equations obtained from (2) by setting the right-hand sides equal to zero and replacing $\mathbf{E}^{sc}(\mathbf{k})$ and $\mathbf{E}^{sc}(\mathbf{k}_h)$ by $\mathbf{E}^{fr}(\mathbf{k})$ and $\mathbf{E}^{fr}(\mathbf{k}_h)$. We obtain from this system a set of homogeneous equations corresponding to (3) (a_{ij} must be taken here to mean $a^{fr}_{ij} = \mathbf{k}_i \cdot \mathbf{E}^{fr}(\mathbf{k}_j)$). It is seen from these equations that a^{fr}_{00} and a^{fr}_{hh} are small in comparison with a^{fr}_{oh} and a^{fr}_{oh}, i.e., the free field in the crystal can also be regarded as transverse. We confine ourselves therefore only to the second and third equations of this system.

The system of two equations obtained in this manner can either 1) have only a trivial solution when its determinant is not equal to zero, or 2) have a nontrivial solution when its determinant is equal to zero. In the former case, in order for the system of homogeneous equations corresponding to (2), to have a nontrivial solution with respect to $\mathbf{E}^{fr}(\mathbf{k})$ and $\mathbf{E}^{fr}(\mathbf{k}_h)$, it is necessary that the determinant of the last system be equal to zero:

$$D_n = (\eta_0 - g_{00})(\eta_h - g_{hh}) - g_{0h}g_{h0} = 0. \tag{9}$$

Then, since $a^{fr}_{ij} = 0$, then $\mathbf{E}^{fr}(\mathbf{k}) \parallel \mathbf{E}^{fr}(\mathbf{k}_h) \parallel \mathbf{k} \times \mathbf{k}_h$. This is the case of the so-called "normal" polarization, for which

$$\mathbf{E}^{fr}(\mathbf{k}_h) = [(\eta_0 - g_{00})/g_{0h}]\mathbf{E}^{fr}(\mathbf{k}).$$

In the second case we have

$$D_p = (\eta_0 - g_{00})(\eta_h - g_{hh}) - \zeta^2 g_{0h}g_{h0} = 0 \tag{10}$$

Then the system of equations corresponding to (2) provides a linear connection between the vectors $\mathbf{E}^{fr}(\mathbf{k})$, $\mathbf{E}^{fr}(\mathbf{k}_h)$, $\mathbf{k}$, and $\mathbf{k}_h$, meaning that these four vectors lie in one plane. This is the case of so-called "parallel" polarization, for which $\mathbf{E}^{fr}(\mathbf{k}) \parallel (\zeta\mathbf{k} - \mathbf{k}_h)$, $\mathbf{E}^{fr}(\mathbf{k}_h) \parallel (\mathbf{k} - \zeta\mathbf{k}_h)$, and

$$|\mathbf{E}^{fr}(\mathbf{k}_h)| = |(\eta_0 - g_{00})/g_{0h}\zeta|\,|\mathbf{E}^{fr}(\mathbf{k})|.$$

The dispersion equations (9) and (10) establish the connection between $\mathbf{k}$ and ω near the Ewald sphere and have two roots relative to k_z.

As a result, the four-dimensional Fourier components of the free-radiation field in the crystal can be written in the form

$$\begin{aligned} \mathbf{E}^{fr}(\mathbf{k}) &= \mathbf{e}_n[E^{fr}_{n1}\,\delta(k_z - \lambda_{n1}) + E^{fr}_{n2}\,\delta(k_z - \lambda_{n2})] \\ &\quad + \mathbf{e}_p[E^{fr}_{p1}\,\delta(k_z - \lambda_{p1}) + E^{fr}_{p2}\,\delta(k_z - \lambda_{p2})], \end{aligned} \tag{11}$$

$$\mathbf{E}^{sc}(\mathbf{k}_h) = \mathbf{e}_n[E_{n1}^{sc}\,\delta(k_{hz} - K_{hz} - \lambda_{n1}) + E_{n2}^{sc}\,\delta(k_{hz} - K_{hz} - \lambda_{n2})](\eta_0 - g_{00})/g_{0h} + \mathbf{e}_{hp}[E_{p1}^{sc}\,\delta(k_{hz} - K_{hz} - \lambda_{p1}) + E_{p2}^{sc}\,\delta(k_{hz} - K_{hz} - \lambda_{p2})](\eta_0 - g_{00})/\zeta g_{0h}.$$

Here $\mathbf{e}_n \parallel \mathbf{k} \times \mathbf{k}_h$ is a unit vector of the "normal" polarization, $\mathbf{e}_p \parallel (\zeta\mathbf{k} - \mathbf{k}_h)$ and $\mathbf{e}_{hp} \parallel (\mathbf{k} - \zeta\mathbf{k}_h)$ are unit vectors of the "parallel" polarization, and $\mathbf{E}_{\alpha i}^{fr}$ (α = n, p, i = 1, 2) are still undetermined amplitudes that must be obtained from the boundary conditions.

The quantities $\lambda_{\alpha i}$ are the roots of the dispersion relation (9) or (10) with respect to k_z for α = n and α = p, respectively. We seek these quantities in the form

$$k_z = \lambda_{\alpha i} = (\omega/c)(1 - \Delta_{\alpha i})\cos\vartheta,$$

assuming the new sought roots $\Delta_{\alpha i}$ to be small, $|\Delta_{\alpha i}| \ll 1$. Here $\vartheta = k_\perp c/|\omega|$ is the emission angle in the central spot. We introduce the frequency ω_B ($\omega_B > 0$) and the scattering angle $2\theta_B$ ($0 < 2\theta_B \leq \pi$) corresponding to the Bragg kinematic condition

$$K_{hz} = (\omega_B/c)(\cos 2\theta_B - 1), \quad K_{\perp h} = (\omega_B/c)\sin 2\theta_B. \tag{12}$$

It is seen from the first relation that $K_{hz} < 0$. We put $|\omega| = \omega_B(1 + \nu)$, where ν is a small relative deviation of the frequency from the kinematic Bragg frequency ω_B, $|\nu| \ll 1$. Solving (9), as mentioned above, we obtain[4,8]

$$\Delta_{ni} = \{\bar{\nu} - g_{00}\cos 2\theta_B - g_{hh} \pm [(g_{00}\cos 2\theta_B - g_{hh} + \bar{\nu})^2 + 4g_{0h}g_{h0}\cos 2\theta_B]^{1/2}\}/4\cos 2\theta_B,$$
$$\bar{\nu} = -2(2\nu - \vartheta^2)\sin^2\theta_B + 2\vartheta(1 - \nu)\sin 2\theta_B\cos\phi, \tag{13}$$

and Δ_{pi} are obtained from Δ_{ni} by replacing $g_{0h}g_{h0}$ by $g_{0h}g_{h0}\cos^2 2\theta_B$. The angle ϕ in the expression for $\bar{\nu}$ is the azimuthal angle between the vectors $\mathbf{k}_\perp$ and $\mathbf{K}_{\perp h}$ in the (x, y) plane, so that

$$2\vartheta(1 - \nu)\sin 2\theta_B\cos\phi = 2\mathbf{k}_\perp\mathbf{K}_{\perp h}c^2/\omega\omega_B.$$

We note that formulas (11) and (13) are valid for all values of the angle $2\theta_B$, with the exception of values close to $\pi/2$.

4. CONDITIONS OF CONTINUITY ON THE BOUNDARIES

In the considered region of x-ray frequencies, the conditions on the boundaries can be approximately written as the conditions for the continuity of the electric field. For simplicity we consider the case of normal entry of a charged particle into a crystal. Then the continuity of the fields should take place at z = 0 and z = l and for arbitrary t and $\boldsymbol{\rho}$ (t is the time and $\boldsymbol{\rho}$ is the transverse component of the radius vector of the point on the boundary). This means that each of the waves corresponding to the vectors $\mathbf{k}$ and $\mathbf{k}_h$ should be continuous.

In addition, depending on whether k_{hz} is positive or negative, one must distinguish between two cases: 1) when the diffracted wave corresponding to the wave vector $\mathbf{k}_h$ emerges from the crystal in the direction of motion of the charged particle, or in other words when $2\theta_B < \pi/2$ (the Laue case), and 2) when the diffracted wave emerges in the opposite direction or $2\theta_B > \pi/2$ (the Bragg case). We consider these two cases separately.

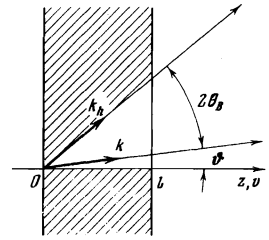

FIG. 1. The Laue case.

1) The Laue case (Fig. 1). In this case we have for the Fourier components $\mathbf{E}^{vac}(\mathbf{k})$ and $\mathbf{E}^{vac}(\mathbf{k}_h)$ of the free field in vacuum, in the region ahead of the crystal (z < 0)

$$\mathbf{E}^{vac}(\mathbf{k}) = \mathbf{E}^{vac}(\mathbf{k}_h) = 0, \tag{14}$$

and in the region behind the crystal (z > l)

$$\mathbf{E}^{vac}(\mathbf{k}) = (E_n^{vac}\mathbf{e}_n + E_p^{vac}\mathbf{e}_p)\,\delta(k_z - \lambda_0),$$
$$\mathbf{E}^{vac}(\mathbf{k}_h) = (E_{hn}^{vac}\mathbf{e}_n + E_{hp}^{vac}\mathbf{e}_{hp})\,\delta(k_{hz} - \lambda_h). \tag{15}$$

Here (see[7])

$$\lambda_0 = \frac{\omega}{c}\left(1 - \frac{k_\perp^2 c^2}{\omega^2}\right)^{1/2},$$
$$\lambda_h = \frac{\omega}{c}\left(1 - \frac{k_{\perp h}^2 c^2}{\omega^2}\right)^{1/2} \approx \frac{\omega}{|\omega|}\left|K_{hz} + \lambda_0 - \frac{\omega\bar{\nu}}{2c\cos 2\theta_B}\right|,$$

E_α^{vac} and $E_{h\alpha}^{vac}$ (α = n, p) are still unknown amplitudes.

The boundary conditions for z = 0 and z = l take the form

$$\int[\mathbf{E}^{sc}(\mathbf{k}) + \mathbf{E}^{sc}(\mathbf{k})]\,dk_z = 0, \quad \int[\mathbf{E}^{sc}(\mathbf{k}_h) + \mathbf{E}^{fr}(\mathbf{k}_h)]\,dk_{hz} = 0,$$
$$\int[\mathbf{E}^{ch}(\mathbf{k}) + \mathbf{E}^{sc}(\mathbf{k})]\exp\{ik_z l\}\,dk_z = \int\mathbf{E}^{vac}(\mathbf{k})\exp\{ik_z l\}\,dk_z,$$
$$\int[\mathbf{E}^{ch}(\mathbf{k}_h) + \mathbf{E}^{sc}(\mathbf{k}_h)]\exp\{ik_{hz} l\}\,dk_{hz} = \int\mathbf{E}^{vac}(\mathbf{k}_h)\exp\{ik_{hz} l\}\,dk_{hz}. \tag{16}$$

Substituting (11) and (15) in the conditions (16) and expanding the quantities $\mathbf{E}^{sc}(\mathbf{k})$ and $\mathbf{E}^{sc}(\mathbf{k}_h)$ defined by formulas (7) in terms of the polarizations, we solve Eqs. (16) with respect to the amplitudes $E_{\alpha i}^{fr}$, E_α^{vac}, and E_h^{vac} (α = n, p, i = 1, 2).

For the normal polarization we obtain

$$E_{n1}^{sc} = \frac{-E_n^{ch}(2\Delta_{n2} + g_{00}) - g_{0h}E_{hn}^{ch}}{2(\Delta_{n2} - \Delta_{n1})},$$
$$E_{n2}^{sc} = \frac{E_n^{ch}(2\Delta_{n1} + g_{00}) + g_{0h}E_{hn}^{ch}}{2(\Delta_{n2} - \Delta_{n1})}$$

and

$$E_n^{vac} = E_n^{ch}\exp\left\{i\left(\frac{\omega}{v} - \lambda_0\right)l\right\} + E_{n1}^{ch}\exp\left\{-\frac{i\omega l\Delta_{n1}}{c}\right\} + E_{n2}^{ch}\exp\left\{-\frac{i\omega l\Delta_{n2}}{c}\right\},$$
$$E_{hn}^{vac} = \left[E_{hn}^{sc}\exp\left\{i\left(\frac{\omega}{v} - \lambda_0\right)l\right\} - E_{n1}^{sc}\exp\left\{-\frac{i\omega l\Delta_{n1}}{c}\right\}\frac{2\Delta_{n1} + g_{00}}{g_{0h}} - E_{n2}^{sc}\exp\left\{-\frac{i\omega l\Delta_{n2}}{c}\right\}\frac{2\Delta_{n2} + g_{00}}{g_{0h}}\right]\exp\left\{-\frac{i\omega l\bar{\nu}}{2c\cos 2\theta_B}\right\}, \tag{18}$$

where E_n^{sc} and E_{hn}^{sc} are the amplitudes of the scattered field of the charge in the $\mathbf{e}_n$ direction and are determined by the formulas

$$E_n^{sc} = \frac{8\pi^2 ei\mathbf{v}[\mathbf{k}\mathbf{k}_h]}{v\omega\,|[\mathbf{k}\mathbf{k}_h]|}\,\frac{g_{00}(\tilde{\eta}_h - g_{hh}) + g_{0h}g_{h0}}{\tilde{\eta}_0 \tilde{D}_n},$$

$$E_{hn}^{sc} = \frac{8\pi^2 ei\mathbf{v}[\mathbf{k}\mathbf{k}_h]}{v\omega\,|[\mathbf{k}\mathbf{k}_h]|}\,\frac{g_{h0}}{\tilde{D}_n}. \qquad (19)^*$$

The quantities η_0, $\tilde{\eta}_h$, and $\tilde{D}_n$ are obtained here from η_0, η_h and D_n respectively by replacing k_z by ω/v, so that

$$\tilde{\eta}_0 = \vartheta^2 + 1 - \beta^2, \quad \tilde{\eta}_h = \vartheta^2 + (1-\beta^2)\cos 2\theta_B - 4\nu \sin^2\theta_B + 2\vartheta \sin 2\theta_B \cos\phi. \qquad (20)$$

The same replacement of k_z by ω/v should be made also in the vectors $\mathbf{k}$ and $\mathbf{k}_h$ of (19).

For parallel polarization, the amplitudes of the free fields inside and outside the crystals, E_{pi}^{fr}, E_p^{vac}, and E_{hp}^{vac} can be obtained from (17) and (18) by replacing the index n in all the quantities by the index p, and by replacing g_{0h} by $g_{0h}\cos 2\theta_B$. The quantities E_p^{sc} and E_{hp}^{sc} are defined here by

$$E_p^{sc} = \frac{\tilde{q}_0\tilde{\zeta}[g_{00}(\tilde{\eta}_h - g_{hh}) + g_{0h}g_{h0}] - \tilde{q}_h[g_{00}(\tilde{\eta}_h - g_{hh}) + \tilde{\zeta}^2 g_{0h}g_{h0}]}{|\zeta\mathbf{k} - \mathbf{k}_n|\,\tilde{D}_p}, \qquad (21)$$

$$E_{hp}^{sc} = \frac{g_{h0}\{\tilde{q}_0[\tilde{\eta}_0 - g_{00}(1-\tilde{\zeta}^2)] - \tilde{\zeta}\tilde{\eta}_0\tilde{q}_h\}}{|\mathbf{k} - \tilde{\zeta}\mathbf{k}_h|\,\tilde{D}_p},$$

where $\tilde{q}_0$ and $\tilde{q}_h$ are defined in (6), $\tilde{D}_p$ is obtained from D_p (see (5)) by replacing η_0 by $\tilde{\eta}_0$ and η_h by $\tilde{\eta}_h$, and $\tilde{\zeta}$ is obtained from ζ by replacing k_z by ω/v. The last substitution should be made also in the tectors $\mathbf{k}$ and $\mathbf{k}_h$.

2) The Bragg case (Fig. 2). In this case the Fourier components of the free field in vacuum take the following form in the region ahead of the crystal

$$\mathbf{E}^{vac}(\mathbf{k}) = 0, \quad \mathbf{E}^{vac}(\mathbf{k}_h) = (E_{hn}^{vac}\mathbf{e}_n + E_{hp}^{vac}\mathbf{e}_{hp})\,\delta(k_{hz} + \lambda_h), \qquad (22)$$

and in the region behind the crystal

$$\mathbf{E}^{vac}(\mathbf{k}_h) = 0, \quad \mathbf{E}^{vac}(\mathbf{k}) = (E_n^{vac}\mathbf{e}_n + E_p^{vac}\mathbf{e}_p)\,\delta(k_z - \lambda_0). \qquad (23)$$

The boundary conditions at z = 0 and z = l are

$$\int [\mathbf{E}^{sc}(\mathbf{k}) + \mathbf{E}^{fr}(\mathbf{k})]\,dk_z = 0,$$

$$\int [\mathbf{E}^{sc}(\mathbf{k}_h) + \mathbf{E}^{fr}(\mathbf{k}_h)]\,dk_{hz} = \int \mathbf{E}^{vac}(\mathbf{k}_h)\,dk_{hz},$$

$$\int [\mathbf{E}^{sc}(\mathbf{k}_h) + \mathbf{E}^{fr}(\mathbf{k}_h)]\exp\{ik_{hz}l\}\,dk_{hz} = 0, \qquad (24)$$

$$\int [\mathbf{E}^{sc}(\mathbf{k}) + \mathbf{E}^{fr}(\mathbf{k})]\exp\{ik_z l\}\,dk_z = \int \mathbf{E}^{vac}(\mathbf{k})\exp\{ik_z l\}\,dk_z.$$

Solving the equations in (24) in analogy with the Laue case, we obtain for the normal polarization

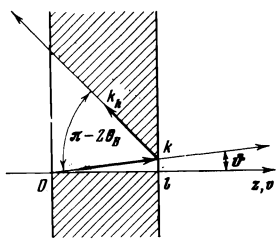

FIG. 2. The Bragg case.

$$E_{n1}^{fr} = \left[-E_n^{ch}(2\Delta_{n2} + g_{00})\exp\left\{-\frac{i\omega l\Delta_{n2}}{c}\right\} - E_{hn}^{ch}g_{0h}\exp\left\{i\left(\frac{\omega}{v} - \lambda_0\right)l\right\}\right]$$
$$\times\left[(2\Delta_{n2} + g_{00})\exp\left\{-\frac{i\omega l\Delta_{n2}}{c}\right\} - (2\Delta_{n1} + g_{00})\exp\left\{-\frac{i\omega l\Delta_{n1}}{c}\right\}\right]^{-1},$$

$$E_{n2}^{fr} = \left[E_n^{ch}(2\Delta_{n1} + g_{00})\exp\left\{-\frac{i\omega l\Delta_{n1}}{c}\right\} + E_{hn}^{ch}g_{0h}\exp\left\{i\left(\frac{\omega}{v} - \lambda_0\right)i\right\}\right]$$
$$\times\left[(2\Delta_{n2} + g_{00})\exp\left\{-\frac{i\omega l\Delta_{n2}}{c}\right\} - (2\Delta_{n1} + g_{00})\exp\left\{-\frac{i\omega l\Delta_{n1}}{c}\right\}\right]^{-1}; \qquad (25)$$

$$E_n^{vac} = E_n^{ch}\exp\left\{i\left(\frac{\omega}{v} - \lambda_0\right)l\right\} + E_{n1}^{fr}\exp\left\{-\frac{i\omega l\Delta_{n1}}{c}\right\} + E_{n2}^{fr}\exp\left\{-\frac{i\omega l\Delta_{n2}}{c}\right\},$$

$$E_{hn}^{vac} = E_{hn}^{ch} - \frac{2\Delta_{n1} + g_{00}}{g_{0h}}E_{n1}^{fr} - \frac{2\Delta_{n2} + g_{00}}{g_{0h}}E_{n2}^{fr}, \qquad (26)$$

where E_n^{sc} and E_{hn}^{sc} are determined by the same formulas (19). As to the parallel polarization, the amplitudes E_{pi}^{fr}, E_p^{vac}, and E_{hp}^{vac}, just as in the Laue case, are obtained from (25) and (26) by replacing the subscript n with p and g_{0h} with $g_{0h}\cos 2\theta_B$. The quantities E_p^{sc} and E_{hp}^{sc} resulting from this replacement are determined by formulas (21).

Substituting $\mathbf{E}^{vac}$ and $E_{h\alpha}^{vac}$ in formulas (15), (22), and (23) we obtain the Fourier components $\mathbf{E}^{vac}(\mathbf{k})$ and $\mathbf{E}^{vac}(\mathbf{k}_h)$, which determine the radiation fields in the central and lateral spots, respectively.

5. DISCUSSION OF RESULTS

As already shown at the end of Sec. 2, we have $|\mathbf{E}^{sc}(\mathbf{k}_h) \ll \mathbf{E}^{sc}(\mathbf{k})|$ far from the Bragg frequencies, and the field of the charge in the crystal reduces to the field of the charge in matter determined by the usual formula of macroscopic electrodynamics[6]. This means, in particular, that far from the Bragg frequencies the free radiation $\mathbf{E}^{vac}(\mathbf{k}_h)$ in vacuum for the lateral spot is small, and for the central spot $\mathbf{E}^{vac}(\mathbf{k})$ is determined by the usual transition-radiation formula that follows from the macroscopic theory that does not take the crystalline structure of matter into account[9]. The foregoing can be seen also from formulas (18) and (26). Indeed, far from the Bragg frequencies, when $|\tilde{\nu}| \gg |g_{ij}|$, we obtain from (13)

$$\Delta_{\alpha 2} \approx -g_{00}/2, \quad \Delta_{\alpha 1} \approx \vartheta/2\cos 2\theta_B,$$

i.e., $|\Delta_{\alpha 1}| \gg |\Delta_{\alpha 2}|$. It follows then from (17) and (25) that in both cases we have $|E_{\alpha 1}^{fr}| \ll |E_{\alpha 2}^{fr}|$ and $E_{\alpha 2}^{fr} = -E_{\alpha}^{sc}$, whereas from (18) and (26) it follows that $|E_{h\alpha}^{vac}| \ll |E_{\alpha}^{vac}|$ and

$$E_{\alpha}^{vac} = E_{\alpha}^{ch}\left[\exp\left\{i\left(\frac{\omega}{v} - \lambda_0\right)l\right\} - \exp\left\{\frac{i\omega l g_{00}}{2c}\right\}\right].$$

Adding the two polarizations E_n^{vac} and E_p^{vac} in accordance with (15), we obtain $\mathbf{E}^{vac}(\mathbf{k})$, and simple calculations show that the expression obtained in this manner for $\mathbf{E}^{vac}(\mathbf{k})$ coincides exactly with the transition-radiation formula that follows from the macroscopic theory[9].

Thus, deviations from the macroscopic formula can occur only for frequencies ω near the Bragg frequencies

$$\omega_B = cK_h^2/2|K_{hz}|. \qquad (27)$$

*$[\mathbf{k}\mathbf{k}_h] \equiv \mathbf{k} \times \mathbf{k}_h$.

Taking different reciprocal-lattice vectors $\mathbf{K}_h$ (multiplied by 2π), we obtain a set of Bragg frequencies corresponding to different Bragg angles

$$\theta_B = \operatorname{arctg}(|K_{hz}| / K_{\perp h}). \tag{28}$$

depending on whether $2\theta_B$ is smaller or larger than $\pi/2$, the radiation will be determined by the formulas for the Laue or Bragg case, respectively.

As seen from (17), (18), (25), and (26), the free radiation fields both inside and outside the crystal are expressed in final analysis in terms of the amplitude of the scattered charge field E_α^{sc} and $E_{h\alpha}^{sc}$. We therefore consider these quantities first.

We examine first separately the case $K_{\perp h} = 0$, i.e., $\sin 2\theta_B = 0$, $\cos 2\theta_B = -1$. This takes place if the Bragg reflection is directed almost exactly backward, when the particle moves perpendicular to some family of crystal planes. We then have from (19) $E_n^{sc} = E_{hn}^{sc} = 0$, and from (21) and (6) we get

$$E_p^{ch} = 8\pi^2 eic^2 k_\perp [g_{00}(\tilde{\eta}_h - g_{hh}) + g_{0h}g_{h0}]/\tilde{\eta}_0 v\omega^2 \tilde{D}_p,$$
$$E_{hp}^{pac} = -8\pi^2 eic^2 k_\perp g_{h0}/v\omega^2 \tilde{D}_p. \tag{29}$$

In the limit when $g_{00} = g_{hh} = g_{0h} = g_{h0}$ (model of pointlike and immobile atoms), we can readily see after some manipulations that the formulas for the free radiation E_p^{vac} and E_{hp}^{vac}, which are obtained from (26) and (25) (after replacing n by p and g_{0h} by $g_{0h}\cos 2\theta_B$), with (29) taken into account, coincide with the corresponding formulas of our paper[2], obtained by the integral-equation method in the same model. It should be borne in mind here that in[2] the Fourier components $\mathbf{E}_{sc}(\mathbf{k}, \omega)$ were defined for the entire space, and the quantity $E_{sc}(z)$ (see formula (19) of[2]) outside the crystal described free radiation. In the present paper, on the other hand, $\mathbf{E}^{sc}(\mathbf{k})$ is defined only for space inside the crystal, and outside the crystal the Fourier components of the free-radiation fields are obtained from the conditions that the one field be the continuation of the other and are designated $\mathbf{E}^{vac}(\mathbf{k})$ and $\mathbf{E}^{vac}(\mathbf{k}_h)$. In the comparison it is therefore necessary to substitute these quantities, which are defined by formulas (22) and (23) of the present paper, into formula (19) of[2]. Then the quantity $(-E_p^{vac}/2\pi)\exp(i\lambda_0 z)$ for forward radiation should coincide with the quantity $E_{sc}(z)$ determined by formula (41) in[2], and for the backward radiation $(E_{hp}^{vac}/2\pi)\exp(-i\lambda_0 z)$ should coincide with $E_{sc}(z)$ of formula (45) of the same paper (the signs of the quantities differ in the first case and are equal in the second because at $\zeta = -1$ we have $\mathbf{e}_p = -\mathbf{k}_\perp/k_\perp$ and $\mathbf{e}_{hp} = \mathbf{k}_\perp/k_\perp$). As noted above, such an agreement is indeed obtained.

We consider now the case of arbitrary $K_{\perp h}$. The quantities $\tilde{D}_n$ and $\tilde{D}_p$ in the denominators of (19) and (21) take, as functions of ν, the form

$$\tilde{D}_\alpha = -4(\tilde{\eta}_0 - g_{00})(\nu - \nu_\alpha' - i\nu_\alpha'')\sin^2\theta_B, \tag{30}$$

where for the normal polarization (α = n)

$$\nu_n' = \{\vartheta^2 + (1-\beta^2)\cos 2\theta_B + 2\vartheta \sin 2\theta_B \cos\phi - \operatorname{Re}[g_{hh} + g_{0h}g_{h0}/(\tilde{\eta}_0 - g_{00})]\}/4\sin^2\theta_B,$$
$$\nu_n'' = -\operatorname{Im}[g_{hh} + g_{0h}g_{h0}/(\tilde{\eta}_0 - g_{00})]/4\sin^2\theta_B, \tag{31}$$

and for the parallel polarization (α = p) the formulas are similar to (31), except that $g_{0h}g_{h0}$ is replaced by $g_{0h}g_{h0}\cos^2 2\theta_B$.

It is seen from (30) that at $\nu = \nu_\alpha'$ the quantities $|\mathbf{E}^{sc}|$ and $|\mathbf{E}_{h\alpha}^{sc}|$ reach maxima with widths in ν on the order of $|\nu_\alpha''|$. Since the imaginary parts of the g_{ij} are much smaller than their real parts, it follows from (31) that $|\nu_\alpha''| \ll |\nu_\alpha'| \ll 1$, i.e., these maxima are quite narrow. Substituting ν' for ν in (20) and then in (19) and (21), we obtain expressions for E_α^{sc} and $E_{h\alpha}^{sc}$ at the maxima. It is easily seen then that the ratio of the amplitudes of $|E_\alpha^{sc}|$ and $|E_{h\alpha}^{sc}|$ at the maximum and away from the ma maximum is of the same order as the ratio of the real and imaginary parts of the g_{ij}, i.e., the indicated maxima are not only quite narrow but also quite high.

The total energy flux passing through a plane z = const that is parallel to the crystal boundaries and located far enough away from the crystal is determined for the central and lateral spots, respectively, by the formulas

$$W = \frac{c}{(2\pi)^6}\int \{|E_n^{vac}|^2 + |E_p^{vac}|^2\}\, d\mathbf{k}_\perp\, d\omega, \tag{32}$$

$$W_h = \frac{c}{(2\pi)^6}\int \{|E_{hn}^{vac}|^2 + |E_{hp}^{vac}|^2\}\, |\cos 2\theta_B|\, d\mathbf{k}_{\perp h}\, d\omega \tag{33}$$

($\mathbf{k}_{\perp h} = \mathbf{k}_\perp + \mathbf{K}_{\perp h}$ is the transverse component of the vector $\mathbf{k}_h$). From now on the integration in the formulas for W and W_h is over positive frequencies ($\omega > 0$).

We consider first the limiting cases of a thin and thick crystal. In the case of a thin crystal, when

$$|\omega l \Delta_{\alpha i}/c| \ll 1 \tag{34}$$

for all values of $\mathbf{k}_\perp$ (or $\mathbf{k}_{\perp h}$) and ν in a given spot, meaning (see (133) generally speaking a simultaneous satisfaction of the conditions

$$|\omega l g_{ij}/c| \ll 1, \quad |\omega l \nu/c| \ll 1, \tag{35}$$

and

$$|\omega l(1-\beta^2)^{1/2}/c| \ll 1 \text{ for } \mathbf{K}_{\perp h} \neq 0,$$
$$|\omega l(1-\beta^2)/c| \ll 1 \text{ for } \mathbf{K}_{\perp h} = 0, \tag{36}$$

we obtain from (18) and (26)

$$E_\alpha^{vac} = 4\pi^2 elg_{00}k_\alpha/\omega\tilde{\eta}_0 \tag{37}$$

and

$$E_{hn}^{vac} = 4\pi^2 elg_{h0}k_{hx}/|\cos 2\theta_B|\,\omega\tilde{\eta}_0,$$
$$E_{hp}^{vac} = 4\pi^2 elg_{h0}(k_{hy} + K_{\perp h})\cos 2\theta_B/|\cos 2\theta_B|\,\omega\tilde{\eta}_0, \tag{38}$$

both in the Laue and in the Bragg case. In (37), in view of the choice of the y-axis direction, we have $k_\alpha = k_x$ at α = n and $k_\alpha = k_y$ at α = p.

Substituting (37) in (32) we find that in the considered case of a thin crystal the formula for the energy flux in the central spot coincides with formula (29) from[1], obtained by solving the integral equation by the iteration method.

For the lateral spot we have according to (33) and (38)

$$W_h = \frac{e^2 l^2}{4\pi^2 c^3}\int \omega^2 |g_{h0}|^2\, d\omega \int \frac{[\sin^2\phi + \cos^2\phi\cos^2 2\theta_B]\vartheta^3 d\vartheta\, d\phi}{(\vartheta^2 + 1 - \beta^2)^2}, \tag{39}$$

where ϕ is the azimuthal angle between the vectors $\mathbf{k}_{\perp h} - \mathbf{K}_{\perp h}$ and $\mathbf{K}_{\perp h}$, and ϑ is the angle of the radiation

in the lateral spot and is equal to the angle between the vector $\mathbf{k}_h$ and the "kinematic" Bragg direction $\{0, -\sin 2\theta_B, \cos 2\theta_B\}$.

It is clearly seen from (39) that the angle width ϑ_{max} of the lateral spot is of the order of $(1 - \beta^2)^{1/2}$ (see[1]). As to the frequency spectrum of the radiation, it is determined in this case by the thinness condition (34), or more accurately by the second condition of (35). At large frequency deviations, formulas (38) cease to hold and it is necessary to use the complete expression (18) or (26) for $E_{h\alpha}^{vac}$. Here, inasmuch as the first condition of (35) remains in focer, we have $|\nu| \gg |g_{ij}|$ and, as indicated at the beginning of the present section, the intensity of the lateral spot will be low.

We consider now the case of a thick crystal, when the conditions

$$|\omega l \operatorname{Im} \Delta_{\alpha i} / c| \gg 1 \tag{40}$$

are satisfied. Again, as in the preceding section, we consider separately the Laue and Bragg cases.

1) Laue case. As seen from (13), since $\cos 2\theta_B > 0$ and the g_{ij} have small positive imaginary parts, we have

$$\operatorname{Im} \Delta_{\alpha 1} < \operatorname{Im} \Delta_{\alpha 2} < 0. \tag{41}$$

Consequently, when the conditions (40) are satisfied, we obtain from (18)

$$E_n^{vac} = E_n^{sc} \exp\{i(\omega/v - \lambda_0)l\}, \quad E_{hn}^{vac} = E_{hn}^{sc} \exp\{i(\omega/v - \lambda_0)l\}$$

and analogous expressions with n replaced by p for E_p^{vac} and E_{hp}^{vac}.

Substituting the obtained expressions in (32) and (33) we find

$$W = \frac{c}{(2\pi)^6} \int \{|E_n^{sc}|^2 + |E_p^{sc}|^2\} d\mathbf{k}_\perp d\omega,$$
$$W_h = \frac{c}{(2\pi)^6} \int \{|E_{hn}^{sc}|^2 + |E_{hp}^{sc}|^2\} |\cos 2\theta_B| d\mathbf{k}_{\perp h} d\omega. \tag{42}$$

As shown above, the quantities $|E_\alpha^{sc}|$ and $|E_{h\alpha}^{sc}|$ have quite narrow and high maxima at frequencies $|\omega_\alpha| = \omega_B(1 + \nu'_\alpha)$ close to the kinematic Bragg frequencies ω_B and called by us in[2] "dynamic" Bragg frequencies. These high and narrow maxima ("dynamic" maxima) exist also in the energy flux (42). Substituting (19) and (21) in (42) and considering the case $K_{\perp h} c/\omega \gg (1 - \beta^2)^{1/2}$, we obtain

$$W = \frac{e^2}{\pi^2 c} \int \left\{ \left| \frac{g_{00}(\tilde{\eta}_h - g_{hh}) + g_{0h} g_{h0}}{\tilde{D}_n} \right|^2 \sin^2\phi + \left| \frac{g_{00}(\tilde{\eta}_h - g_{hh}) + g_{0h} g_{h0} \cos^2 2\theta_B}{\tilde{D}_p} \right|^2 \cos^2\phi \right\} \frac{\vartheta^3 d\vartheta\, d\phi}{\tilde{\eta}_0^2} d\omega,$$
$$W_h = \frac{e^2}{\pi^2 c} \int |g_{h0}|^2 \left\{ \frac{\sin^2\phi}{|\tilde{D}_n|^2} + \frac{\cos^2 2\theta_B \cos^2\phi}{|\tilde{D}_p|^2} \right\} \cos 2\theta_B\, \vartheta^3 d\vartheta\, d\phi\, d\omega. \tag{43}$$

As seen from these formulas, the energy fluxes W and W_h, given the values of ϑ and ϕ, have in general two dynamic maxima corresponding to the minima of the denominators of $|\tilde{D}_n|^2$ and $|\tilde{D}_p|^2$. The relative widths $\Delta\omega/\omega_B$ of these maxima, as already mentioned, are of the order of ν''_n and ν''_p, respectively, and the (relative) distance between them, as seen from (31), is of the order of $\operatorname{Re}[g_{0h}g_{h0}/(\tilde{\eta}_0 - g_{00})]$. The positions of these maxima depend in turn on ϑ and ϕ (see (31)). The indicated maxima are therefore superimposed in the frequency spectrum integrated with respect to ϑ from zero to $\vartheta_{max} \sim (1 - \beta^2)^{1/2}$ and with respect to ϕ from zero to 2π, and the resultant curve of the frequency spectrum may not have a simple form.

2) Bragg case. We obtain from (13)

$$\operatorname{Im} \Delta_{\alpha 1} + \operatorname{Im} \Delta_{\alpha 2} = -(g_{00}'' \cos 2\theta_B + g_{hh}'')/4\cos 2\theta_B \geqslant 0$$

(since $\cos 2\theta_B < 0$ and $g''_{00} \approx g''_{hh} > 0$) and

$$\operatorname{Im} \Delta_{\alpha 2} < 0 < \operatorname{Im} \Delta_{\alpha 1}. \tag{44}$$

It follows then from (26), with allowance for (25), that

$$E_n^{vac} = [E_n^{sc} + E_{hn}^{sc}\, g_{0h}/(2\Delta_{n1} + g_{00})] \exp\{i(\omega/v - \lambda_0)l\},$$
$$E_{hn}^{vac} = E_{hn}^{sc} + (2\Delta_{n2} + g_{00}) E_n / g_{0h} \tag{45}$$

with analogous expressions obtained for E_p^{vac} and E_{hp}^{vac} from (45) by replacing n by p and g_{0h} by $g_{0h} \cos 2\theta_B$. Substituting the obtained expressions in (32) and (33), we obtain formulas for the energy fluxes W and W_h in the Bragg case. Just as in the Laue case, the frequency spectrum of the lateral spots contains narrow and high dynamic maxima. These maxima are not present, however, in the central spot.

We consider finally a crystal of intermediate thickness, when it is necessary to use the general formulas for E_α^{vac} and $E_{h\alpha}^{vac}$ (18) or (26) with allowance for formulas (17) or (25) and (19), (21). Usually one obtains then complicated interference patterns that depend on the concrete crystal characteristics g_{ij}. Without presenting a detailed analysis of the angular and frequency spectra of the radiation in this case, we make only a few remarks.

1) Laue case. As already noted, in the Laue case we obtain the inequalities (41). The quantity $|\operatorname{Im} \Delta_{\alpha 2}|$, as seen from (13), can be here quite small when $|\tilde{\nu}| \ll |g_{ij}|$ if the value of $|g_{0h}| \approx |g_{h0}|$ is close to $|g_{00}| \approx |g_{hh}|$. This means that the term containing $\exp\{-i\omega l \Delta_{\alpha 2}/c\}$ in the radiation-field amplitude (18) attenuates quite weakly with increasing l.

The presence of a weakly damped electromagnetic wave in the crystal near the Bragg frequency is known as "anomalous passage" or the "Bormann effect"[10]. When a charged particle passes through a crystal dynamic effects occur near the Bragg frequencies and the appearance of an "anomalously passing part" in the produced radiation is valid.

We see from (10) that the radiation both in the central spot and in the lateral ones is the result of a superposition of three wave fields, the scattered field of the charge and two free fields emerging from the crystal. Each of these fields has its own phase factor that depends in an oscillatory manner on the crystal thickness l. The nature of the oscillatory dependences of the radiation intensities on the crystal thickness l is the same as in the well known "pendulum effect" (see, e.g.,[11]). In our case, however, this phenomenon is made more complicated by the presence of a scattered charge field.

It should be added to the foregoing that all the phase factors depend on the angle ϑ, and the $\Delta_{\alpha i}$ depend also on the angle ϕ, the polarization α, and the relative frequency deviation ν. In the general case, therefore, the dependence of the intensities of the dynamic maxima on l can be quite complicated.

2) Bragg case. The remarks made above concerning the Laue case are valid in principle also for the Bragg

case. Now, however, the anomalously passing wave is missing from the lateral spot.

It should also be noted in addition that near the crystal boundaries the free radiation field interferes also with the proper field $\mathbf{E}_{ch}$ of the charge. Owing to the difference between the phase velocities of these fields, the region where these fields interfere is bounded to the "formation zone"[7]. In vacuum, for the central spot, the formation zone has a dimension of the order $c/\omega(1-\beta^2)$, and for the lateral spots it is of the order of $c/\omega(1-\cos 2\theta_B) \sim c/\omega$, i.e., it is much less (at $2\theta_B \neq \pi$).

Let us estimate the height and width of the aforementioned dynamic maxima in the case of scattering by electrons. For example, in the case of beryllium, for quanta of energy 6—20 keV, the ratio of the radiation-field amplitudes at the maximum and off maximum is of the order of $(1-5)\times 10^3$, while the width of the maximum at the specified values of ϑ and ϕ is of the order of $10^{-4}-10^{-5}$ eV.

We note finally that the entire calculation presented above can be used also in the case when the scattering is from nucleons of the nuclei. To this end, it suffices to use the corresponding expressions for the g_{ij}.

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