ELECTROMAGNETIC RADIATION IN A MOVING MEDIUM

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The radiation intensity has been determined from an arbitrary source in an unbounded medium moving with a velocity greater than the velocity of light in the medium. All types of waves propagating in the moving medium are discussed, and drag on the light by the medium is taken into account when the latter is moving faster than the velocity of light.

WHEN a medium is moving faster than the velocity of light, in addition to ordinary electromagnetic waves with an index of refraction N_1 there appears a second type of electromagnetic waves with an index of refraction N_2. The latter arise as the result of drag on the light by the moving medium. The regions of propagation of waves with refractive indices N_1 and N_2 are two coaxial cones whose solid angles face the direction of motion of the medium.

The appearance of the new waves substantially affects the radiation of an arbitrary source around which the moving medium flows. If the velocity of the medium relative to the source is greater than the phase velocity of light in the medium at rest, the radiation intensity depends strongly on the relative velocity of this motion and the structure of the source. Here a multipole expansion of the radiation field, as a rule, loses its meaning. The motion of the medium also affects the angular distribution of the radiation intensity.

According to the Umov-Poynting theorem[1,2] the rate of change of electromagnetic energy in an unbounded moving medium is equal to the power expended by external currents[1)],

$$d\mathscr{E}/dt = -\int J_\alpha(\mathbf{x},t)E_\alpha(\mathbf{x},t)\,dV, \tag{1}$$

where $\mathbf{E}$ is the intensity of the electric field produced by external currents with volume density $\mathbf{J}$. For a monochromatic source

$$J_i(\mathbf{x},t) = j_i(\mathbf{x},\omega)e^{i\omega t} + j_i^*(\mathbf{x},\omega)e^{-i\omega t}$$

the radiation intensity (1) averaged over a period is

$$\frac{d\mathscr{E}}{dt} = -\frac{\omega}{4\pi^3}\operatorname{Im}\int j_i^*(\mathbf{k},\omega)j_m(\mathbf{k},\omega)L_{im}(\mathbf{k},\omega)\,d\mathbf{k}. \tag{2}$$

In the case of an isotropic medium moving with velocity $\mathbf{v}$, the Green's function L_{im} takes the form

$$L_{im} = \frac{4\pi}{c^2}\frac{u_iu_m\varkappa/(\varkappa+1) - g_{im} + K_{im}}{k_n^2 + \varkappa(k_nu_n)^2 + i\delta},$$
$$K_{im} = \frac{1}{\varkappa+1}\left(\frac{k_iu_m + k_mu_i}{k_nu_n} - \frac{k_ik_m}{(k_nu_n)^2}\right), \tag{3}$$

where $\kappa = \epsilon - 1$, ϵ is the dielectric permittivity of the medium at rest, c is the velocity of light in vacuum, k_i is the 4-dimensional wave vector, g_{im} is the metric tensor, and u_i is the 4-velocity of the medium. The latter two quantities are determined by the relations

$$g_{00} = 1, \quad g_{\alpha\beta} = -\delta_{\alpha\beta}, \quad g_{0\alpha} = g_{\alpha 0} = 0;$$
$$u_0 = \frac{1}{(1-v^2/c^2)^{1/2}}, \quad u_\alpha = \frac{v_\alpha}{c(1-v^2/c^2)^{1/2}}.$$

The dielectric permittivity ϵ describes the electric and magnetic properties, and therefore the magnetic permeability μ of the medium at rest is set equal to unity[1]. The bypassing of the poles in the Green's function (3) is determined by the infinitely small imaginary term $i\delta$. For $\omega > 0$ we have $\delta < 0$. The value of δ changes sign together with the frequency ω. The frequency distribution of the energy $d\mathscr{E}(\omega)$ radiated during the entire time of action of the arbitrary source $J_i(\mathbf{x}, t)$ is given by the right-hand side of Eq. (2), multiplied by $d\omega/2\pi$. In this case the quantity $j_i(\mathbf{k}, \omega)$ in the variable arguments $\mathbf{k}$ and ω is the Fourier component of the external four-dimensional current $j_i(\mathbf{x}, t)$.

Equation (2) is the relativistic generalization of the radiation intensity in a medium at rest[3]. The integrand in Eq. (2) is simplified if we use the charge conservation law $k_ij_i(\mathbf{k}, \omega) = 0$.

We will write the denominator of the Green's function (3) in the form

$$k_n^2 + \varkappa(k_nu_n)^2 = (kc - \omega N_{\mathbf{k}\omega})(kc + \omega N_{-\mathbf{k}\omega})[\varkappa(\mathbf{ku}/k)^2 - 1],$$

where we have introduced the designation

$$N_{\mathbf{k}\omega} = \frac{[1 + \varkappa u_0^2 - \varkappa(\mathbf{ku}/k)^2]^{1/2} - \varkappa u_0(\mathbf{ku}/k)}{1 - \varkappa(\mathbf{ku}/k)^2},$$
$$\varkappa = \varepsilon - 1 \equiv N'^2 - 1. \tag{4}$$

In the presence of dispersion the refractive index of the medium at rest $N' = N'(\mathbf{k}', \omega')$ depends on the wave vector $\mathbf{k}'$ and the frequency ω' of the radiation in the medium at rest. The quantities $\mathbf{k}'$ and ω' are expressed in terms of the wave vector $\mathbf{k}$ and frequency ω in the moving medium by the Lorentz transformation equations for the wave 4-vector.

The first pole

$$kc - \omega N_{\mathbf{k}\omega} = 0 \tag{5}$$

of the Green's function (3) in the moving medium describes radiation of direct waves with vector $\mathbf{k}$ and refractive index $N_1 = N_1(\theta)$, where

$$N_1(\theta) = \frac{[1 + \varkappa u_0^2(1 - \beta^2\cos^2\theta)]^{1/2} - \varkappa u_0^2\beta\cos\theta}{1 - \varkappa u_0^2\beta^2\cos^2\theta}. \tag{6}$$

[1)]The Greek vector indices take on values 1, 2, and 3, and the Latin indices values 0, 1, 2, and 3. The index 0 denotes the time component of the 4-vector. The following summation rule is adopted: $a_\alpha b_\alpha = a_1b_1 + a_2b_2 + a_3b_3$, $a_ib_i = a_0b_0 - a_\alpha b_\alpha$.

Here $\cos\theta = \mathbf{k}\cdot\mathbf{v}/kv$ and $\beta = v/c$. The refractive index N' of the medium at rest, which enters into κ, is taken for the corresponding fixed direction, if the medium is dispersive or anisotropic. At the point of the pole the function (4) is equal to the refractive index (6) of the moving medium, so that Eq. (5) goes over to the form $kc = \omega N_1$.

If $\epsilon > 1$ and the velocity v of motion of the medium is less than the critical velocity $v^2 < c^2/\epsilon$, then the refractive index (6) is finite and positive. For velocities greater than light $v^2 > c^2/\epsilon$ there exist directions of the wave vector $\mathbf{k}$ for which $N_1 = \infty$, if we neglect the dispersion of the medium at rest. The angle θ_0 for which $N_1(\theta_0) = \infty$ is $\theta_0 = \arccos(-1/\sqrt{\kappa}u_0\beta)$. In this case the refractive index (6) is positive in the range of angles $\theta_0 > \theta \geq 0$.

The second pole

$$kc + \omega N_{-\mathbf{k}\omega} = 0 \tag{7}$$

of the Green's function (3) refers to backward waves, if $v^2 < c^2/\epsilon$. The second pole (7) contributes to the radiation intensity (2) for a velocity of the medium greater than light $v^2 > c^2/\epsilon$. In this case it describes the radiation of direct waves with wave vector $\mathbf{k}$ and refractive index

$$N_2 = -N_1(\pi - \theta), \tag{8}$$

where the function $N_1(\theta)$ for any θ is given by Eq. (6), and the refractive index $N_2 = N_2(\theta)$ is determined and positive in the region $\pi - \theta_0 > \theta \geq 0$. Waves with refractive index (8) have anomalous Doppler frequencies[4,5]. At the pole Eq. (7) becomes $kc = \omega N_2$.

After the remarks which have been made, the radiation intensity of transverse waves (2) is easy to convert to a form convenient for applications,

$$\frac{d\mathscr{E}}{dt} = \int \frac{\omega^2 N_1^2(\varkappa|j_i u_i|^2/(\varkappa+1) - |j_i|^2)}{c^3[1+\varkappa u_0^2(1-\beta^2\cos^2\theta)]^{1/2}} \sin\theta\, d\theta + \int \frac{\omega^2 N_2^2(\varkappa|j_i u_i|^2/(\varkappa+1) - |j_i|^2)}{c^3[1+\varkappa u_0^2(1-\beta^2\cos^2\theta)]^{1/2}} \sin\theta\, d\theta, \tag{9}$$

where θ is the angle between $\mathbf{k}$ and $\mathbf{v}$. The modulus of the wave vector in $j_i = j_i(\mathbf{k}, \omega)$ is $k = \omega N_1/c$ in the first interval and $k = \omega N_2/c$ in the second. The integration is performed over the region of angles where the refractive indices N_1 and N_2 of the moving medium are determined and positive. The second term in Eq. (9) arises only for motion of the medium faster than light.

In the presence of dispersion in the medium at rest, it is necessary to represent the expression $\kappa = N'^2 - 1$ as a function of the angle of integration θ, using for this the formulas for the aberration of light. In this case inclusion of dispersion in the refractive index N' of the medium at rest removes the infinity in the refractive indices N_1 and N_2 of the moving medium. Existence of dispersion can also lead to the result that several branches of the waves correspond to a given frequency ω[4,5]. Then the radiation intensity (9) will consist of integrals taken individually for each branch of the radiated waves.

As follows from Eqs. (9), (6), and (8), there exist directions of the radiated waves for which the refractive indices (6) and (8) become rather large and the corresponding wavelengths small. This leads to the result that the ordinary multipole expansion of the radiation loses its meaning in this case, since the next term of the multiple expansion may turn out to be larger than the preceding term. In this case the radiation intensity depends strongly on the structure of the radiator.

In a magnetized plasma the refractive index N' (of the medium at rest) sometimes assumes extremely large numerical values, and therefore the effects found above can appear for nonrelativistic velocities of the plasma as a whole.

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