# MULTIPLE SCATTERING OF CHANNELED PARTICLES

V. A. RYABOV

Physics and Power Engineering Institute

Submitted March 24, 1972

Zh. Eksp. Teor. Fiz. 63, 1096–1106 (September, 1972)

A method is developed for discussion of directional effects arising in interaction of a fast charged particle with a single crystal, based on use of the path integral formalism. The proposed method permits the effects of multiple scattering of the particle by the nuclei and electrons of the crystalline target to be taken into account. It is shown that in its general form the scattering problem does not reduce to the problem of motion of a particle in a continuous potential averaged over the thermal vibrations of the lattice atoms. A specific analysis is carried out for the case of plane channeling in the approximation of a parabolic interaction potential. The role of multiple scattering is brought out in establishing statistical equilibrium in the transverse momentum of the particle in the channel. The minimal nuclear reaction yield is obtained as a function of the crystal temperature and the depth of the scattering layer for single crystals of any thickness. The results of the theory are in satisfactory agreement with the experimental results and with computer calculations.

## 1. INTRODUCTION

THE motion of a fast charged particle in a crystal is the result of a large number of correlated collisions due to the ordered location of the target nuclei. Under certain conditions the particle can remain for a long time far from atomic strings or planes, i.e., can be channeled. With motion into the interior of the crystal the stable channeling mode is destroyed—dechanneling occurs, after which the periodic structure of the target no longer affects the subsequent behavior of the particle. This phenomenon is brought about by the thermal vibrations of the lattice atoms, the discreteness of the medium along the channel, collisions with electrons and defects, and so forth. Reliable theoretical evaluations of the effect of these factors on the motion of the particle in the channel are necessary for solution of the important question: how does the range of a channeled particle differ from the range of the same particle in an amorphous medium. In addition, multiple scattering effects determined to a significant degree the dependence of the yield of a nuclear reaction on the thickness of the scattering layer, knowledge of which is necessary for interpretation of numerous experiments on channeling.

The dechanneling effect appears particularly strongly in the minimal nuclear reaction yield $\chi$ in directions parallel to low-index axes and planes of the crystal[1]. Near the surface, $\chi$ is an oscillating function of the ion penetration depth as the result of the establishment of equilibrium in the transverse momentum of the particle. In deeper layers of the crystal the oscillations are damped and the yield gradually increases to the value corresponding to an amorphous medium.

Until recently the theory of directional effects arising in interaction of a fast charged particle with a crystal utilized the statistical-mechanics approach proposed by Lindhard[2]. The applicability of this approach can be justified only in cases in which statistical equilibrium in the transverse phase space of the particle is established. However, there are a number of problems involving scattering of ion beams in thin single crystals, for which this approach is not suitable[3,4]. The theoretical interpretation of axial dechanneling in thick single crystals, developed by Lindhard[2] and Ellegard and Lassen[5] is essentially based on use of a Gaussian angular distribution function of the particles in the crystal, which does not actually take into account the oscillating behavior of the particle in the channel.

It is therefore clear that only the accurate solution of the dynamical problem of ion scattering by nuclei, electrons, and lattice defects will permit description from a unified point of view of all the observed phenomena. Up to the present time such a program has not been accomplished, and all calculations of nuclear reaction intensity have been based on more or less successful simplified models[5,6]. In particular, the continuous potential approximation[2,5] in calculations of the dechanneling probability does not permit inclusion of the discreteness of the medium along the channel, which has an appreciable effect in the case of plane channeling.

The purpose of the present work is to construct such a theory of directional effects, which would be free from the approximations mentioned above. This can apparently be achieved most simply in the Brownian-particle model with variable first and second moments of the scattering angle. An important feature of the theory developed below is the analytic description of the near-surface region with a nonequilibrium distribution function, in contrast to the crude estimates and numerical calculations carried out up to the present time[7].

## 2. DISTRIBUTION FUNCTION OF FAST IONS IN A SINGLE CRYSTAL

We will assume that the entrance surface of the crystal is perpendicular to some family of crystallographic planes and a flux of ions with energy E is incident at an angle $\theta_0 \ll 1$ (the X axis is normal to the family of planes, and the Z axis is normal to the entrance sur-

face). We will limit ourselves to study of the projection of the motion on the XZ plane with neglect of energy loss. We will assume that atoms distributed in the crystal planes uniformly with a density Nd (d is the distance between planes, and N is the number of atoms per $cm^3$) oscillate along the X axis with a mean square amplitude $\overline{u^2}$. Strictly speaking, this discussion corresponds to the case of beam incidence far from any close-packed directions in the atomic plane. Let us determine the probability that a particle with entrance coordinate $x_0$, traversing a single-crystal layer of thickness L, will turn out at the point x to deviate from the initial direction by an angle $\theta - \theta_0$.

Taking into account only nuclear collisions, the equation for the ion trajectory x(z) can be written in the form

$$\frac{d^2x}{dz^2} = f(z), \quad f(z) = \sum_i \theta_i \delta(z - z_i), \quad 0 \leqslant z_i \leqslant L \tag{2.1}$$

with boundary conditions

$$\begin{aligned} x(0) &= x_0, \quad \dot{x}(0) = \theta_0, \\ x(L) &= D, \quad \dot{x}(L) = \theta, \end{aligned} \tag{2.2}$$

where $\theta_i$ is the sudden deviation of the particle in the i-th binary collision. At first we will consider only scattering in one plane, later taking into account the contribution of other planes.

The combined action of a large number of collisions results in the particle motion in the crystal becoming close to periodic with a characteristic length $\lambda \gg d$. For channeled particles $\lambda$ has the meaning of the wavelength of the oscillations in the channel and has a value $\sim 10^3$ Å.

Let us break down the particle path into segments of length $\Delta z \ll \lambda$, so that in each of them x can be considered constant. This breakdown permits us to introduce in each individual segment $\Delta z$ an average change in scattering angle $R_1(x)$ which depends on the distance to the plane, which determines the macroscopic change in the average direction of motion of the particle, and also the mean square fluctuation of the angle $R_2(x)$ per unit path-length:

$$R_\alpha(x) = \frac{1}{\Delta z} \overline{\sum_i \theta_i^\alpha}, \quad \alpha = 1, 2. \tag{2.3}$$

The summation in (2.3) is carried out over all collisions with atoms of the plane in the interval (z, z + $\Delta$z), and the bar over the sum indicates averaging over the position of the atoms. Converting from the summation of (2.3) to integration, we obtain

$$R_\alpha(x) = Nd \left\langle \int \theta^\alpha(x, y)\, dy \right\rangle = Nd \iint \theta^\alpha(x + u, y)\, W(u)\, dy\, du, \tag{2.4}$$

where W(u) is the distribution function of thermal displacements of the atoms:

$$W(u) = (2\pi\overline{u^2})^{-1/2} \exp(-u^2 / 2\overline{u^2}). \tag{2.5}$$

The noise function f(z) in the right-hand side of Eq. (2.1) is determined by a set of random quantities—the coordinates $z_i$ and scattering angles $\theta_i$. Generalizing the well known result of Gaussian noise theory (see the Appendix), we can obtain the probability for observing a given function f(z) in Eq. (2.1):

$$P_f(f(z)) = \exp\left\{ -\frac{1}{2} \int_0^L \frac{[f(z) - R_1(x(z))]^2}{R_2(x(z))}\, dz \right\}. \tag{2.6}$$

It is characteristic that for the case of small angle scattering it is not necessary to know exactly the probability distribution for the scattering angle. The fact that the expression for $P_f$ in (2.6) involves only the first two moments of the distribution means in our problem the neglect of low-multiplicity collisions, which become important in the case of close approach of the ion trajectory to the atomic plane.

Since Eq. (2.1) couples the particle coordinate x and the random function f with a linear function, the probability distribution for values of the functions x(z) coincides with that for the function f(z):

$$P_x(x(z)) = \text{const} \cdot \exp\left\{ -\frac{1}{2} \int_0^L \frac{[\ddot{x} - R_1(x)]^2}{R_2(x)}\, dz \right\}. \tag{2.7}$$

Integration of (2.7) over all trajectories with the given boundary conditions (2.2) gives the formal solution of the problem:

$$P(x_0, \theta_0, D, \theta) = \text{const} \cdot \int \exp\left\{ -\frac{1}{2} \int_0^L \frac{[\ddot{x} - R_1(x)]^2}{R_2(x)}\, dz \right\} Dx(z). \tag{2.8}$$

Here the normalization constant must be determined from the obvious relation

$$P(x_0, \theta_0, D, \theta)\,|_{L=0} = \delta(D - x_0)\,\delta(\theta - \theta_0). \tag{2.9}$$

The intensity of a nuclear reaction for an arbitrary angle of incidence of the flux on the crystal surface $\theta_0$, normalized to the yield in an amorphous medium, can be represented in the form

$$I(\theta_0) = \int dx_0 \int d\theta \int dD\, P(x_0, \theta_0, D, \theta)\, W(D). \tag{2.10}$$

In regard to the angular distribution of the particles, in the general case we have the following expression:

$$P_{\theta_0}(\theta) = \int dx_0 \int dD\, P(x_0, \theta_0, D, \theta). \tag{2.11}$$

Let us now determine the form of the functions $R_\alpha(x)$ necessary for the subsequent discussion. Using the classical formula for the scattering angle[2], we can show that

$$R_1(x) = -\langle \partial Y / \partial x \rangle / 2E, \tag{2.12}$$

where Y(x) is the continuous planar potential of the static lattice[4]. It is interesting to note that the distribution along the ion trajectories determines not the potential $\langle Y \rangle$ averaged over the thermal vibrations, as is usually assumed (see, for example, refs. 2 and 7), but the average of its derivative. In the plane case the difference in order of averaging is unimportant, since $Y \sim e^{-x/a}$ (a is the screening constant). In axial channeling this fact is apparently one of the causes of the small discrepancy between Lindhard's theory and the computer calculations[8].

The scattering angle $\theta$ for various types of potential can be written approximately in the form

$$\theta = A \left\{ \frac{br^2}{ax} \left[ 1 + c \left( \frac{r}{a} \right)^2 \right] \right\}^{-1}, \quad A = Z_1 Z_2 e^2 / Ea, \quad r = \sqrt{x^2 + y^2}, \tag{2.13}$$

where $Z_1e$ and $Z_2e$ are the charges of the ion and atom of the target, and b and c are constants depending on the form of the potential. Thus, these constants in the case of Lindhard's potential[2] are b = 1, c = 1/3, and in the case of Moliere's potential: $b \approx 1.6$, $c \approx 1$.

Substituting (2.13) into (2.4), it is easy to find

$$R_2(x) \approx R_0\left[F(t)+\frac{\overline{u^2}}{2a^2}F''(t)\right], \qquad (2.14)$$

where

$$t = x/a, \quad R_0 = {}^1/_2\pi a A^2 N d/b^2,$$

$$F(t)=\frac{1}{t}\left[1-\frac{4t^2}{c^2}+\frac{t^3(5c^2+4t^2)}{c^2(t^2+c^2)^{3/2}}\right]. \qquad (2.15)$$

The final expression for the distribution function (2.8) is obtained by addition to $R_\alpha(x)$ of the independent contributions of other atomic planes:

$$R_\alpha'(x)=\sum_{n=-\infty}^{\infty} R_\alpha(x-nd). \qquad (2.16)$$

The approach developed above can easily be generalized to the axial case[1]. However, we will not dwell here on discussion of the features of axial channeling. All of the principal aspects of the theory can be followed in the example of plane channeling.

## 3. SCATTERING IN THE NEAR-SURFACE REGION OF A SINGLE CRYSTAL

In the plane channeling mode, the flux of ions is localized between two atomic planes. We will consider a parabolic potential for interaction of a particle with the channel walls, so that

$$\frac{1}{2E}\langle Y(x)\rangle = \frac{\omega^2 x^2}{2}, \qquad (3.1)$$

where x is measured from the center of the channel.

The particle leaves the channel (is dechanneled) if as the result of a number of successive collisions with atoms of the neighboring planes its transverse energy becomes greater than the height of the potential barrier: $E_\perp > Y(d/2)$. Since our model describes only channeled particles, it follows from this inequality that the initial distribution in entrance coordinates should have the form of a "shelf" of width $2(1 - 4\theta_0^2/\omega^2 d^2)^{1/2}$.

We note further that for thin single crystals of thickness $L \lesssim z_{1/2}$ ($z_{1/2}$ is the distance at which half of the ions leave the channel) the transverse energy of the particle, and consequently also the amplitude of the oscillations in the channel, do not change appreciably in the greater part of the path[9]. This means that the behavior of $R_2(x)$ along the trajectory is determined to a significant degree by the initial transverse energy of the particle: $E_{\perp 0} = E\theta_0^2 + Y(x_0)$. Therefore as a first approximation we can consider the angular divergence constant over the entire trajectory of the particle and dependent only on the initial oscillation amplitude $x_{m0} = Y^{-1}(E_{\perp 0})$.

On this basis we can write expression (2.6) in the form

$$P(x_0, \theta_0, D, \theta) = \text{const} \cdot \int \exp\left\{-\frac{1}{2R(x_{m0})}\int_0^L (\ddot{x}+\omega^2 x)^2\, dz\right\} Dx(z), \qquad (3.2)$$

where R(x) is equal to the sum of the contributions to the angular divergence from the two atomic planes. Since the greatest contribution to the integral in (2.6) is from the segments of the trajectory near the atomic planes of length $\sim\lambda/4 = \pi/2\omega$, R(x) can be represented approximately in the form

$$R(x) = {}^1/_2[R_2(x + {}^1/_2 d) + R_2(x - {}^1/_2 d)]. \qquad (3.3)$$

The integral in (3.2) can be calculated by a method which recalls in its general features the well known method of Laplace. Following this method, we can take outside the functional integration sign the factor containing the entire dependence of P in (3.2) on the ends of the trajectory (2.2). This factor is the integral over the extremal trajectory $\tilde{x}(z)$, determined from the condition that the first variational derivative of the functional in the exponent of (3.2) is zero[10]. The remaining integral over the trajectories, which has a Gaussian form, depends only on the path length L and thus can be included in the normalization constant.

It is easy to see that the condition mentioned for calculation of $\tilde{x}(z)$ leads to the following equation:

$$\dddot{\tilde{x}} + 2\omega^2\ddot{\tilde{x}} + \omega^4\tilde{x} = 0. \qquad (3.4)$$

The solution of this equation has the form

$$\tilde{x}(z) = (A_1 + B_1 z)\cos\omega z + (A_2 + B_2 z)\sin\omega z, \qquad (3.5)$$

where the constants $A_i$ and $B_i$ are determined by the boundary conditions (2.2). Substituting (3.5) into the exponent in (3.2), with inclusion of (2.9) we finally obtain

$$P(x_0, \theta_0, D, \theta) = \frac{\omega}{\pi RL}\exp\left\{-\frac{1}{RL}\left[(\theta_0\cos\omega L - \theta_0 + \omega D\sin\omega L)^2 + (\omega D\cos\omega L - x_0\omega - \theta\sin\omega L)^2 + O\left(\frac{1}{L}\right)\right]\right\}. \qquad (3.6)$$

Knowing the angular distribution of the particles (2.11) and taking into account (3.6), we can evaluate how rapidly equilibrium is achieved in the transverse momentum $\sqrt{2mE}\,\theta$, i.e., the range in which the channeled particle "forgets" its initial phase. As in ref. 2, we can take as a measure of this length the change of the average (over the angular distribution (2.11)) transverse momentum. We note here that the rate of mixing of the phases is strongly nonuniform over the channel cross section. Thus, for the part of the trajectories which lie near the channel walls, equilibrium is established already for $L \sim L_p = \omega^2 d^2/4R_0$. This leads to the fact that the periodic dependence of the minimal yield $\chi = I(0)$ on L disappears at a depth $L > L_p$. At the same time, for particles localized near the channel axis ($\theta_0 = 0$), the equilibrium distribution is established in a range $L \sim z_{1/2} \sim \omega^2 d^2/4R(0)$. It is therefore easy to understand that the periodic nature of the motion and, consequently, the asymmetry in the angular distribution of the channeled particles is destroyed at a penetration depth

$$L \sim \omega^2 d^2/4R\left[\frac{d}{2a}\left(1-\frac{2\theta_0}{\omega d}\right)\right].$$

This is obviously an exaggerated estimate, since in our discussion the anharmonicity of the potential is not included. The role of the latter is of two kinds. On the one hand, ions with small transverse momentum located in an anharmonic potential well can in principle have a substantial oscillation amplitude, which leads to intense dechanneling. On the other hand, the existence of a wide range of channeled-particle wavelengths in the presence of an anharmonic potential itself accelerates the process of phase mixing.

For calculation of the minimal yield of a nuclear reaction for $L \lesssim L_p$ we will use expression (2.10). Setting

[1] In this connection we must make the following remark. Derivation of the expression analogous to (2.6) in the axial case must be carried out with inclusion of correlations between the thermal vibrations of the lattice atoms, which somewhat complicates the problem.

$$\gamma = \omega^2 d^2 / 4R(x_0)L, \quad x_0' = 2x_0/d,$$

we find

$$\chi = \frac{2}{\sqrt{\pi}} \int_{-1}^{1} \frac{dx_0'}{(2\sigma + \gamma^{-1})^{1/2}} \exp\left[ -\frac{\gamma (x_0' \cos \omega L + 1)^2}{1 + 2\sigma\gamma} \right], \quad \sigma = 4\overline{\omega^2}/d^2. \tag{3.7}$$

In Fig. 1 we have shown the form of the function $\chi$ calculated according to Eq. (3.7) for protons with energy E = 0.4 MeV incident parallel to a (11$\bar{1}$) channel in aluminum. As $\omega$ we chose the frequency of oscillations for trajectories with $x_0 \sim d/2$ in an anharmonic potential well[11], which provide the greatest contribution to the integral in (3.7). The oscillations of the yield are due to the periodically varying density of particles near the nuclei of the crystals. For $L > L_p$, the yield oscillations disappear and the value of $\chi$ monotonically increases to the value corresponding to an arbitrary direction.

Near the crystal surface the effects of multiple scattering are still weakly expressed and the simplest analytic evaluation of the minimal yield, based on the continuous potential approximation, gives a value $\chi = 2a/d$.[2] However, the experimental value of $\chi$ measured for thin single crystals[10] is appreciably greater than this value. The theory developed above leads to better agreement with experiment even in the framework of the simple model proposed. In Fig. 2 we have shown curves for the minimal yield, averaged over the layer thickness 340—1370 Å, as a function of incident-particle energy and thermal-vibration amplitude of the atoms of a tungsten crystal. Comparison of the results of computer calculations[7] and the approach developed here permits the limits of applicability of our method to be defined. It is evident from Fig. 1 that the oscillations of the curve corresponding to the harmonic-oscillator model are damped at an appreciably slower rate. This is explained by the effect mentioned above of anharmonicity of the potential, which was taken into account in the computer calculations[7].

## 4. PLANE DECHANNELING

We will use the results of the preceding section to calculate the value of $\chi$ in the interval $L_p < L \lesssim z_{1/2}$. It should be emphasized that Eq. (3.7) is valid only for $L \sim L_p$, i.e., in the region where the number of dechanneled particles is still small. For $L \gg L_p$ the situation changes. The contribution of channeled particles to $\chi$ drops as $\sim 1/\sqrt{LR_0}$ (see Eqs. (2.10) and (3.6)) and the further behavior of $\chi$ is determined mainly by dechanneled ions. The contribution of the latter cannot

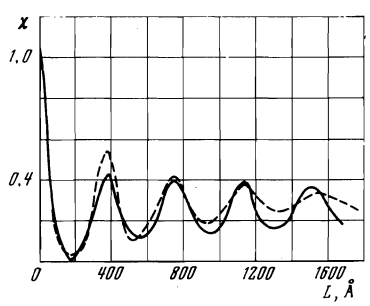


FIG. 1. Behavior of the minimal yield of a nuclear reaction near the surface of a single crystal. The solid line is the theoretical curve, and the dashed line is the curve obtained by the Monte Carlo method [7].

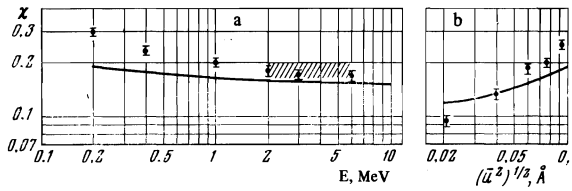


FIG. 2. Minimal yield (a) as a function of incident-particle energy and (b) as a function of thermal-vibration amplitude of the atoms. The cross-hatched region represents the experimental results [12]. The points show the results of computer calculation [7].

be calculated from Eq. (2.10), since this is an independent problem for solution of which it is necessary to introduce various assumptions of a model nature. For what follows, however, a phenomenological approach is convenient[5]. In particular, the quantity of interest to us can be represented in the form

$$\chi = 1 - N_c, \tag{4.1}$$

where $N_c$ is the fraction of channeled particles. It is easy to see that a similar approximation describes the behavior of particles with small amplitudes of oscillation in the channel, whose range before leaving it is $L \sim (\omega^2 d^2/4R_0)e^{x/a} \gg L_p$. In this case we can neglect the range of the dechanneled ion up to the moment of collision with an atom of the target which is accompanied by a nuclear reaction or by large-angle scattering.

Equation (4.1) can be represented in the form

$$\chi = \int_{-1}^{1} dx_0' \iint_{\rho_\perp > \gamma(d/2)} dx\, d\theta\, P(x_0, \theta_0 = 0,\ x, \theta). \tag{4.2}$$

After transformation from the variables x, $\theta$ to the variables $\rho$, $\varphi$ in polar coordinates, and taking into account (4.2), we have

$$\chi = \frac{1}{\pi} \int_{-1}^{1} dx_0' \int_{1}^{\infty} \rho\, d\rho \int_{0}^{\pi} \exp\{-\gamma(x_0')[x_0'^2 + \rho^2 - 2x_0\rho\cos(\varphi - \omega L)]\}\, d\varphi. \tag{4.3}$$

In the region of variation of L considered, the integral over the angle variable can be expanded in a series in powers of $L_p/L$ and we can limit ourselves to the first term of the expansion

$$\chi = \int_{-1}^{1} \exp[-\gamma(x_0')(1 + x_0'^2)]\, dx_0'. \tag{4.4}$$

Up to this point our discussion has concerned only the nuclear mechanism of dechanneling. However, collisions with electrons for $L \gg L_p$ also make an appreciable contribution to dechanneling. Inclusion of electron collisions in the general scheme of the theory, since they are not localized, reduces to a change in the mean square scattering angle. Thus, the additional term besides $R_2(x)$ in Eq. (2.14) resulting from collisions with atomic electrons can be evaluated from the known formula for Rutherford scattering[2]:

$$R_{2e}(x) = (m/2ME) S_e n(x), \tag{4.5}$$

where $S_e$ is the stopping cross section per electron, m and M are the electron and ion masses, and n(x) is the electron density of the continuous atomic plane. The latter is uniquely related to the atomic potential and, in the last analysis, to the scattering angle in collision of an ion with a lattice atom. Using Eq. (2.3) for the scattering angle, we can easily show that

$$n(x) = \frac{Z_2 \sqrt{c}\, Nd}{2ab} \left[ 1 + c\left( \frac{x}{a} \right)^2 \right]^{-3/2}. \tag{4.6}$$

We finally obtain

$$R_{2e}(x)=R_0\frac{\sqrt{c}}{bZ_2}\left[1+c\left(\frac{x}{a}\right)^2\right]^{-1/2}\ln\frac{4mE}{MI}, \qquad (4.7)$$

where I is the mean excitation potential, $I \sim 10Z_2$ [eV]. From comparison of (4.7) and (2.13) it is evident that, beginning with a distance to the plane of the order of several screening radii, the electron contribution to the angular broadening begins to dominate the nuclear contribution. This important fact has already been discussed repeatedly (see, for example, refs. 2 and 7), but up to the present time no corresponding estimates have been made of the dependence of the minimal yield on crystal thickness with inclusion of the spatial nonuniformity of $R_{2e}(x)$.

The $\chi$ values shown in Fig. 3, which were obtained by means of (4.4) and (4.7), reproduce reasonably well both the qualitative dechanneling features pointed out above and the experimentally observed pattern[12]. The value of $\omega$ in these calculations was chosen equal to the frequency of oscillations of an ion with the minimal transverse energy. This permits evaluation of the contribution to dechanneling of trajectories lying near the channel axis. An averaged continuous potential in the atomic plane in the Moliere form[11] was used in the calculation.

The calculation showed that inclusion of electron collisions in the discussion led to a decrease in $z_{1/2}$ by almost an order of magnitude. To this we must add that the stopping effect also leads to a decrease in $z_{1/2}$. However, accurate inclusion of this effect in our discussion would lead to severe mathematical difficulties. We can attempt to obtain only an upper limit of the stopping effect, by reducing the incident-particle energy by the average energy loss. For protons with E = 3 MeV scattered in a tungsten crystal, the energy loss per unit length is $\sim 100$ keV/$\mu$. The distance $z_{1/2}$, as can be seen from Fig. 3, reaches a value of $\sim 4$ $\mu$. Thus, the estimate desired can be found by setting E = 2.5 MeV in (2.14) and (4.7). The results of the calculation show a weak dependence of $\chi$ on the stopping power.

The theoretical curves, as can be seen from Fig. 3, lie somewhat below the experimental values[2)]. This fact, on the one hand, is due to certain features of the experiment which we have not taken into account, namely the divergence of the ion beam, scattering by defects, the mosaic structure of the single crystal, and so forth. On the other hand, the approach used contains a number of approximations, in particular the assumption of functional independence of the angular divergence of the trajectories (see Eqs. (3.1) and (3.2)). It would be possible to attempt to improve the results obtained by choosing as $R_2(x_m)$ in (3.2) its average value along the trajectory (3.5). However, in our discussion the anharmonicity of the interaction potential of the ion with the channel walls was not taken into account. Since the contribution of the latter to dechanneling is not completely clear, "improvement" of R(x) in terms of the harmonic oscillator model would represent excessive accuracy.

[2)] A similar result obtained on the basis of solution of the Fokker-Plank equation for a flux of particles in a plane channel is given by Pokhil and Tulinov [13].

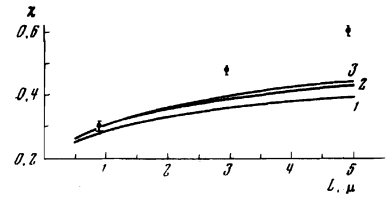


FIG. 3. Plane dechanneling: 1—yield curve without inclusion of electron collisions, 2—with inclusion of electron collisions, 3—with inclusion of stopping. The experimental points were taken from ref. 12.

The author is grateful to A. F. Tulinov and G. P. Pokhil for discussion of the results of this work.

## APPENDIX

We will present a systematic derivation of the formula for the probability distribution of the random function f(z) in Eq. (2.1) with allowance for Eq. (2.3).

After traversal of a segment of length $\Delta z \ll d/\theta$ the particle is deflected along the x axis by an amount $\Delta x \sim \theta\Delta z$. This fact is valid only in the case of high incident-particle energies ($Z_2Z_1e^2/aE \ll 1$) and substantially simplifies the problem. We can consider the ion trajectory in each separate interval (z, z + $\Delta$z) as an almost straight line with a constant separation from the atomic plane.

It is convenient to carry out the discussions which follow on the assumption of a finite number of collisions. For this we will assume that the ion interacts with atoms of the plane which are located at a distance from it not exceeding $(x^2 + y_a^2)^{1/2}$, where $y_a \gg a$. Thus, the total number of collisions is $K \approx 2Ndy_a\Delta z$, and in each collision there occurs a deflection by an angle $\theta$ determined by some probability distribution $p_z(\theta)d\theta$.

The characteristic functional for the probability distribution of the functions f(z) in the interval (z, z + $\Delta$z) has the following form[12]:

$$\Phi_z[\eta]=\exp\left\{-\frac{K}{\Delta z}\int_z^{z+\Delta z}(1-Q_z[\eta(s)])\,ds\right\}, \qquad (A.1)$$

where the generating function $Q_z$ for the moments of the distribution $p_z(\theta)$ in the small-angle approximation can be represented as

$$Q_z[\eta]\approx 1+i\eta\bar{\theta}-{}^1/_2\eta^2\overline{\theta^2}. \qquad (A.2)$$

The first and second moments of the distribution are determined in turn by the expressions

$$\overline{\theta^\alpha}=\frac{1}{K}\sum_{i=1}^{K}\theta_i^\alpha, \quad \alpha=1,2, \qquad (A.3)$$

where the line over the sum indicates averaging over the location of the atoms.

Going to the limit $y_a \to \infty$ and replacing the summation in (3.2) by integration, we obtain

$$\Phi_z[\eta]=\exp\left\{-\int_z^{z+\Delta z}\left[i\eta(s)R_1(z)-\frac{\eta^2(s)R_2(z)}{2}\right]ds\right\}. \qquad (A.4)$$

Here $R_\alpha[x(z)]$ are determined by Eqs. (2.12) and (2.14). Combining the independent contributions of all scatterings in the interval (0, L) and letting $\Delta z$ go to zero, we find

$$\Phi[\eta]=\exp\left\{-\int_0^L\left[i\eta(s)R_1(s)-\frac{\eta^2(s)R_2(s)}{2}\right]ds\right\}. \qquad (A.5)$$

Thus, the density distribution corresponds to the characteristic functional (A.5) has the form (2.5).

[1] A. F. Tulinov, Usp. Fiz. Nauk 87, 585 (1965) [Sov. Phys.-Uspekhi 8, 864 (1966)].
[2] J. Lindhard, Kgl. Danske Vindenskab. Selskab, Mat-Fys. Medd. 34, No. 14 (1965).
[3] Yu. N. Zhukova, G. A. Iferov, A. F. Tulinov, and V. Ya. Chumanov, Zh. Eksp. Teor. Fiz. 63, 217 (1972) [Sov. Phys.-JETP 36 (1973) (in press)].
[4] S. Datz, C. D. Moak, T. S. Noggle, B. R. Appleton, and H. O. Lutz, Phys. Rev. 179, 315 (1969).
[5] C. Ellegard and N. O. Lassen, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 35, 16 (1967).
[6] M. A. Kumakhov, Phys. Letters 33A, 133 (1970).
[7] J. H. Barret, Phys. Rev. B3, 1527 (1971).
[8] V. A. Ryabov, Kandidatskaya dissertatsiya (PhD. Dissertation), Nuclear Physics Institute, Moscow, 1971.
[9] M. Robinson, Phys. Rev. 179, 327 (1969).
[10] R. P. Feynman and A. R. Hibbs, Quantum Mechanics and Path Integrals, New York, McGraw-Hill, 1965. Russian translation, Mir, 1968.
[11] B. R. Appleton, C. Erginsoy, and W. M. Gibson, Phys. Rev. 161, 330 (1967).
[12] J. A. Davis, J. Denhartog, and J. L. Whitton, Phys. Rev. 165, 345 (1968).
[13] G. P. Pokhil and A. F. Tulinov, Fiz. Tverd. Tela 14, 1927 (1972) [Soviet Phys.-Solid State 14, No. 7 (1973) (in press)].

Translated by C. S. Robinson
117