THE SUPERCONDUCTOR–EXCITONIC DIELECTRIC PHASE TRANSITION IN A SEMIMETAL

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The superconducting and dielectric state of a semimetal is considered by taking account of polarization of the lattice. Lattice polarization leads to the result that the exciton gap associated with large band overlap is determined by the frequency of the longitudinal optical phonons and may turn out to be of the same order of magnitude as the superconducting gap. In the presence of different effective masses in the conduction band and in the valence band, a change of the pressure and the associated change of the band overlap lead to a superconductor to excitonic dielectric phase transition. The case of an impurity semimetal, where exciton pairing and Cooper pairing of the "excess" electrons (holes) can simultaneously exist, is also considered. It is shown that two superconducting states exist, with different dependences of the gap on the concentration. A first-order phase transition is possible between these states.

RECENTLY there has been an appreciable increase of interest in theoretical and experimental investigations of metal-dielectric phase transitions. The model of a semimetal-excitonic dielectric phase transition, which was proposed by Keldysh and Kopaev,[1] is a widely known model of such a transition. A number of investigations exist in which not only the properties of a pure excitonic dielectric are discussed,[2] but also the properties of an excitonic dielectric containing impurities.[3] The existence of two phase-transition points in an impurity semimetal was predicted,[3] the impurity states in an excitonic dielectric have been investigated,[4] and it has also been shown that superconductivity can exist in an excitonic dielectric containing excess electrons or holes.[5]

However, the polarization of the lattice was not taken into consideration in any of the articles devoted to consideration of an excitonic dielectric; it may essentially modify the nature of both the electron-hole and the electron-electron interactions, and it may also change the magnitude of the dielectric energy gap. In fact, the effective interaction between electrons and holes with allowance for the lattice polarization is given by $4\pi e^2/\epsilon(\omega, q)q^2$ ($\epsilon(\omega, q)$ represents the total dielectric constant of the crystal), and is evaluated by taking account of both the lattice vibrations and the free electrons and holes. In a polar semimetal, where due to the interaction the frequencies of the longitudinal and transverse phonons do not coincide, the lattice contribution to the dielectric constant becomes negative for $\omega_l > \omega > \omega_t$, and this leads to a change in the sign of the potential for the interaction of electrons and holes in this frequency interval. At the same time the effective electron-electron repulsion is changed to an attraction in this frequency interval, and as is well known this leads to the formation of Cooper pairs.

The change in the sign of the effective interaction as a function of the frequency in principle makes the simultaneous existence of exciton and Cooper pairings possible. In the present article we shall show that, depending on the extent of the band overlap (pressure), either exciton pairing or Cooper pairing will turn out to be energetically more favorable, and a dielectric-superconductor phase transition of first order will occur at a certain critical value of the band overlap. Such a phase transition occurs only when the effective masses of the electrons and holes are markedly different, when the masses are identical the exciton pairing is always energetically more favorable. The dielectric superconductor phase transition in an impurity semimetal is also considered in the present article. Here in the region of existence of the excitonic dielectric, both exciton and Cooper pairings exist simultaneously; this corresponds to superconductivity of the excess electrons occupying the conduction band of the excitonic dielectric. However, since a given electron participates simultaneously in the two pairings (Cooper and exciton pairings), the magnitude of the energy gap associated with a Cooper pair depends on the phase difference between the wave functions of the Cooper pair and the exciton pair, that is, an unusual coherence effect exists.

1. FORMULATION OF THE PROBLEM

We shall consider a semimetal with two overlapping bands: valence and conduction bands occurring at different points of k-space, with effective masses m_1 and m_2. In addition to the direct Coulomb interaction between the electrons, we shall also take the interaction of the electrons with longitudinal optical phonons[1)] into consideration. The direct Coulomb interaction and the interaction with phonons already lead, even in the Born approximation, to the diagram shown in Fig. 1c in addition to the diagrams shown in Figs. 1a and 1b (the wavy lines shown in Figs. 1a, 1b, and 1c denote either the Coulomb propagator $4\pi e^2/\epsilon_\infty q^2$ or the phonon propagator). However, owing to the orthogonality of the Bloch functions of different bands at different points of k-space, diagrams that contain at least one vertex at which the band state of the electron changes (e.g., Fig. 1c) will have an additional degree of smallness in comparison with diagrams of the type shown in Figs. 1a and 1b; the parameter characterizing this additional degree of smallness is $qa \ll 1$, where $q \sim p_F$ is the

[1)] We assume that the interaction with acoustic phonons is not strong enough to lead to the formation of Cooper pairs.

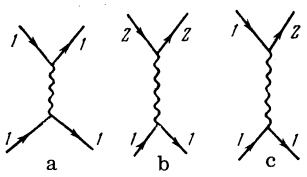

FIG. 1

momentum transfer and a^{-1} is the distance between the band extrema and is of the order of the reciprocal lattice constant.[2)] We shall therefore only consider vertices of the type shown in Figs. 1a and 1b. Then the propagator of the effective interaction of electrons with each other (the electrons being in a single band or in different bands) has the form[6]

$$D(\omega, \mathbf{q}) = \frac{4\pi e^2}{\varepsilon(\omega) q^2 - 4\pi e^2 \Pi(\omega, \mathbf{q})}, \tag{1}$$

where $\pi(\omega, q)$ is the polarization operator, and

$$\varepsilon(\omega) = \varepsilon_\infty (\omega^2 - \omega_l^2) / (\omega^2 - \omega_e^2)$$

is the dielectric constant of the lattice, ϵ_∞ denotes the high-frequency dielectric constant, and ω_l and ω_t are the frequencies of the longitudinal and transverse phonons, respectively.

Let us introduce the electron Green's functions:

$$\begin{aligned} G_{ik}^{\alpha\beta}(x - x') &= -i\langle T(\Psi_i^\alpha(x)\Psi_k^{+\beta}(x'))\rangle, \\ F_{ik}^{\alpha\beta}(x - x') &= \langle T(\Psi_i^\alpha(x)\Psi_k^\beta(x'))\rangle, \end{aligned} \tag{2}$$

where the superscripts α and β characterize the spin state of an electron, and the subscripts i and k characterize the band states. Since the interaction does not depend on the spin, then the spin is conserved at each vertex—just as is true for the band state.[3)] Below we shall only consider singlet exciton pairing. This means that $G_{ik}^{\alpha\beta}(x - x') = \delta_{\alpha\beta} G_{ik}(x - x')$. In addition, by the usual method one can show[7] that in the presence of singlet superconducting pairing the anomalous functions $\hat{F}_{ik}$ and $\hat{F}_{ik}^+$ are proportional to the matrix $\hat{I}$, which is antisymmetric with respect to its spin indices:

$$F_{ik}^{\alpha\beta}(x - x') = -I^{\alpha\beta} F_{ik}(x - x'), \quad (\hat{I}^2)_{\alpha\beta} = -\delta_{\alpha\beta}. \tag{3}$$

By using the property (3) we obtain a system of Dyson equations for the functions $\hat{G}$ and $\hat{F}$, which will now be matrices only with respect to the band indices; this system has the following form (the arguments of the functions are omitted in order to shorten the notation):

$$\begin{aligned} (\omega - \xi_i(\mathbf{p})) G_{ik} - i\Delta_{ij} G_{jk} - i\Sigma_{ij} F_{jk}^+ &= \delta_{ik}, \\ (\omega + \xi_i(\mathbf{p})) F_{ik}^+ + i\Sigma_{ij}^+ G_{jk} + i\Delta_{ij} F_{jk}^+ &= 0, \end{aligned} \tag{4}$$

where we have introduced the self-energy parts

$$\Sigma_{ij}(p) = \int \frac{d^4p'}{(2\pi)^4} F_{ij}(p - p') D(p'); \tag{5}$$

$$\Delta_{12}(p) = \int \frac{d^4p'}{(2\pi)^4} G_{12}(p - p') D(p'); \tag{6}$$

$$\Delta_{11} = \Delta_{22} = 0, \quad p = (\omega, \mathbf{p}),$$

which satisfy the symmetry relations[4)]

$$\Delta_{12}(\omega, \mathbf{p}) = -\Delta_{21}^*(\omega, \mathbf{p}), \quad \Sigma_{ij}^*(\omega, p) = \Sigma_{ji}^+(\omega, p). \tag{7}$$

Here $\xi_i(p) = (p^2 - p_{Fi}^2)/2m_i$ are the unperturbed band energies, reckoned from the chemical potential level; m_i and p_{Fi} denote the effective mass and Fermi momentum in the corresponding bands; in order to be definite we assume that $m_1 > 0$ and $m_2 < 0$. In a pure semimetal $p_{F1} = p_{F2}$. For identical signs of the masses (for example, for the case of two conduction bands) Δ_{12} vanishes, and the system of equations treated by Geĭlikman and Pashitskiĭ[8] is obtained from (4).

2. THE EXCITONIC DIELECTRIC-SUPERCONDUCTOR PHASE TRANSITION IN AN INTRINSIC SEMIMETAL

Let us consider semimetals having equal concentrations of electrons and holes. In this case the system of equations (4) together with the self-consistency equations (5) and (6) admit the following types of solutions:

1. The trivial solution $\Sigma_{ik} = \Delta_{12} = 0$ corresponding to the unperturbed semimetallic state;

2. $\Delta_{12} \neq 0$ and $\Sigma_{ik} = 0$ corresponding to an excitonic dielectric;

3. $\Sigma_{11} \neq 0$ or $\Sigma_{22} \neq 0$, and $\Sigma_{12} = \Delta_{12} = 0$, corresponding to a superconductor.

In principle solutions in which $\Sigma_{ik} \neq 0$ and $\Delta_{12} \neq 0$ might be possible. However, in the presence of equal numbers of electrons and holes such solutions are absent in any case if the self-energy parts differ markedly in their magnitudes. This follows directly from the investigation of impurity superconductivity (see Sec. 3), when one lets the concentration of excess electrons tend to zero.

First let us consider the superconducting state in band I, for example. In this case the system of equations (4) has the usual form of the Gor'kov equations, which have the following solution for the superconducting gap Σ_{11}:[6]

$$\Sigma_{11} = 2\omega_l \exp(-1/\lambda_x); \tag{8}$$

$$\lambda_x = \alpha_1 \left[\frac{\varepsilon_\infty}{\varepsilon_c} - \left(1 + \alpha_1 \ln \frac{\omega_m}{\omega_l}\right)^{-1} \right],$$

$$\alpha_1 = \frac{m_1 e^2}{4\pi p_F \varepsilon_\infty} \ln \frac{p_F^2}{\varkappa_D^2}, \quad \omega_m = \frac{(p_F^2 \varkappa_D)^{2/3}}{m_1}. \tag{9}$$

Here $1/\epsilon_c = 1/\epsilon_\infty - 1/\epsilon_0$; ϵ_0 is the static dielectric constant; $\kappa_D = [2(m_1 + |m_2|) p_F e^2/\pi\epsilon_0]^{1/2}$ is the reciprocal Debye length. The solution (8) is obtained to within terms of the order of

$$\ln(p_F^2/\varkappa_D^2) / \ln(\varkappa_D p_F / m\Sigma_{11}) \ll 1 \tag{10}$$

and differs from the formula given by Gurevich et al.[6] only by the expression for ω_m. This difference in the value of ω_m is associated with the fact that besides the frequency dependence of $\Sigma_{11}(\omega)$, which is the same as in[6], we have also considered the logarithmic dependence of $\Sigma_{11}(\omega, p)$ and the kernel of the integral equation on $p - p_F$, which thus enables us to determine the factor in front of the exponential to within the accuracy indicated above.

2) $\hbar = 1$ everywhere below.

3) Neglecting interband transitions, the physical results turn out not to depend on the distance a^{-1} between the extrema in k-space. [2] We shall assume for simplicity that both extrema are located at the point $\mathbf{k} = 0$, that is, in each band the quasimomentum is reckoned from the bottom of the band.

4) Just like Kopaev, [5] we assume that the exciton pair is formed by an electron and a hole with total quasimomentum equal to zero. It is possible that in an impurity semimetal it may be energetically more favorable if the pairs were to possess nonvanishing average values of the quasimomentum. This question requires special examination.

The expression for the superconducting gap in band II differs from expressions (8) and (9) only by interchanging the indices: $1 \rightleftarrows 2$.

One can obtain an expression for the dielectric gap from the system of equations (7) by a similar method. Here we have the following result, to the same accuracy as described by (10):

$$\Delta = \omega_l \frac{1+\beta}{\beta^{1/2}} \exp\left(-\frac{1}{\lambda_\Delta}\right), \qquad (11)$$

$$\lambda_\Delta = 2\alpha_l \left[\left(1 - 2\alpha_l \ln\frac{\omega_m}{\omega_l}\right)^{-1} - \frac{\varepsilon_\infty}{\varepsilon_c}\right].$$

Here

$$\alpha_l = \alpha_1/(1+\beta), \quad \beta = m_1/m_2.$$

The expression for the exciton gap differs from the expression for the superconducting gap by the change in the sign of the interaction constant α. From (11) it is seen that for large band overlaps the dielectric gap is always smaller than the frequency of a longitudinal optical phonon. If there is no coupling between the electrons and the longitudinal phonons, i.e. if $\epsilon_c^{-1} = 0$, then expression (11) goes over into the well-known expression given in[1], which is distinguishable by having a different factor in front of the exponential:

$$\Delta = \omega_m \frac{1+\beta}{\beta^{1/2}} \exp\left(-\frac{1}{2\alpha_l}\right). \qquad (12)$$

Since the gain in the ground state energy associated with pairing (both for exciton pairing as well as for superconducting pairing) is proportional to the square of the energy gap, while the temperatures of the dielectric-semimetal and superconductor-semimetal transitions are proportional to the energy gap, then in order to qualitatively explain the phase diagram of the system it is sufficient to investigate the relationship between the gaps for different degrees of overlap, that is, for different values of p_F. The superconducting gap increases with an increase of the effective mass. In order to be definite, let us take $m_1 > |m_2|$. Then in order to determine the phase diagram it is sufficient to investigate the relationship between Σ_{11} and Δ for different values of p_F.

The condition for applicability of the theory is that the quantities λ_Δ and λ_Σ must be positive. This gives the inequality

$$1 - \frac{2\alpha_1}{1+\beta}\ln\frac{\omega_m}{\omega_l} < \frac{\varepsilon_c}{\varepsilon_\infty} < 1 + \alpha_1 \ln\frac{\omega_m}{\omega_l}. \qquad (13)$$

Since the factors appearing in front of the exponentials are essentially identical, it follows that $\Sigma_{11} > \Delta$ if $\lambda_\Sigma > \lambda_\Delta$. Then the superconducting gap is larger than the exciton gap upon fulfilment of the condition

$$y^2 - {}^1/_2(\beta - 1)y - {}^1/_2(1 - \varepsilon_c/\varepsilon_\infty)(1+\beta) < 0, \qquad (14)$$

where $y = \alpha_1 \ln(\omega_m/\omega_l)$ and only depends on p_F, that is, it only depends on the overlap. From inequality (13) it follows that if $\beta = 1$, i.e., if the masses are the same, then inequality (14) is never satisfied, and the superconducting state is metastable for any degree of overlap. If $\beta \neq 1$, then a region $y_1 < y < y_2$ exists in which the superconducting state is the ground state, and the dielectric state is metastable. As follows from inequality (14), the critical degrees of overlap y_1 and y_2 are determined by only the ratio of the masses associated with given values of ϵ_0 and ϵ_∞. Thus, the phase diagram (T vs. P) associated with unequal electron and hole masses has the form shown in Fig. 2 (we assume that the overlap of the bands increases with increasing pressure). The solid lines AB and DC are the lines corresponding to first order phase transitions between the dielectric and superconducting states, but the solid line EBCF represents the line of second-order phase transitions. The points B and C are triple points. In the region where superconductivity exists (below the curves BC), switching-on a magnetic field will lead to a superconductor-dielectric phase transition, but it does not lead to a superconductor-semimetal phase transition since in this region the metastable dielectric phase will always be energetically more favorable than the semimetallic phase.

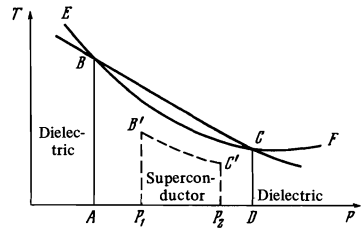

FIG. 2

Switching on a magnetic field leads to the points P_1 and P_2 moving closer together and to the appearance of a line B'C' representing a first-order phase transition between the superconducting and dielectric states; this line drops down with increasing values of the magnetic field so that for $H > H_{cr}$ only the dielectric phase exists over the entire range of pressures.

3. SUPERCONDUCTIVITY IS AN IMPURITY EXCITONIC DIELECTRIC

In this section we shall investigate the case when superconducting and exciton pairings simultaneously exist. Mathematically this means that the system of equations (4)—(6) has a solution in which both the exciton self-energy part Δ_{12} as well as the superconducting self-energy part Σ_{ik} do not vanish. Kopaev showed[5] within the framework of the BCS theory that such a possibility can be realized in an impurity excitonic dielectric, when the numbers of electrons and holes are not mutually equal. Physically such solutions imply that the excess electrons in an excitonic dielectric can be bound into Cooper pairs. It will be shown below that two physically different solutions exist with nonzero values of Σ and Δ, these solutions corresponding to different phase differences between the complex functions Σ_{ik} and Δ_{12}, that is, an unusual coherence effect exists associated with the fact that each electron simultaneously participates in the formation of both exciton and Cooper pairs. Therefore, the energy of the pairs depends on the phase difference between the wave functions. One of the solutions has been previously derived by Kopaev.[5] The other solution differs from Kopaev's solution both with regard to the energy and by the fact that the interband, self-energy part Σ_{12} vanishes for different masses of the electrons and holes. For both solutions there is a range of values of the parameters where they determine the ground state of the system.

For simplicity we shall confine our treatment to the case when the effective masses of electron and hole are equal. Then

$$\xi_1 = \xi - \delta\mu, \quad \xi_2 = -\xi - \delta\mu,$$

where ξ denotes the energy of the electrons in the conduction band, reckoned from the position of the Fermi level in the pure semimetal, and $\delta\mu$ denotes the correction to the chemical potential due to the excess concentration of electrons ($\delta\mu > 0$). If the effective masses of the electrons and holes are equal, then $|\Sigma_{11}| = |\Sigma_{22}| = \Sigma$.

The general solution of the system of equations (4) is very complicated and we shall not present it here. We shall only describe a procedure for finding a system of equations for the determination of the self-energy parts. We shall assume that the value of Σ_{ik} are small in comparison with the quantities Δ, $\delta\mu$, and $\Delta - \delta\mu$. In this case the corrections of order Σ_{ik} are essential only in the range of values of ξ where at least one of the poles of the Green's function goes to zero in the zeroth approximation with respect to Σ_{ik}. Then, solving the system of integral equations for the self-energy parts by a method similar to the method of Gurevich et al.,[6] we obtain the following algebraic system of equations:

$$\begin{gathered} 1/\lambda_\Delta = {}^1/_2 J_1^+(\omega_l), \\ \Sigma/\lambda = {}^1/_2 J_3[\Sigma(2\delta\mu^2 - \Delta^2) + \Sigma\Delta^2 e^{i(\alpha+\beta)} + 2\Delta\delta\mu|\Sigma_{12}|e^{i\alpha}] \\ + J_2[\Sigma\delta\mu + \Delta|\Sigma_{12}|e^{i\alpha}], \\ -|\Sigma_{12}|/\lambda_{12} = {}^1/_2 J_1^-(\omega_l)|\Sigma_{12}| + J_3[{}^1/_2|\Sigma_{12}|(\Delta^2 + \delta\mu^2) \\ + \Delta\Sigma(e^{-i\alpha} + e^{i\beta})\delta\mu] - {}^1/_2 J_2 \Delta\Sigma(e^{i\beta} - e^{-i\alpha}). \end{gathered} \tag{15}$$

Here

$$J_1^{\pm}(\omega_l) = \int_{-\omega_l}^{\omega_l} \frac{d\xi}{\omega_+ + \omega_-}\left(1 \pm \frac{\xi^2}{\omega_+\omega_-}\right),$$

$$J_2 = \int_{-\infty}^{\infty} \frac{\xi\, d\xi}{\omega_+ + \omega_-}\frac{1}{\omega_+\omega_-}, \quad J_3 = \int_{-\infty}^{\infty} \frac{d\xi}{\omega_+ + \omega_-}\frac{1}{\omega_+\omega_-},$$

and $1/\lambda = 1/\lambda_\Sigma - 1/\lambda_\Delta$. The constants λ_Δ and λ_Σ are defined by expressions (11) and (9) respectively, and the interband interaction constant

$$\lambda_{12} = \alpha(1 - \varepsilon_\infty/\varepsilon_c) = \alpha\varepsilon_\infty/\varepsilon_0 \tag{16}$$

does not, in contrast to λ_Δ and λ_Σ, contain the Coulomb logarithm.

To the quadratic approximation in Σ_{ik} the spectrum of the system has the form

$$\omega_\pm^2 = [\delta\mu \pm (\xi^2 + \Delta^2)^{1/2}]^2 + \Lambda^2(\xi), \tag{17}$$

$$\Lambda^2(\xi) = \Sigma^2 + |\Sigma_{12}|^2 \pm \frac{1}{\delta\mu(\xi^2+\Delta^2)^{1/2}}\left[\Delta^2\Sigma^2 \sin^2\frac{\alpha+\beta}{2} + |\Sigma_{12}|^2\xi^2 + 2|\Sigma_{12}|\Sigma\Delta\left(\xi \sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} - \delta\mu\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}\right)\right], \tag{18}$$

If ξ_0 denotes the point corresponding to the minimum of the function $\omega_-^2(\zeta)$, then $\Lambda = \Lambda(\xi_0)$ is the effective superconducting gap for electrons, which depends on the difference between the phases α and β which are related to the phases of Δ_{ik} and Σ_{ik}:

$$\alpha = \varphi_{11} - \varphi_{21} - \psi_{21}, \quad \beta = \varphi_{21} - \varphi_{22} + \psi_{21}, \quad \Delta_{ik} = \Delta\exp[-i(\psi_{ik} + \pi/2)],$$
$$\Sigma_{ik} = |\Sigma_{ik}|\exp[i(\varphi_{ik} - \pi/2)].$$

Since Λ, Δ, and $|\Sigma_{ik}|$ are by definition real positive quantities, then the phase factors $e^{i\alpha}$ and $e^{i\beta}$ must also be real and equal to ± 1. The integral J_2 then vanishes because of the symmetry of the spectrum.

The integrals $J_1^{\pm}(\omega_l)$ and J_3 are easily calculated; then, taking account of only the dominant term in $J_1^{\pm}(\omega_l)$, we have

$$\begin{gathered} J_1^+(\omega_l) = 2\ln\frac{\omega_l}{\delta\mu + (\delta\mu^2 - \Delta^2)^{1/2}}, \\ J_1^-(\omega_l) = -\frac{(\delta\mu^2 - \Delta^2)^{1/2}}{\delta\mu}\ln\frac{2(\delta\mu^2 - \Delta^2)}{\Lambda[\delta\mu + (\delta\mu^2 - \Delta^2)^{1/2}]}, \\ J_3 = \frac{1}{\delta\mu(\delta\mu^2 - \Delta^2)^{1/2}}\ln\frac{2(\delta\mu^2 - \Delta^2)}{\Lambda[\delta\mu + (\delta\mu^2 - \Delta^2)^{1/2}]}, \\ \Lambda = \Lambda(\xi_0), \quad \xi_0 = (\delta\mu^2 - \Delta^2)^{1/2}. \end{gathered} \tag{19}$$

Substituting the expressions for $J_1^{\pm}$ and J_3 into the system of equations and recognizing that the phase factors are equal to ± 1, we find that we have two physically different types of solutions:

1. $e^{i\alpha} = -e^{i\beta} = \pm 1$, $\quad \alpha + \beta = (2n+1)\pi$.

The expression for the effective gap Λ is of the form

$$\Lambda = \frac{2(\delta\mu^2 - \Delta^2)}{\delta\mu + (\delta\mu^2 - \Delta^2)^{1/2}}\exp\left[-\frac{1}{\lambda}\frac{(\delta\mu^2 - \Delta^2)^{1/2}\delta\mu}{\delta\mu^2 - \lambda_{12}\Delta^2/\lambda}\right]. \tag{20}$$

This case was treated by Kopaev,[5] who assumed that the constants λ_{12} and λ_Σ are identical. In our model this equality does not hold.

If the quantities Δ and $\delta\mu$ are expressed in terms of the value of the exciton gap Δ_0 in a pure semimetal and the concentration δn of excess electrons (see[3]), then we obtain

$$\Lambda = \frac{\Delta_0}{2}\left(\frac{\delta n}{n_{cr}}\right)^2\exp\left[-\frac{1}{\lambda}\frac{(1 - \delta n/2n_{cr})\delta n/2n_{cr}}{(1 - \lambda_{12}/\lambda)(1 - \delta n/2n_{cr})^2 + \lambda_{12}(\delta n/n_{cr})^2/\lambda}\right] \tag{21}$$

where n_{cr} denotes the critical concentration of excess electrons, that is, the value for which the gap vanishes:

$$n_{cr} = \Delta_0 m p_F/\pi^2.$$

The properties of this solution have been discussed in detail by Kopaev.[5]

2. $e^{i\alpha} = e^{i\beta} = \pm 1$, $\quad \alpha + \beta = 2n\pi$

In this case the interband self-energy part vanishes (however, the anomalous Green's function F_{12} does not vanish) and the expression for the effective gap has the form

$$\Lambda = \frac{2(\delta\mu^2 - \Delta^2)}{\delta\mu + (\delta\mu^2 - \Delta^2)^{1/2}}\exp\left(-\frac{1}{\lambda}\frac{\delta\mu}{(\delta\mu^2 - \Delta^2)^{1/2}}\right), \tag{22}$$

or

$$\Lambda = \frac{\Delta_0}{2}\left(\frac{\delta n}{n_{cr}}\right)^2\exp\left(-\frac{1}{\lambda}\frac{2n_{cr} - \delta n}{\delta n}\right).$$

From expressions (21) and (22) it is seen that when δn tends to zero, $\Lambda \to 0$, that is, solutions do not exist in a pure semimetal when Σ_{ik} and Δ_{ik} are not simultaneously equal to zero and differ markedly in magnitude.

The state with the larger energy gap is energetically more favorable. From a comparison of expressions (20) and (22) it is seen that solution 1 (Kopaev's solution) is energetically more favorable for $\lambda > \lambda_{21}$, and the second solution is energetically more favorable for $\lambda < \lambda_{21}$. Since the interaction constants λ and λ_{21} depend on the overlap between the bands of the semimetal, which varies as the pressure changes, a first-order phase transition from one superconducting state to the other is possible with respect to the pressure.

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