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QUANTUM KINETIC EQUATIONS FOR A NONIDEAL GAS AND A NONIDEAL PLASMA

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A derivation is given of the quantum kinetic equations for a nonideal Boltzmann gas and a nonideal plasma, without the assumption that the interaction of the particles is small. The collision integrals take into account the effects of the retardation of the distribution functions and the effect of an external electric field. The kinetic equations for a nonideal plasma differ from the quantum Boltzmann equation in that they take the averaged dynamical polarization of the plasma into account. Because of this, the collision integrals for a plasma converge at large and small distances. The method of obtaining the quantum kinetic equations for a nonideal gas and plasma is analogous to that described previously[8,9] for classical systems. Because of this, the paper is principally devoted to the main ways in which the quantum theory differs from the classical theory. It is shown that it is especially important to take account of the strong interactions over small distances when calculating the non-dissipative characteristics of a nonideal plasma, since then the contributions of bound states whose life-times are shorter than the mean time between collisions are taken into account. Calculations of the internal energy for nonequilibrium states are performed. For a local-equilibrium state, an expression valid in any approximation of perturbation theory is obtained. It is shown that this expression takes into account the contribution of the bound states. The effective field is calculated, with allowance for the quantum corrections, for arbitrary ratio of the de Broglie wavelength to the Debye radius.

1. INTRODUCTION

In this paper we study the quantum kinetic equations for a nonideal gas and nonideal plasma. The term "nonideal" means that, within the framework of the given model (the binary-collision approximation and polarization approximation), we take into account not only the dissipative, but also the non-dissipative contributions of the interaction of the particles.

The first attempt to take non-dissipative effects into account in the quantum kinetic Boltzmann equation was made by Green[1]. A more consistent derivation of the quantum kinetic equation for a gas is given in the work of Kadanoff and Baym[2] and Baerwinkel and Grossmann[3,4]. On the basis of this equation a momentum-transport equation was obtained[4] in which the pressure occurs with the second virtual coefficient taken into account. The virial coefficient is expressed in terms of the T-matrix.

Quantum kinetic equations for a nonideal plasma have not, apparently, been studied up to the present time. The problem of obtaining kinetic equations for a nonideal partially ionized plasma is very complicated because of the necessity of taking the bound states into account. In the present paper, we study the kinetic equations for a completely ionized plasma. For this case, one usually uses kinetic equations which are a quantum generalization of the Balescu-Lenard equation[5-7]. In these, the dynamical polarization is taken into account and this ensures screening of the interaction of the charged particles at large distances. At small distances, however, the contribution of correlations to the collision integral is calculated from perturbation theory in the coupling parameter (the Born approximation).

In this paper, the quantum kinetic equation for a nonideal plasma is studied. Rather than take account of the dynamical polarization exactly, which leads to extremely complicated equations, we take only the averaged polarization into account. However, at small distances, the collision integral takes into account the contributions of all the terms of perturbation theory in the interaction. The strong interactions of the particles at small distances are thereby also taken into account. This is especially important in the calculation of the non-dissipative characteristics of a nonideal plasma, since in this case one also takes into account the contributions of the bound states whose lifetimes are much shorter than the time between collisions.

Since the method used in our work to obtain the quantum kinetic equations is analogous to that described in the papers of one of the authors for the classical nonideal gas[8] and for the classical nonideal plasma[9], the main attention here is given only to the principal ways in which the quantum theory differs.

2. THE BOLTZMANN KINETIC EQUATION FOR A NONIDEAL GAS

We shall consider briefly the derivation of the quantum kinetic equation for a nonideal gas in the framework of the binary-collision model, without using the formalism of Green functions and the T-matrix. This makes it possible to write down immediately the corresponding equations for a nonideal plasma with the dynamical polarization taken into account.

In the binary-collision approximation, the chain of equations for the density matrix of one, two, etc., particles reduces to a system of equations for the one- and two-particle density matrices. For a spatially uniform multi-component gas, the equations for the one-particle functions can be written in the form

$$\partial f_a(\mathbf{p}_a, t)\,/\,\partial t = I_a(\mathbf{p}_a, t). \tag{2.1}$$

Here $f_a(p_a, t)$ is the momentum distribution function for particles of component a, and I_a is the collision integral. This is connected with the two-particle density matrix $\rho_{ab}(\mathbf{r}_a', \mathbf{r}_a'', \mathbf{r}_b', \mathbf{r}_b'', t)$ by the relation

$$\begin{gathered} I_a = \frac{i}{\hbar}\sum_b N_b \int [\Phi_{ab}(|\mathbf{r}_a'' - \mathbf{r}_b|) - \Phi_{ab}(|\mathbf{r}_a' - \mathbf{r}_b|)] \\ \times \rho_{ab}(\mathbf{r}_a', \mathbf{r}_a'', \mathbf{r}_b, \mathbf{r}_b, t) \exp\left[-\frac{i}{\hbar}(\mathbf{r}_a' - \mathbf{r}_a'')\mathbf{p}_a\right]\frac{d\mathbf{r}_a'\, d\mathbf{r}_a''\, d\mathbf{r}_b}{V^3}. \end{gathered} \tag{2.2}$$

The equation for ρ_{ab} in the binary-collision approximation has the form

$$\begin{gathered} i\hbar\frac{\partial\rho_{ab}}{\partial t} + \left[\frac{\hbar^2}{2m_a}(\Delta_{\mathbf{r}_a'} - \Delta_{\mathbf{r}_a''}) + \frac{\hbar^2}{2m_b}(\Delta_{\mathbf{r}_b'} - \Delta_{\mathbf{r}_b''})\right]\rho_{ab} \\ - [\Phi_{ab}(|\mathbf{r}_a' - \mathbf{r}_b'|) - \Phi_{ab}(|\mathbf{r}_a'' - \mathbf{r}_b''|)]\,\rho_{ab} \\ = i\hbar\frac{\partial}{\partial t}\int f_a(\mathbf{p}_a, t) f_b(\mathbf{p}_b, t) \exp\left[\frac{i}{\hbar}(\mathbf{r}_a' - \mathbf{r}_a'')\mathbf{p}_a \right. \\ \left. + \frac{i}{\hbar}(\mathbf{r}_b' - \mathbf{r}_b'')\mathbf{p}_b\right]\left[\frac{V}{(2\pi\hbar)^3}\right]^2 d\mathbf{p}_a d\mathbf{p}_b \\ \equiv i\hbar\frac{\partial}{\partial t}\rho_{ab}^{(0)}(\mathbf{r}_a', \mathbf{r}_a'', \mathbf{r}_b', \mathbf{r}_b'', t), \end{gathered} \tag{2.3}$$

where $\rho_{ab}^{(0)}$ is the matrix ρ_{ab} disregarding the correlations of the particles.

The functions f_a and ρ_{ab} are normalized as follows

$$\int f_a \frac{V}{(2\pi\hbar)^3}d\mathbf{p}_a = 1, \qquad \int \rho_{ab}(\mathbf{r}_a, \mathbf{r}_a, \mathbf{r}_b, \mathbf{r}_b)\frac{d\mathbf{r}_a d\mathbf{r}_b}{V^2} = 1. \tag{2.4}$$

Eqs. (2.1)–(2.3) correspond to the starting equations (4) and (12) of the paper by Green[1] and, in the classical case, to Eqs. (11) and (12) from[8].

We expand the function ρ_{ab} in the eigenfunctions $\psi_{\mathbf{p}_a\mathbf{p}_b}(\mathbf{r}_a, \mathbf{r}_b)$ of the Hamiltonian of a pair of particles. They are determined by the equation

$$\left[-\frac{\hbar^2}{2m_a}\Delta_{\mathbf{r}_a} - \frac{\hbar^2}{2m_b}\Delta_{\mathbf{r}_b} + \Phi_{ab}(|\mathbf{r}_a - \mathbf{r}_b|)\right]\psi_{\mathbf{p}_a\mathbf{p}_b}(\mathbf{r}_a, \mathbf{r}_b) = E_{\mathbf{p}_a\mathbf{p}_b}\psi_{\mathbf{p}_a\mathbf{p}_b}(\mathbf{r}_a, \mathbf{r}_b);$$

$$\int \psi^*_{\mathbf{p}_a''\mathbf{p}_b''}(\mathbf{r}_a, \mathbf{r}_b)\,\psi_{\mathbf{p}_a'\mathbf{p}_b'}(\mathbf{r}_a, \mathbf{r}_b)\,d\mathbf{r}_a d\mathbf{r}_b = \left[\frac{(2\pi\hbar)^3}{V^3}\right]^2 \delta(\mathbf{p}_a' - \mathbf{p}_a'')\,\delta(\mathbf{p}_b' - \mathbf{p}_b''),$$

$$\psi_{\mathbf{p}_a\mathbf{p}_b}(\mathbf{r}_a, \mathbf{r}_b, t) = \exp(-iE_{\mathbf{p}_a\mathbf{p}_b}t/\hbar)\,\psi_{\mathbf{p}_a\mathbf{p}_b}(\mathbf{r}_a, \mathbf{r}_b). \tag{2.5}$$

We write the expansion of the function ρ_{ab} in the form

$$\begin{gathered} \rho_{ab}(\mathbf{r}_a', \mathbf{r}_a'', \mathbf{r}_b', \mathbf{r}_b'') = \left[\frac{V}{(2\pi\hbar)^3}\right]^4 \int \rho_{ab}(\mathbf{p}_a', \mathbf{p}_a'', \mathbf{p}_b', \mathbf{p}_b'') \\ \times \psi_{\mathbf{p}_a'\mathbf{p}_b'}(\mathbf{r}_a', \mathbf{r}_b')\,\psi_{\mathbf{p}_a''\mathbf{p}_b''}(\mathbf{r}_a'', \mathbf{r}_b'')\,d\mathbf{p}_a' d\mathbf{p}_a'' d\mathbf{p}_b' d\mathbf{p}_b''. \end{gathered} \tag{2.6}$$

In the approximation of free motion,

$$\psi_{\mathbf{p}_a\mathbf{p}_b}(\mathbf{r}_a, \mathbf{r}_b)\Big|_{\mathbf{r}_a - \mathbf{r}_b \to \infty} = \frac{1}{V}\exp\left[\frac{i}{\hbar}(\mathbf{r}_a\mathbf{p}_a + \mathbf{r}_b\mathbf{p}_b)\right].$$

From (2.3), using the expansion (2.6), we find

$$\begin{gathered} \left(i\hbar\frac{\partial}{\partial t} - (E_{\mathbf{p}_a'\mathbf{p}_b'} - E_{\mathbf{p}_a''\mathbf{p}_b''})\right)\rho_{ab}(\mathbf{p}_a', \mathbf{p}_a'', \mathbf{p}_b', \mathbf{p}_b'') \\ = i\hbar\frac{\partial}{\partial t}\rho_{ab}^{(0)}(\mathbf{p}_a', \mathbf{p}_a'', \mathbf{p}_b', \mathbf{p}_b''). \end{gathered} \tag{2.7}$$

We denote the relaxation time of the function f_a by τ_{rel} and the time of the collision process by τ_0. We write the solution of Eq. (2.7) for times

$$\tau_0 \ll \tau \equiv t - t_0 \ll \tau_{\mathrm{rel}},$$

where t_0 is the initial time. In a time $\tau \gg \tau_0$, the initial correlations are weakened. Taking this into account, we can write the solution of Eq. (2.7) in the form

$$\begin{gathered} \rho_{ab} = \exp[-i(E_{\mathbf{p}_a'\mathbf{p}_b'} - E_{\mathbf{p}_a''\mathbf{p}_b''})\tau/\hbar]\,\rho_{ab}^{(0)}(\mathbf{p}_a', \mathbf{p}_a'', \mathbf{p}_b', \mathbf{p}_b'', t - \tau) \\ + \int_0^\tau \exp[-i(E_{\mathbf{p}_a'\mathbf{p}_b'} - E_{\mathbf{p}_a''\mathbf{p}_b''})\tau'/\hbar]\frac{\partial}{\partial t}\rho_{ab}^{(0)}(\mathbf{p}_a', \mathbf{p}_a'', \mathbf{p}_b', \mathbf{p}_b'', t - \tau')\,d\tau'. \end{gathered} \tag{2.8}$$

Using the solution (2.8) and the expansion (2.6), we find the density matrix ρ_{ab} in the coordinate representation. We introduce for the transition matrix the notation

$$\begin{gathered} A(-\tau) = \int \psi_{\mathbf{p}_a'\mathbf{p}_b'}(\mathbf{r}_a', \mathbf{r}_b')\,\psi^*_{\mathbf{p}_a''\mathbf{p}_b''}(\mathbf{r}_a'', \mathbf{r}_b'')\,\psi^*_{\mathbf{p}_a'\mathbf{p}_b'}(\mathbf{q}_a', \mathbf{q}_b', -\tau) \\ \times \psi_{\mathbf{p}_a''\mathbf{p}_b''}(\ {}_a'', \mathbf{q}_b'', -\tau)\,d\mathbf{p}_a' d\mathbf{p}_a'' d\mathbf{p}_b' d\mathbf{p}_b''\left[\frac{V}{(2\pi\hbar)^3}\right]^4. \end{gathered} \tag{2.9}$$

Then

$$\begin{gathered} \rho_{ab}(\mathbf{r}_a', \mathbf{r}_a'', \mathbf{r}_b', \mathbf{r}_b'', t) = \int \Big\{A(-\tau)\,\rho_{ab}^{(0)}(\mathbf{q}_a', \mathbf{q}_a'', \mathbf{q}_b', \mathbf{q}_b'', t - \tau) \\ + \int_0^\tau A(-\tau')\frac{\partial}{\partial t}\rho_{ab}^{(0)}(\mathbf{q}_a', \mathbf{q}_a'', \mathbf{q}_b', \mathbf{q}_b'', t - \tau')\,d\tau'\Big\}\,d\mathbf{q}_a'\,d\mathbf{q}_a''\,d\mathbf{q}_b'\,d\mathbf{q}_b''. \end{gathered} \tag{2.10}$$

In the classical theory, this expression is analogous to the expression (13) from[8].

We substitute the solution (2.10) into the expression (2.2). As a result, we obtain an expression for I_a in terms of the first distribution functions ($\rho_{ab}^{(0)}$ is defined in (2.3)). The expression (2.10) can be simplified if we confine ourselves to the first approximation in $\tau_0\partial t$, i.e., to the first approximation in the retardation. We represent the collision integral in the form of a sum of two parts

$$I_a = I_{a1} + I_{a2}. \tag{2.11}$$

The first term corresponds to the zeroth approximation in the retardation. In this approximation, it follows from (2.10) that

$$\rho_{ab}{}^1 = \int A(-\infty)\,\rho_{ab}^{(0)}(\mathbf{q}_a', \mathbf{q}_a'', \mathbf{q}_b', \mathbf{q}_b'', t)\,d\mathbf{q}_a'\,d\mathbf{q}_a''\,d\mathbf{q}_b'\,d\mathbf{q}_b''.$$

For $\tau \to \infty$ in the expression for $A(-\tau)$,

$$\begin{gathered} \psi^*_{\mathbf{p}_a'\mathbf{p}_b'}(\mathbf{q}_a', \mathbf{q}_b', -\tau)\,\psi_{\mathbf{p}_a''\mathbf{p}_b''}(\mathbf{q}_a'', \mathbf{q}_b'', -\tau) \to \\ \to \frac{1}{V}\exp\left[-\frac{i}{\hbar}(\mathbf{p}_a'\mathbf{q}_a' + \mathbf{p}_b'\mathbf{q}_b') + \frac{i}{\hbar}(\mathbf{p}_a''\mathbf{q}_a'' + \mathbf{p}_b''\mathbf{q}_b'')\right], \end{gathered}$$

since, in the zeroth approximation in the retardation, correlation of the initial states is completely absent.

As a result, (taking the definition (2.3) into account), after integrating over $\mathbf{q}_a'$, $\mathbf{q}_a''$, $\mathbf{q}_b'$, $\mathbf{q}_b''$, and $\mathbf{p}_a'$, $\mathbf{p}_a''$, $\mathbf{p}_b'$, $\mathbf{p}_b''$, we obtain in the zeroth approximation in the retardation

$$\begin{gathered} \rho_{ab}{}^1(\mathbf{r}_a', \mathbf{r}_a'', \mathbf{r}_b', \mathbf{r}_b'', t) = \frac{V^4}{(2\pi\hbar)^6}\int \psi_{\mathbf{p}_a\mathbf{p}_b}(\mathbf{r}_a', \mathbf{r}_b'')\,\psi_{\mathbf{p}_a\mathbf{p}_b}(\mathbf{r}_a', \mathbf{r}_b'') \\ \times f_a(\mathbf{p}_a, t) f_b(\mathbf{p}_b, t)\,d\mathbf{p}_a\,d\mathbf{p}_b. \end{gathered} \tag{2.12}$$

From (2.12) and (2.2), we find an expression for the first part of the collision integral. We write it in the form

$$\begin{gathered} I_{a1}(\mathbf{p}_a, t) = \frac{i}{\hbar}\frac{V^2}{(2\pi)^6\hbar^3}\sum_b N_b \int [\Phi_{ab}(|\mathbf{r}_a - \mathbf{r}_b - {}^1\!/_2\hbar\boldsymbol{\gamma}_a|) \\ - \Phi_{ab}(|\mathbf{r}_a - \mathbf{r}_b + {}^1\!/_2\hbar\boldsymbol{\gamma}_a|)]\exp(-i\boldsymbol{\gamma}_a\mathbf{p}_a) \\ \times \psi_{\mathbf{p}_a'\mathbf{p}_b}(\mathbf{r}_a + {}^1\!/_2\hbar\boldsymbol{\gamma}_a, \mathbf{r}_b)\,\psi^*_{\mathbf{p}_a'\mathbf{p}_b}(\mathbf{r}_a - {}^1\!/_2\hbar\boldsymbol{\gamma}_a, \mathbf{r}_b) \\ \times f_a(\mathbf{p}_a', t) f_b(\mathbf{p}_b, t)\,d\boldsymbol{\gamma}_a d\mathbf{p}_a' d\mathbf{p}_b d\mathbf{r}_b. \end{gathered} \tag{2.13}$$

This expression coincides with that obtained in the paper by Green.[1] In the paper by Hoffman et al.[10], it is shown that the expression (2.13) can be brought to the usual form of a Boltzmann collision integral with a quantum cross-section. Allowance for the retardation gives corrections to the usual Boltzmann collision integral, which take into account the nonideality of the gas.

The expression for ρ_{ab} in the first approximation in $\tau_0 \partial/\partial t$ has the form

$$\rho_{ab}^{2} = -\frac{\partial}{\partial t}\int_0^\infty d\tau \int d\mathbf{q}_a' \, d\mathbf{q}_a'' \, d\mathbf{q}_b' \, d\mathbf{q}_b'' \times \tau \frac{d}{d\tau} A(-\tau)\, \rho_{ab}^{(0)}(\mathbf{q}_a', \mathbf{q}_a'', \mathbf{q}_b', \mathbf{q}_b'', t). \tag{2.14}$$

Substitution of this expression into (2.2) gives the required expression for I_{a2}. As in the classical theory, the integral I_{a2} determines the contribution of the interaction to the non-dissipative characteristics of the nonideal gas.

3. THE TRANSPORT EQUATIONS AND THERMODYNAMIC FUNCTIONS FOR A NONIDEAL GAS

As in the classical approximation, the collision integral possesses the properties ($\varphi_a = 1, \mathbf{p}_a$)

$$\sum_a n_a \int \varphi_a(\mathbf{p}_a) I_a \, d\mathbf{p}_a = 0.$$

These equalities ensure the conservation of the number of particles and of the total momentum.

We shall study the energy conservation law in more detail. The energy conservation law with allowance for the pair interaction follows from the kinetic equation (2.1). Calculations analogous to those carried out in Balescu's book[5] lead to the following result for the internal-energy density:

$$U = \sum_a n_a \int \frac{\mathbf{p}_a^2}{2m_a} f_a \frac{V}{(2\pi\hbar)^3} d\mathbf{p}_a + \frac{1}{2}\sum_{ab} n_a n_b \int \Phi_{ab} \rho_{ab}^{1}(\mathbf{r}_a, \mathbf{r}_a, \mathbf{r}_b, \mathbf{r}_b, t)\, d(\mathbf{r}_a - \mathbf{r}_b). \tag{3.1}$$

Here $\rho_{ab}^{1}(\mathbf{r}_a, \mathbf{r}_a, \mathbf{r}_b, \mathbf{r}_b)$ is the distribution function of the coordinates of two particles in the zeroth approximation in the retardation. An expression for it follows from (2.12):

$$\rho_{ab}^{1}(\mathbf{r}_a, \mathbf{r}_a, \mathbf{r}_b, \mathbf{r}_b, t) = \frac{V^4}{(2\pi\hbar)^6}\int |\psi_{\mathbf{p}_a \mathbf{p}_b}(\mathbf{r}_a, \mathbf{r}_b)|^2 f_a(\mathbf{p}_a, t) f_b(\mathbf{p}_b, t)\, d\mathbf{p}_a \, d\mathbf{p}_b. \tag{3.2}$$

In the expression (3.1) for the internal-energy density, the interaction determines not only the second term (the potential energy). The point is that the first distribution function, like any thermodynamic function, can be expanded in the density. In the first approximation in the density, we obtain from Eqs. (2.1)—(2.3) the following expression ($\mathbf{x} = (\mathbf{r}, \mathbf{p})$):

$$f_a(\mathbf{p}_a, t) = C\left[f_a^0 + \sum_b n_b \int g_{ab}(\mathbf{x}_a, \mathbf{x}_b)\, d\mathbf{r}_a \, d\mathbf{r}_b \frac{d\mathbf{p}_b}{(2\pi\hbar)^3}\right]. \tag{3.3}$$

The constant C is determined from the normalization condition. f_a^0 is the distribution function of the ideal gas, and $g_{ab} = f_{ab} - f_a f_b$ is the correlation function.

Using the expression for C, we can write the expression (3.3) in the first approximation in the density in the form

$$f_a(\mathbf{p}_a, t) = f_a^0 + \sum_b n_b \int \left\{ \int g_{ab}(\mathbf{x}_a, \mathbf{x}_b, t) \frac{V\, d\mathbf{p}_b}{(2\pi\hbar)^3} - \int g_{ab}(\mathbf{x}_a, \mathbf{x}_b, t)\left[\frac{V}{(2\pi\hbar)^3}\right]^2 d\mathbf{p}_a \, d\mathbf{p}_b f_a^0(\mathbf{p}_a, t)\right\} \frac{d\mathbf{r}_a \, d\mathbf{r}_b}{V}. \tag{3.4}$$

We can substitute into the right-hand side the expression for g_{ab} in the zeroth approximation in the density and retardation.

The difference between $f_a(p, t)$ and the ideal-gas distribution function f_a^0 is due to correlations of two types: quantum correlations of the distributions of the coordinates and momenta of the particles, and classical correlations caused by the interaction. In a local-equilibrium state, only the quantum fluctuations remain. Indeed, we shall obtain an expression for f_a exact to $\hbar^2$ inclusive. This can be done by two methods: 1) using the equation for the quantum distribution function in the Wigner representation (this follows from Eq. (2.3)), solve it by the method of successive approximations, taking the classical distribution f_{ab}^{cl} as the initial function; 2) use Eq. (2.1) with the collision integral (2.2) to determine the quantum corrections to f_a.

In the binary-collision approximation, both methods of calculation lead to the result

$$f_a^{qu}(\mathbf{p}_a) = f_a^{cl} + \sum_b n_b \frac{\hbar^2}{24kT}\int\left(\frac{\partial \Phi_{ab}}{\partial \mathbf{r}_a}\frac{\partial}{\partial \mathbf{p}_a}\right)^2 \times f_a^{cl} g_{ab}^{cl}(\mathbf{r}_a, \mathbf{r}_b) \frac{d\mathbf{r}_a \, d\mathbf{r}_b}{V}, \tag{3.5}$$

which coincides with that obtained by Green[1]. Here,

$$g_{ab}^{cl}(\mathbf{r}_a, \mathbf{r}_b) = \exp(-\Phi_{ab}/kT) - 1$$

is the pair correlation function of the Boltzmann gas in the classical approximation.

From (3.5) we find the known expression (cf. Sec. 33 of the book by Landau and Lifshits[11]) for the mean kinetic energy in a state of local equilibrium

$$\int \frac{\mathbf{p}_a^2}{2m_a} f_a^{id} \, dp_a = \frac{3}{2}kT + \frac{\hbar^2}{24 m_a kT}\sum_b n_b \int \frac{\partial^2 \Phi_{ab}}{\partial \mathbf{r}_a^2} g_{ab}^{cl} \frac{d\mathbf{r}_a \, d\mathbf{r}_b}{V}.$$

The expression (3.1) for the internal-energy density can be represented in the form

$$U = U_{id} + U_{corr}. \tag{3.6}$$

Here U_{id} is the energy density of the ideal gas. The quantity U_{corr} is determined by the potential energy (the last term in (3.1)) and by the correlation part of the average kinetic energy.

We shall study the expression for U_{corr} in the case of local statistical equilibrium. It follows from (3.4) that the correlation correction to the total kinetic energy of all the components is

$$U_{corr}^{kin} = \frac{1}{2}\sum_{ab} n_a n_b V \int \left(\frac{\mathbf{p}_a^2}{2m_a} + \frac{\mathbf{p}_b^2}{2m_b} - 3kT\right) f_{ab}^0 \frac{d\mathbf{x}_a \, d\mathbf{x}_b}{(2\pi\hbar)^6}, \tag{3.7}$$

where $f_{ab}^0(\mathbf{x}_a, \mathbf{x}_b)$ is the equilibrium quantum distribution function.

The function

$$U_{corr} = U_{corr}^{kin} + U_{pot}$$

can be expressed in terms of the simpler distribution function of the coordinates, $\rho_{ab}(\mathbf{r}_a, \mathbf{r}_a, \mathbf{r}_b, \mathbf{r}_b)$. This is defined by the expression (3.2). From (3.1) and (3.7),

$$U_{corr} = \frac{V}{2}\sum_{ab} n_a n_b \int (H_{ab} - 3kT) f_{ab}^0 \frac{d\mathbf{x}_a \, d\mathbf{x}_b}{(2\pi\hbar)^6}.$$

Putting $\Phi_{a,b}(\lambda) = \lambda \Phi_{ab}$, we write this expression in the form

$$U_{corr} = \frac{V}{2}\sum_{ab} n_a n_b \int_0^1 d\lambda \frac{\partial}{\partial \lambda}\int H_{ab}(\lambda) f_{ab}^0 \frac{d\mathbf{x}_a \, d\mathbf{x}_b}{(2\pi\hbar)^6}. \tag{3.8}$$

On the other hand

$$\frac{\partial}{\partial\lambda}\int H_{ab}(\lambda) f_{ab}{}^{0}\frac{d\mathbf{x}_a\,d\mathbf{x}_b}{(2\pi\hbar)^6} = -\frac{\partial^2}{\partial\beta\,\partial\lambda}\ln \operatorname{Sp}\exp[-\beta\hat{H}_{ab}(\lambda)] \tag{3.9}$$
$$= \frac{\partial}{\partial\beta}\beta\int\frac{\Phi_{ab}(\lambda)}{\lambda} f_{ab}{}^{0}\frac{d\mathbf{x}_a\,d\mathbf{x}_b}{(2\pi\hbar)^6} = \frac{\partial}{\partial\beta}\beta\int\frac{\Phi_{ab}(\lambda)}{\lambda}\rho_{ab}{}^{0}\frac{d\mathbf{r}_a\,d\mathbf{r}_b}{V}.$$

Substituting (3.9) into (3.8) and taking the definition of the potential energy into account (cf. (3.1)), we obtain

$$U_{\text{corr}} = \frac{1}{2}\frac{\partial}{\partial\beta}\beta\sum_{ab} n_a n_b\int_0^1\frac{d\lambda}{\lambda}\Phi_{ab}(\lambda)\rho_{ab}{}^{0}\frac{d\mathbf{r}_a\,d\mathbf{r}_b}{V}$$
$$= \frac{\partial}{\partial\beta}\beta\int_0^1\frac{d\lambda}{\lambda}U_{\text{pot}}(\lambda,\beta). \tag{3.10}$$

The distribution function in the coordinates is defined by the expression (3.2).

The latter formula in (3.10) coincides with the known result from equilibrium quantum statistical theory (cf., e.g., Appendix 7 in Balescu's book[5]). The expression (3.10) determines the second virial coefficient in the expansion of the internal energy in the density.

4. KINETIC EQUATIONS AND THERMODYNAMIC FUNCTIONS FOR A NONIDEAL PLASMA

In the kinetic equations for an ideal plasma, the interaction at small distances is usually treated in an approximation of perturbation theory in the interaction. Allowance for the dynamical polarization ensures screening at large distances[5-7]. For a nonideal plasma, the strong interactions of charged particles at small distances play a significant role. The derivation of the quantum kinetic equations for a nonideal plasma with simultaneous allowance for the strong interactions at short distances and for the dynamical polarization is the purpose of the present Section.

Taking exact account of the dynamical polarization leads to very complicated equations; we therefore confine ourselves to taking the averaged dynamical polarization into account. This enables us to use as our starting point again the system of equations (2.1)—(2.3), with the difference that in Eq. (2.3) for the function ρ_{ab} the Coulomb potential $\Phi_{ab}(r) = e_a e_b/r$ is replaced by an effective potential $\tilde{\Phi}_{ab}$ that takes into account the averaged dynamical polarization. In expression (2.2), the potential Φ_{ab} remains unchanged. This means that the polarization only affects the form of the function ρ_{ab} (or of the correlation function g_{ab}).

The form of the effective potential is determined from the following considerations. The correlation function determining the form of the collision integral in the zeroth approximation in the retardation is proportional to $\Phi_{ab}(k)/|\epsilon(\mathbf{k}\cdot\mathbf{v}, \mathbf{k})|^2$. Here $\epsilon(\omega, \mathbf{k})$ is the quantum dielectric permittivity, given by the expression

$$\varepsilon(\omega,\mathbf{k}) = 1 + \sum_a\frac{4\pi e_a{}^2 n_a}{k^2}\int\frac{[f_a(\mathbf{p}_a + {}^1/_2\hbar\mathbf{k}) - f_a(\mathbf{p}_a - {}^1/_2\hbar\mathbf{k})]}{\omega - \mathbf{k}\mathbf{v} + i\Delta}\frac{V\,d\mathbf{p}_a}{(2\pi\hbar)^3}.$$

Hence, one of the possible definitions of an effective potential that takes into account the averaged dynamical polarization is

$$\tilde{\Phi}_{ab}(k) = \Phi_{ab}(k)\frac{1}{2}\frac{\sum_c e_c{}^2 n_c}{\sum_d e_d{}^2 n_d}.$$

$$\times\int\frac{f_c(\mathbf{p}_c + {}^1/_2\hbar\mathbf{k}) + f_c(\mathbf{p}_c - {}^1/_2\hbar\mathbf{k})}{|\varepsilon(\mathbf{k}\mathbf{v},\mathbf{k})|^2}\frac{V\,d\mathbf{p}_a}{(2\pi\hbar)^3}. \tag{4.1}$$

If the averaging in (4.1) is performed over a function f_c corresponding to local equilibrium, and quantum effects in the dynamical polarization are not taken into account, then it follows from (4.1) that

$$\tilde{\Phi}_{ab}(k) = \frac{\Phi_{ab}(k)}{(1 + r_d{}^{-2}k^{-2})}, \qquad \tilde{\Phi}_{ab}(r) = \frac{e_a e_b}{r}\exp\left(-\frac{r}{r_d}\right), \tag{4.2}$$

i.e., the effective potential coincides with the Debye potential.

We remark that other methods of introducing the effective potential are possible. For example,

$$\tilde{\Phi}_{ab}(k) = \Phi_{ab}(k)/\varepsilon(0, k).$$

On introduction of an effective potential into Eq. (2.3), $\Phi_{ab} \to \tilde{\Phi}_{ab}$. In accordance with this, the eigenfunctions of (2.5) are replaced by the eigenfunctions $\tilde{\psi}_{\mathrm{p_a p_b}}(\mathrm{r_a}, \mathrm{r_b})$ of a system of two particles with interaction potential $\tilde{\Phi}_{ab}$. It follows from this that all the subsequent expressions of Sec. 2 will also be valid, in the framework of the model under consideration, for a nonideal plasma, if in them we carry out the replacement

$$\psi_{\mathbf{p}_a\mathbf{p}_b}(\mathbf{r}_a,\mathbf{r}_b) \to \tilde{\psi}_{\mathbf{p}_a\mathbf{p}_b}(\mathbf{r}_a,\mathbf{r}_b).$$

We denote the expression for the density matrix thus altered by $\tilde{\rho}_{ab}$. This matrix then determines all the expressions for the thermodynamic functions for a nonideal plasma.

The simplest expression for the effective potential $\tilde{\Phi}_{ab}$ is the expression (4.2). We shall show to what extent the expressions for the thermodynamic functions calculated in the framework of a model of a nonideal plasma with this effective potential agree with the corresponding expressions of the equilibrium quantum statistical theory. It follows from formula (3.10) that the correlation part U_{corr} of the internal energy can be expressed in terms of the potential energy U_{pot}. We can also write the corresponding expression for the free energy[2]

$$F_{\text{corr}} = \int_0^1\frac{d\lambda}{\lambda}U_{\text{pot}}(\beta,\lambda).$$

It follows from the expression (3.1) that

$$U_{\text{pot}} = \frac{1}{2}\sum_{ab} n_a n_b\int\Phi_{ab}(|\mathbf{r}_a - \mathbf{r}_b|)\tilde{\rho}_{ab}{}^{1}(\mathbf{r}_a,\mathbf{r}_a,\mathbf{r}_b,\mathbf{r}_b)\,d(\mathbf{r}_a - \mathbf{r}_b), \tag{4.3}$$

where $\tilde{\rho}^1_{ab}$ is given by expression (3.2), if in it $\tilde{\psi}$.

Even with the simplest potential (4.2), the exact solution of the equation for $\tilde{\psi}_{\mathrm{p_a p_b}}$ is evidently unknown; therefore, we shall represent the expressions for the thermodynamic functions in the form of series in the Born parameter

$$\xi_{ab} = \frac{-e_a e_b}{kT\lambda_{ab}}; \qquad \lambda_{ab} = \left(\frac{1}{m_a} + \frac{1}{m_b}\right)\hbar\left(\frac{2kT_a}{m_a} + \frac{2kT_b}{m_b}\right)^{-1/2}.$$

In the Born approximation for a local equilibrium state, the Fourier component of the function $\tilde{\rho}^1_{ab}$ is determined by the expression

$$\tilde{\rho}_{ab}{}^{1}(\mathbf{k}) = (2\pi)^3\delta(\mathbf{k}) - \frac{m_a + m_b}{m_a kT_b + m_b kT_a}\tilde{\Phi}_{ab}(k)\,{}_1F_1\left(1, \frac{3}{2}, -\frac{1}{4}\lambda_{ab}{}^2 k^2\right), \tag{4.4}$$

where ${}_1F_1$ is the confluent hypergeometric function.

From (4.3) and (4.4),

$$U_{\text{pot}}=-2\pi\sum_{ab}n_an_be_a^2e_b^2\frac{m_a+m_b}{m_akT_b+m_bkT_a}r_dG\left(\frac{\lambda_{ab}}{r_d}\right),\tag{4.5}$$

$$G(\eta)=\frac{1}{\pi e_ae_br_d^2}\int_0^\infty d\rho\,\rho^2\int_0^\infty dk'\,\tilde{\Phi}_{ab}(k')e^{-\rho^2}\left\{\frac{2\rho}{\eta k'}\sin(\eta k'\rho)-\cos(\eta k'\rho)\right\},$$
$$k'=r_dk.\tag{4.6}$$

The expressions (4.4) and (4.6) are valid for arbitrary potential Φ_{ab}. For the Debye potential,

$$G(\eta)=\pi^{1/2}\eta^{-1}\{1-\exp(\eta^2/4)[1-\Phi(\eta/2)]\}.\tag{4.7}$$

Hence, for $\eta\ll 1$,

$$G(\lambda_{ab}/r_d)=1-\pi^{1/2}\lambda_{ab}/4r_d+O(\lambda_{ab}^2/r_d^2).\tag{4.8}$$

From (4.8) and (4.5) we obtain an expression, which, for an isothermal plasma, coincides with the result of the equilibrium theory[12-15]. Thus, in the first approximation in λ_{ab}/r_d, using the Debye effective potential, one obtains in the equilibrium limit the correct classical expressions for the thermodynamic functions and the correct quantum corrections of order (λ_{ab}/r_d).

We note that the expression (4.8) is practically independent of the choice of the effective potential $\tilde{\Phi}_{ab}$.

For isothermal local equilibrium for $(\lambda_{ab}/rd)\ll 1$, it is possible to carry out a calculation of the thermodynamic functions in any approximation in the Born parameter, by using the method developed by Ebeling et al.[13,14]. For the internal energy density, one obtains the expression

$$U=\frac{3}{2}kT\sum_a n_a-\frac{kT}{8\pi r_d^3}-\pi kT\sum_{ab}n_an_b\lambda_{ab}^3\left\{-\frac{5}{8}\pi^{1/2}\xi_{ab}^2\right.$$
$$\left.-\left(a+\ln\frac{\lambda_{ab}}{r_d}\right)\xi_{ab}^3+\pi^{1/2}\sum_{m=4}^{\infty}(m+3)\frac{\zeta(m-2)}{\Gamma(m/2)+1}\left(\frac{1}{2}\xi_{ab}\right)^m+O\left(\frac{\lambda_{ab}}{r_d}\right)\right\};\tag{4.9}$$

where $\zeta(x)$ is the Riemann zeta-function, $\Gamma(x)$ is the gamma-function, C is Euler's constant, and $a=\frac{1}{2}C+\ln 2-\frac{1}{3}$. This result agrees with the result of the equilibrium theory[13,14]. The only difference lies in the numerical value of the coefficient a in (4.9). This difference, however, is not important, since for a symmetric plasma ($e_e=-e_i$), the term with ξ_{ab}^3 in (4.9) vanishes.

Formula (4.9) implicitly takes into account the contribution of the bound states whose lifetime is much shorter than the time between collisions. In order to separate out this contribution more explicitly, we transform the expression (4.9) for a hydrogen plasma. We make use of the formula

$$\lambda_{ie}^3\sum_{m=4,6,8\ldots}^{\infty}(m+3)\frac{\zeta(m-2)}{\Gamma(m/2)+1}\left(\frac{1}{2}\xi_{ie}\right)^m$$
$$=\frac{\partial}{\partial\beta}\left\{\lambda_{ie}^3\sum_{s=2}^{\infty}\frac{1}{s!}\left(\frac{1}{4}\xi_{ie}^2\right)^s\sum_{n=1}^{\infty}\frac{1}{n^{2s-2}}\right\}=\frac{\partial}{\partial\beta}\left\{\lambda_{ie}^3\sum_{n=1}^{\infty}n^2(e^{-\beta E_n}-1+\beta E_n)\right\}.\tag{4.10}$$

Here we have taken into account that $\xi_{ie}^2/4n^2=-\beta E_n$, where E_n are the energy levels.

Formula (4.10) determines the total contribution to (4.9) from terms of a series in even powers of ξ_{ab}, starting from ξ_{ab}^4. This contribution determines the contribution of the bound states. We see that it is determined by the derivative of the hydrogen-atom partition function, truncated by the Planck-Larkin method (cf.[15] and the review[16]).

We denote the contribution of the other terms of the series in ξ_{ab} by $K^*(\beta)$. Then the expression for the internal energy of the hydrogen plasma can be written in the form

$$U=\frac{3}{2}kT(2n_e)-\frac{kT}{8\pi r_d^3}\tag{4.11}$$
$$-n_e^2\frac{\partial}{\partial\beta}\left[8\pi^{3/2}\lambda_{ie}^3\sum_{n=1}^{\infty}n^2(e^{-\beta E_n}-1+\beta E_n)+K^*(\beta)\right]+\ldots$$

The residue K^* depends weakly on temperature. Analytic and numerical estimates of this function give the good interpolation formula

$$K^*(\beta)\approx-4\pi^{3/2}\lambda_{ie}^3|\beta E_1|,$$

where E_1 is the ground-state energy of the hydrogen atoms.

To conclude this Section, we note that the second quantum distribution function in the coordinates, $\tilde{\rho}_{ab}^1(r_a, r_a, r_b, r_b)$, is finite for all $r=|r_a-r_b|$. The decrease at large distances is due to the allowance for the screening. That it is finite at short distances is due to quantum effects. For example, for a local-equilibrium state of a non-isothermal plasma with the Debye effective potential, from formula (4.4) we find

$$\tilde{\rho}_{ab}^1(r=0)=1-\frac{\pi^{1/2}(m_a+m_b)e_ae_b}{(m_akT_b+m_bkT_a)\lambda_{ab}}\exp\left(\frac{\lambda_{ab}^2}{4r_d^2}\right)\left[1-\Phi\left(\frac{\lambda_{ab}}{2r_d}\right)\right],$$

where $\Phi(x)$ is the error function.

For an isothermal plasma for $\lambda_{ab}/r_d\ll 1$, this expression takes the form

$$\tilde{\rho}_{ab}^1(r=0)=1-\pi^{1/2}\frac{e_ae_b}{\lambda_{ab}kT}$$

and coincides with the corresponding results of the work of Kelbg[17] and of Trubnikov and Elesin[12].

5. EFFECTIVE FIELD FOR A NONIDEAL PLASMA

Up to this point, in obtaining the collision integrals we have neglected the influence of external fields on the act of collision. But this effect plays a definite role in a nonideal plasma. The presence of an external field leads to the result that a third term, which is due to the electric field, appears in the collision integral, i.e., the kinetic equation has the form

$$\frac{\partial f_a}{\partial t}+e_a\mathbf{E}\frac{\partial f_a}{\partial \mathbf{p}_a}=I_{a1}+I_{a2}+I_{a3}.\tag{5.1}$$

Here the term I_{a3} is determined by formula (2.2) in which must be substituted the part of the density matrix that is proportional to the external field. The corresponding contribution to the density matrix can be written in the form

$$\tilde{\rho}_{ab}^3(\mathbf{r}_a',\mathbf{r}_a'',\mathbf{r}_b',\mathbf{r}_b'',t)=\frac{1}{i\hbar}\int_0^\infty d\tau\int d\mathbf{q}_a'\,d\mathbf{q}_a''\,d\mathbf{q}_b'\,d\mathbf{q}_b''\{\tilde{\Phi}_{ab}(|\mathbf{q}_a'-\mathbf{q}_b'|)$$
$$-\tilde{\Phi}_{ab}(|\mathbf{q}_a''-\mathbf{q}_b''|)\}\{\tilde{A}^E(-\tau)-\tilde{A}(-\tau)\}\rho_{ab}^{(0)}(\mathbf{q}_a',\mathbf{q}_a'',\mathbf{q}_b',\mathbf{q}_b'',t),\tag{5.2}$$

where the function $\tilde{A}$ is the ordinary transition matrix for two particles interacting through a potential Φ_{ab}, and $\tilde{A}^E$ is the corresponding transition matrix in the presence of an external field E. The momentum-transport equation for a plasma in an external field has the form

$$\frac{\partial}{\partial t}\langle \mathbf{p}_a\rangle^{\text{eff}} = e_a\mathbf{E}_a^{\text{eff}} + \int d\mathbf{p}_a\mathbf{p}_a I_{a1}\frac{V}{(2\pi\hbar)^3}, \tag{5.3}$$

in which

$$\frac{\partial}{\partial t}\langle \mathbf{p}_a\rangle^{\text{eff}} = \frac{\partial}{\partial t}\int d\mathbf{p}_a\mathbf{p}_a f_a\frac{V}{(2\pi\hbar^3)} - \int d\mathbf{p}_a\mathbf{p}_a I_{a2}\frac{V}{(2\pi\hbar)^3}, \tag{5.4}$$

$$\mathbf{E}_a^{\text{eff}} = \mathbf{E} + \frac{1}{e_a}\int d\mathbf{p}_a\,\mathbf{p}_a I_{a3}\frac{V}{(2\pi\hbar)^3} \tag{5.5}$$

In the local-equilibrium approximation, the last term in (5.4) goes to zero.

In the first order of perturbation theory in the Born parameter, for a local-equilibrium state we obtain from (5.2) and (5.5) the expression

$$\mathbf{E}_a^{\text{eff}} = \mathbf{E} - \frac{2}{e_a\hbar(2\pi)^3}\sum_b n_b\int_0^\infty d\tau\int d\mathbf{k}\,\mathbf{k}\Phi_{ab}(k)\tilde{\Phi}_{ab}(k)\exp\left\{-\frac{1}{2}k^2\tau^2\left(\frac{kT_a}{m_a} + \frac{kT_b}{m_b}\right)\right\}\sin\left\{\frac{1}{2}\mathbf{k}\mathbf{E}\left(\frac{e_a}{m_a} - \frac{e_b}{m_b}\right)\tau^2\right\}\sin\left\{\frac{1}{2}\hbar k^2\left(\frac{1}{m_a} + \frac{1}{m_b}\right)\tau\right\}. \tag{5.6}$$

Investigation of formula (5.6) shows that the quantity $|\mathrm{E}_a^{\text{eff}} - \mathrm{E}|/\mathrm{E}$ decreases with increase of the field E.

In a weak field, in the first approximation in E, the expression (5.6) takes the form

$$\mathbf{E}_a^{\text{eff}} = \mathbf{E}\left\{1 - \frac{4\pi e_a r_d}{3}\sum_b n_b e_b^2 \times\frac{(m_a + m_b)(e_a m_b - e_b m_a)}{(kT_a m_b + kT_b m_a)^2}K\left(\frac{\lambda_{ab}}{r_d}\right)\right\}, \tag{5.7}$$

$$K(\eta) = \frac{\pi^{1/2}}{2\eta}\left\{1 + \pi^{-1/2}\eta - \left(1 + \frac{\eta^2}{4}\right)e^{\eta^2/4}\left[1 - \Phi\left(\frac{\eta}{2}\right)\right]\right\} = 1 - \frac{3}{8}\pi^{1/2}\eta + O(\eta^2).$$

With the condition $\lambda_{ab}/r_d \ll 1$, we have for the field acting on the electrons

$$\mathbf{E}_e^{\text{eff}} = \mathbf{E}\left\{1 - \frac{e^2 kT_i}{3r_d kT_e(kT_e + kT_i)} + \frac{\pi^{3/2}\hbar n_e e^4}{2kT_e(2m_e kT_e)^{1/2}} + \ldots\right\}. \tag{5.8}$$

In the classical limit $\hbar \to 0$, the formula (5.8) coincides with the result of Kadomtsev[18]. We note that the factor

$$\tfrac{1}{3}(1 + 2^{-1/2})^{-1}.$$

was obtained[19] in the classical hydrodynamic limit in place of the factor $\tfrac{1}{3}$ in the second term of formula (5.8).

The correction to the external field is usually of the order of the plasma parameter. For a dense plasma, this parameter can be of the order of unity[16]. Hence, it follows that the relaxation field in a dense plasma gives a significant correction to the external field. The corrections to the external field reduce the conductivity of the plasma by the factor $\mathrm{E}_e^{\text{eff}}/\mathrm{E}$ compared with the "ideal" plasma. It is possible that this effect, together with allowance for the effect of screening on the collision integral I_{a1} which takes account of the dissipative processes, can explain the observed systematically low experimental data for the conductivity of a dense plasma compared with the usual theory[20-22].

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96