# INTERACTION BETWEEN A MODULATED RELATIVISTIC BEAM AND A PLASMA

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Results of an experimental and theoretical investigation of collective interaction between a modulated relativistic electron beam and a plasma are presented. The dependences of the modulated relativistic electron beam energy spectra at the exit of the interaction region, averaged over the pulse duration, on the plasma density are measured. The time variation of "instantaneous" energy spectra is also determined for a plasma density for which collective interaction between the beam and plasma is maximal. Moreover, the frequency spectrum and phase velocity of the excited oscillations and also the time variation of the microwave electric-field amplitude are measured. Coherent energy losses of a modulated relativistic beam and excitation of collective oscillations in a plasma due to the Vavilov-Cerenkov effect are observed. It is shown that oscillations with a narrow frequency spectrum can be excited in long plasma guides  $(l \sim 2 m)$  by employing relativistic modulated beams. The electric fields thus induced are mainly concentrated inside the plasma waveguide.

## 1. INTRODUCTION

In the relativistic energy region, the increments of the Cerenkov instabilities decrease<sup>[1,2] 1)</sup> as a consequence of the weakening of the reaction of the induced field on the motion of the beam particles (automodulation of the beam by the field) because of the relativistic increase of the beam electron mass. It is of interest, therefore, to investigate the possibility of increasing the intensity of the collective interaction of relativistic beams with a plasma.

As is well known, the effectiveness of exciting collective oscillations with a current of charged particles is determined by three factors: the radiation intensity in the elementary effect, the degree of coherence of the elementary radiators, and the intensity of the bunching of these radiators by the field of the induced oscillations<sup>[7-11]</sup></sup>. This determines, by the same token, the possible ways of increasing the intensity of the collective interactions in the relativistic energy region. One such way is the use of grouping of the beam in bunches by external or collective fields, which ensures a high degree of coherence in the spontaneous emission of the beam particles (within the boundaries of the produced bunches and between them). Then the automodulation of the beam by the radiation field becomes superfluous; this eliminates in turn the increase in the rise time of the field amplitude with increasing beam-electron energy, which is characteristic of the induced interaction process. Moreover, the weakening of the reaction of the induced field on the beam due to the relativistic electron mass increase ensures in the given case an increase in the maximum amplitude of the induced field<sup>[12]</sup>. Another way consists of creating the conditions for exciting instabilities that are based on the

elementary particle-field interaction processes in which the radiation intensity grows with the increase of the relativistic factor  $\gamma$  (for example, reflection from a moving mirror<sup>[10,13]</sup>, synchrotron or transition radiation<sup>[11]</sup>). Finally, by ensuring the conditions for excitation of transverse instabilities, in which the bunching intensity is determined by the transverse electron mass, it is possible to weaken the decrease, inherent in longitudinal instabilities, of the increment with increasing beam energy.

In this paper we set forth the results of an experimental and theoretical investigation of the collective coherent interaction between a plasma and a relativistic modulated electron beam from a linear electron accelerator.

#### 2. EXPERIMENT

To obtain the relativistic electron beam we used a linear electron accelerator with an iris-loaded wave-guide<sup>[14]</sup>. The beam parameters were: energy W = 2 MeV, current I<sub>0</sub> = 1 A, pulse width  $\tau \approx 2 \mu \text{sec}$ , modulation (packet-repetition) frequency f<sub>0</sub> = 2805 MHz, and beam diameter ~10 mm.

A block diagram of the apparatus is shown in Fig. 1.

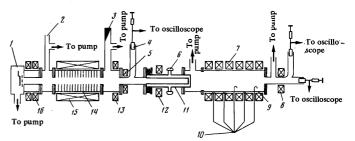


FIG. 1. Block diagram of the apparatus: 1-electron gun, 2-vacuum waveguide line, 3-microwave load, 4-Faraday cylinder, 5, 8, 12, 13, 16-beam correction and focusing systems, 6-electrodynamic valve, 7, 15-solenoids, 9-interaction chamber, 10-microwave diagnostics elements, 11-plasma gun, 14-iris-loaded waveguide.

<sup>&</sup>lt;sup>1)</sup>Here, as shown previously  $[^3]$  (see also  $[^{4-6}]$ ), the increase in the instability evolution time does not lead to a decrease in the effectiveness of the energy exchange between the beam and the plasma.

The plasma source is a coaxial plasma gun analogous to that described in<sup>[15]</sup>. The internal electrode of the gun is hollow, with a diameter  $\sim 20$  mm, for admitting the beam into the interaction chamber. The latter is a glass tube of diameter 100 mm and length 2 mm, placed in a constant, longitudinal, homogeneous, external magnetic field of intensity up to 2000 Oe. The beam is introduced into the interaction chamber through the plasma gun with the aid of correction and focusing systems. The initial pressure in the system is  $10^{-6}$  Torr. The plasma density was determined by measuring microwave signal cutoff at 10, 3, and 0.8 cm, and also with the aid of radio interferometers with wavelengths 3 and 0.8 cm. The experiments were performed on a decaying plasma whose density varied between  $n_{max} = 10^{13} \text{ cm}^{-3}$  to  $n_{min} \approx 10^{10} \text{ cm}^{-3}$ . The optimal plasma density, at which the modulation frequency  $\omega_{M} \equiv 2\pi f_{0}$  is close to the Langmuir plasma frequency  $\omega_{\rm p} \equiv (4\pi n_{\rm p} {\rm e}^2/{\rm m})^{1/2}$ , is equal to  $10^{11} {\rm \, cm^{-3}}$ .

In the experiments we measured the dependence, of the energy spectra of the beam at the exit from the interaction region, averaged over the duration of pulse<sup>[16]</sup>, on the plasma density, and determined in the same way the time variation of the "instantaneous" energy spectra (averaged over a time interval  $\Delta t \approx 0.15 \ \mu \text{sec} \ll \tau$ ) for the plasma density at which the collective interaction of the beam with the plasma was maximal. In addition, we measured the frequency spectrum and phase velocity of the induced oscillations and also the time of variation of the amplitude of the microwave electric field.

The energy spectra of beam electrons passing through the interaction chamber was measured with the aid of a magnetic analyzer at the exit from the interaction chamber (see Fig. 1). For every fixed value of the energy W, we made at least 10 measurements of the electron current I at the exit from the analyzer. The rms error in the current measurements did not exceed 2 to 3%. The beam energy distribution functions thus obtained, averaged over the pulse duration, are shown in Fig. 2 for several values of plasma density and for no plasma (Imax is the maximum value of the current in the energy spectrum of the beam without the plasma). As is evident from this diagram, the beam energy losses have a resonant dependence on the plasma density. The maximum loss is observed at a plasma density  $n_p\approx 10^{11}~\text{cm}^{-3}$  (Fig. 2c) and the shift of the maximum of the energy spectrum of the electron beam as a result of the interaction with the plasma reaches 150-250 keV, which is about 10% of the initial beam energy. A considerable fraction of the beam electrons  $(\sim 15\%)$  is accelerated by roughly 100 keV. At plasma densities greater or smaller than the optimum value  $(n_p \approx 10^{12} \text{ cm}^{-3} \text{ and } n_p \approx 5 \times 10^{10} \text{ cm}^{-3})$  the energy loss of beam electrons is much less (Figs. 2b and 2d), while for  $n_p \gtrsim 5 \times 10^{12} \text{ cm}^{-3}$  and  $n_p \leq 10^{10} \text{ cm}^{-3}$  the loss vanishes (Figs. 2a and 2e). Thus, as expected, the maximum effectiveness of beam-plasma interaction is observed when the beam modulation frequency  $\omega_{\rm M}$ is close to the plasma Langmuir frequency  $\omega_{\rm D}$ .

The energy loss of the beam in a plasma is essentially dependent on the beam current. Figure 3 shows the beam energy distribution function at the exit from the plasma, measured at the optimum plasma density

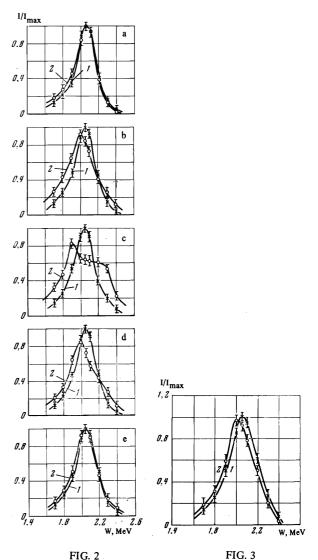


FIG. 2. Electron energy spectra at a beam current = 1 A. Curves 1-without plasma, curves 2-with plasma:  $a-n_p \sim 5 \times 10^{12}$  cm<sup>-3</sup>,  $b-n_p \sim 10^{12}$  cm<sup>-3</sup>,  $c-n_p \sim 10^{11}$  cm<sup>-3</sup>,  $d-n_p \sim 5 \times 10^{10}$  cm<sup>-3</sup>,  $e-n_p \sim 10^{10}$  cm<sup>-3</sup>.

FIG. 3. Electron energy spectra at a beam current 0.5 A and  $n_D \sim 10^{11} \text{ cm}^{-3}$ : 1-with plasma, 2-without plasma.

 $(n_p \approx 10^{11} \text{ cm}^{-3})$  for a beam current  $I_0 = 0.5 \text{ A}$  (curve 2). A comparison of Figs. 2c and 3 shows that halving the beam current leads to a substantial decrease in energy losses even at the optimum value of the plasma density<sup>2)</sup>.

For the case of maximum beam-plasma interaction  $(n_p \approx 10^{11} \text{ cm}^{-3})$ , we determined in the same way the instantaneous beam energy distribution functions (resolution time ~0.15  $\mu$ sec, which is small compared to the pulse duration  $\tau \approx 2$  msec). To this end, oscillograms of the beam electron current at the exit from the magnetic analyzer were obtained at fixed values of

<sup>&</sup>lt;sup>2)</sup> The strong dependence of the loss on the beam current and on the plasma density explains the negative result gotten by Mendell and Holt, where the conditions for coherent beam-plasma interaction ( $\omega_p \approx \omega_M$ ) were not met and the beam current was too small ( $I_0 \approx 0.15$  A).

the electron energy (3-4 oscillograms for 8-10 energy)values). Typical current oscillograms are shown in Fig. 4. The energy dependences of the current were then constructed from these oscillograms for each instant of time t, reckoned from the start of the beamcurrent pulse. The resulting distributions are shown in Figs. 5a-5d for instants of time t equal to 0.3, 0.5, 1.0 and 1.6 msec (curves 2; curves 1 refer to the case without plasma). For comparison, Fig. 2 shows the beam distribution functions at the exit from the interaction chamber, averaged over the pulse duration, in the presence of the plasma ( $n_p\approx\,10^{11}\;\text{cm}^{-3})$  and without it. With the aid of the "instantaneous" spectra it is possible to construct the time dependence of the maximum of the beam electron energy spectrum at the exit from the plasma. Such a dependence is shown in Fig. 6a. It is seen from the figure that in the initial stage the beam energy loss in the plasma increases almost linearly in time and reaches its maximum at t = 0.5msec. During that time about 1500 bunches pass through the plasma. During the last instants of time the losses decrease, and accelerated particles appear towards the end of the pulse (t  $\gtrsim 1.3-1.5 \ \mu \text{ sec}$ ).

In the course of the experiments we also measured the high-frequency field induced in the plasma as a result of collective interaction with the relativistic modulated electron beam. We measured the field component  $E_{Z}$  inside and outside the plasma waveguide. Field measurements outside the plasma were carried out by means of a section of rectangular 10-cm waveguide placed outside the glass tube and oriented with the broad side perpendicular to the axis of the plasma resonator, as well as with the aid of an electric probe (oriented along the  $E_z$  component of the field, which may be situated either inside or outside the plasma waveguide. The probe is a section of HF cable terminating in an antenna prong  $(l \sim \lambda/4)$  and placed inside a glass tube of 10 mm diameter, which can be moved across the plasma waveguide. To measure the field distribution along the axis of the system, three such

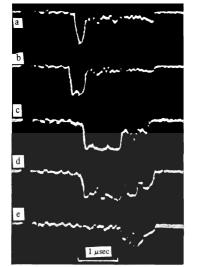


FIG. 4. Current oscillograms of an electron beam that has traversed the plasma, for different sections of the spectrum: a-W = 1.7 MeV, b-W = 1.8 MeV, c-W = 2.0 MeV, d-W = 2.2 MeV, e-W = 2.3 MeV.

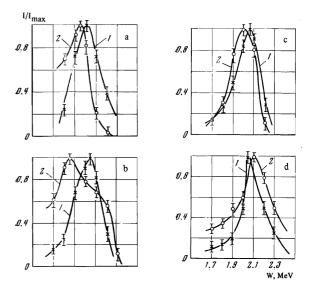


FIG. 5. Time variation of beam spec trum without (1) and with (2) plasma: a-t = 0.3 msec, b-t = 0.5 msec, c-t = 1.0 msec, d-t = 1.6 msec.

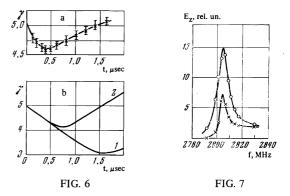


FIG. 6. Time dependence of energy loss of beam traversing a plasma: a-experimental, b-theoretical;  $1-\kappa = 0$ ,  $2-\kappa = 4 \times 10^{-4}$ .

FIG. 7. Frequency spectra of field component  $E_z$ : O-with plasma, X-without plasma (n = 10<sup>11</sup> cm<sup>-3</sup>).

probes were introduced into the chamber. The probe was connected with a coaxial cable to a resonant wavemeter (with half-bandwidth  $\Delta f \sim 1$  MHz), whose signal was fed to the oscilloscope. The measured frequency spectrum of the oscillations induced in the plasma, at a fixed position of the probe within the chamber, is shown in Fig. 7. The maximum of the spectrum corresponds to the accelerator working frequency  $(f_0)$ = 2805 MHz), while the half-width of the spectrum in the presence of the plasma ( $\Delta f_p \approx 10-12$  MHz) does not exceed the half-width of the spectrum without the plasma ( $\Delta f_0 = 8$  MHz) by more than a factor of onehalf. Such an insignificant difference in the spectra shows that it is possible to induce oscillations with narrow frequency spectra in a plasma with the aid of modulated relativistic beams.

With the aid of the probes we measured also the amplitude (in relative units) of the field induced by the beam for different values of the plasma density. The wave meter was set at a frequency  $f_0 = 2805$  MHz, corresponding to the intensity maximum in the frequency

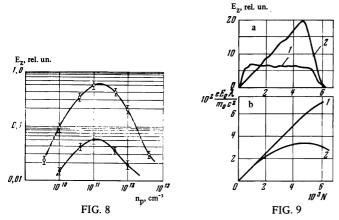


FIG. 8. Field amplitude  $E_z$  at different distances from the axis of the waveguide, vs the plasma density: O-r = 2 cm,  $\bullet-r = 5 \text{ cm}$ . FIG. 9. Time variation of field amplitude  $E_z(n_p \sim 10^{11} \text{ cm}^{-3})$ : a-

experiment (1-without plasma, 2-with plasma), b-theory (1- $\kappa = 0$ , 2- $\kappa = 4 \times 10^{-4}$ ).

spectrum of the signal. Figure 8 shows the so measured dependence of the field amplitude  $E_Z$  inside the interaction chamber on the plasma density. As expected, it shows a resonant character with the maximum field amplitude corresponding to a plasma density  $n_p \sim 10^{11}$  cm<sup>-3</sup>, at which the most effective deceleration of the beam in the plasma is observed (see Fig. 2).

Measurement of the longitudinal field component  $E_Z$  by external probes showed that outside the interaction chamber the amplitude of that field is notably less (by an approximate factor 10) than inside the plasma resonator (Fig. 8).

With the same probes we measured the time dependence of the field amplitude  $E_z$  in the plasma at the optimum value of the plasma density  $(n_p \sim 10^{11} \text{ cm}^{-3})$ . The results are shown in Fig. 9. For comparison, the same figure also shows the oscillogram of the field in the chamber with the plasma absent. As seen from Fig. 9, in the presence of plasma the field amplitude increases linearly in a time  $t \sim 1.2-1.4$  msec, which exceeds by more than two orders of magnitude the transit time of beam particles through the interaction region (~ $6.5 \times 10^{-9}$  sec.). After reaching a maximum, the field amplitude falls off quickly, and at exactly that instant accelerated particles appear in the energy spectrum at the exit from the plasma (see Fig. 6a).

We measured the phase velocity of the wave excited by the beam-plasma interaction. The measurements were made in two ways: with a correlation meter and with a phase analyzer (phase meter)<sup>[18]</sup>. In the first case the signals from two probes oriented along the  $E_z$ component of the field and situated 7 cm apart along the interaction chamber were summed and then fed into a square-law detector. A coaxial line of variable length was placed in the circuit of the first probe. By varying the length of the coaxial line it was possible to measure the wavelength in the absence of plasma,  $\lambda_b$ , and for any given plasma density,  $\lambda_g$ , i.e., to measure the phase velocity of the wave  $V_{ph} = \lambda_g f_0$ .

In the second case, the signals from the abovementioned probes were fed to the entrance of a 10-cm differential phase analyzer (phasemeter). With the aid of a variable-length coaxial line in the circuit of the first probe, the system was adjusted to give a zero signal at the exit from the phasemeter in the absence of plasma. In the presence of the plasma, the signal from the second probe was shifted in phase relative to the first. It was possible to restore the zero reading of the output signal by varying the length of the coaxial line. From the difference in lengths of the coaxial line it was possible to determine the phase shift  $\Delta \varphi$  and then to calculate the phase velocity of the move  $V_{ph} = c \left[1 + \Delta \varphi / \varphi_0\right]^{-1}$  where  $\varphi_0$  is the initial phase.

The measurements thus carried out showed that in the case of maximum interaction of a relativistic electron beam with a plasma the phase velocity of the wave in the plasma agrees with the velocity of the beam within the limits of experimental error (10-15%).

The results presented above pertain to the case when the plasma density attenuated from the beginning of the wave guide to the end by a factor of six in a length of one meter. Decreasing the density differential in this sector by a factor 2.5 increased the loss to 300-350 keV. The electron energy distribution function was then similar to that shown in Fig. 2c (the plasma density at the center of the interaction region was  $n \sim 10^{11}$  cm<sup>-3</sup>).

We also measured the dependence of the losses on the energy spread of the beam electrons. At an energy  $W \sim 3.5$  MeV and an energy spread  $\Delta W \sim 1$  MeV at the entrance to the interaction region the beam energy distribution function at the exit showed no real change.

### 3. THEORY

The theory of Cerenkov radiation of a particle moving along the axis of a gyrotropic plasma waveguide was considered in<sup>[19]</sup>. It turned out that for not too strong magnetic fields  $(\omega_H^2 < \omega_p^2)$  the radiation spectrum consists of a surface wave with frequency  $\omega < \omega_p / \sqrt{2}$  and volume moves whose frequencies  $\Omega_n$  have a point of condensation in the neighborhood of the upper hybrid frequency  $\Omega_h \equiv (\omega_H^2 + \omega_p^2)^{1/2}$ 

$$\Omega_n = \Omega_h \left( 1 - \frac{2\omega_p^2}{\Omega_h^2} \frac{\omega_H^2 a^2}{\lambda_n^2 c^2} \right), \qquad \frac{\omega_H^2 a^2}{\lambda_n^2 c^2} \ll 1.$$
 (1)

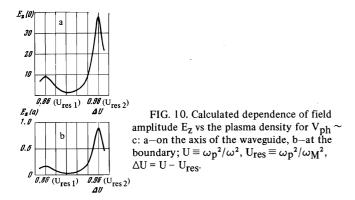
Here a is the radius of the waveguide,  $\lambda_n$  are the transverse wave numbers, which, for the conditions we are considering ( $a \approx 5$  cm,  $\gamma_0 \equiv (1 - \beta_0^2)^{-1/2} = 5$ ,  $n_p \approx 10^{11}$  cm<sup>-3</sup>, H<sub>0</sub> = 500 Oe) are, as expected from the dispersion equation<sup>[19]</sup>, close to the roots of the Bessel function<sup>3)</sup> J<sub>0</sub>(x):

$$J_0(\lambda_n) = \lambda_n^{-1} J_1(\lambda_n) \cdot \operatorname{const} + O(\lambda_n^{-1})$$

At a large running plasma density  $(\omega_p a > c)$  the losses due to excitation of the surface wave are exponentially small, so that the main component in the Cerenkov radiation is given by the volume wave. The corresponding amplitude of the wave field on the waveguide axis turns out to be

$$E_{i}^{(n)} = e/\lambda^{2}\lambda_{n}, \qquad (2)$$

<sup>&</sup>lt;sup>3)</sup>Numerical solution of the dispersion equation for the parameters indicated gives  $\lambda_1 \approx 2.30$ ; at  $n \ge 2$  the corresponding correction falls off in proportion to  $n^{-3/2}$ .



where e is the particle charge. Outside the plasma waveguide the fields of these waves fall off exponentially with the exponent  $\kappa_{\perp} \equiv 1/\gamma \pi$ . The amplitudes of the field component  $E_z$  of these waves, determined by their values at the surface of the plasma waveguide, prove to be small:

$$E_{z}^{(n)}(r=a)/E_{z}^{(n)}(r=0) = O(\lambda_{n}^{-*/2}).$$
(3)

The maximum amplitude of the field excited by the modulated beam can also be calculated from the formulas in<sup>[19]</sup>. Taking into account the finite loss in the plasma with the aid of the effective collision frequency  $\nu$ , we obtain the following expression for the amplitude of the field on the axis of the waveguide:

$$E_{z}^{(n)}(r=0) = \frac{vA_{n}}{[\omega_{p}^{2} - (\omega_{p}^{n})^{2}]^{2}/\omega_{p}^{4} + v^{2}B_{n}}, \qquad (4)$$

$$A_{n} = \frac{\lambda_{n}\omega_{p}^{2}Ne}{\omega_{H}^{2}a^{2}(\omega_{p}^{n})^{2}}, \quad B_{n} = \frac{2\lambda_{n}^{2}\omega_{p}^{4}c^{2}}{\omega_{H}^{2}\omega_{n}^{6}a^{2}}, \qquad \omega_{p}^{n} \equiv (\Omega_{n}^{2} - \omega_{H}^{2})^{\frac{1}{2}},$$

where N is the number of particles in a bunch.

To permit comparison with an experiment in which the modulation frequency  $\omega_M$  was fixed and the plasma density varied, we have included in (4) the possible deviation of the plasma frequency  $\omega_p$  from the corresponding resonant value  $\omega_p^n$ . Plots of the field amplitude against the plasma density calculated from (4) are shown in Fig. 10 for  $\nu \approx 5 \times 10^7 \text{ sec}^{-1}$ . As seen from Fig. 10 and from (4), this dependence has the form of a resonant curve with Lorentz line shape, while the resonances with  $n \ge 3$  are smeared out, and the first resonance has the largest amplitude. In addition, the field amplitude at the surface of the plasma waveguide (r = a) proves to be smaller than on the axis (r = 0) by a factor 40 even for the first resonance.

The relations presented above are valid for not too large beam currents, when the amplitude of the excited field is such that the shift  $\Delta z$  of a bunch under the action of the field during the time of passage through the plasma does not exceed the wavelength:

$$\Delta z = eE_0 l^2 / 2m_{\parallel} c^2 \ll \lambda, \qquad m_{\parallel} = m_0 \gamma^3. \tag{5}$$

If this inequality is not fulfilled, as happens in the case of pair collisions with sufficiently low frequencies and not too large beam currents, then the reaction of the field on the motion of the beam particles must be taken into account to find the excited field.<sup>4)</sup> If we integrate the kinetic equation for the beam by the method of characteristics and express the beam current in terms of the Lagrange trajectories of the electrons<sup>[6,12]</sup>, then we obtain, in the limiting case of little thermal broadening, the following equations for the amplitude R and the phase  $\psi$  of a plasma wave synchronous with the beam:

$$\frac{dR}{d\theta} = -2\mu \frac{\Gamma^{\gamma_2}}{R_1^2} \int_0^{\pi} d\xi I(\xi) \left[R_2^2 \Gamma^{\gamma_2} \sin^2 \varphi -\gamma R_2 \sin \varphi \left(2 + R_2^2 \sin^2 \varphi\right)^{\gamma_2} \operatorname{sign} \cos \xi\right],$$

$$\int d\xi = -2\mu \frac{\Gamma^{\gamma_2}}{R_1^2} \int_0^{\pi} d\xi I(\xi) \left[R_2^2 \Gamma^{\gamma_2} \sin^2 \varphi\right],$$
(6a)

$$R\left[\frac{u_{5}}{d\theta}-\varkappa(\theta)\right] = \mu_{0}^{f} d\xi I(\xi) \left\{ \Gamma[\sin\xi\cos\varphi \\ \times(1-\sin^{2}\xi\cos^{2}\varphi)^{\frac{1}{2}}-\sin\xi\cos\xi] + (6b) \right.$$
$$\left. \frac{\gamma^{\frac{1}{2}}}{R^{\frac{1}{2}}} \int_{1}^{\infty} \frac{dx(1-2\sin^{2}\xi\cos^{2}x)(1+R_{2}^{2}\sin^{2}x)}{\left[(1-\sin^{2}\xi\cos^{2}x)(2+R_{2}^{2}\sin^{2}x)\right]^{\frac{1}{2}}} \cdot \operatorname{sign}\cos\varphi \right\},$$

where  $\theta = \omega_M t$ ,  $R \equiv eEk_{\parallel}/m_0 \Omega^2$ ,  $\zeta \equiv \psi/2 + \pi/4$ ,  $2\xi \equiv \omega_M t_0$  is the phase of entry of the beam particle into the resonator,  $\Gamma \equiv \gamma^2 - 1$ ,  $\mu \equiv 4\pi e^2 I_0/m_0 \omega_M^3 Sl$ ,  $R_1^2 \equiv \Gamma R/\gamma$ ,  $R_2^2 \equiv R_1^2 \sin^2 \xi$ ,  $S \approx \pi a^2$ ,  $I(\xi)$  is the shape of the beam current at the entry into the plasma,  $I_0$  is the average beam current, and  $\kappa(\theta) \equiv \omega_{\Pi}(\theta)/\omega_M - 1$  is the deviation of the natural frequency of the plasma waveguide from the modulation frequency of the beam. Here the dependence of the amplitude  $\varphi$  of the phase oscillations of the beam particles on the amplitude R and phase  $\zeta$  of the field, and also on the entry phase  $\xi$ of the particle is determined by the equation

$$\pi \mathscr{L} - \gamma^{2} \{ [\arccos(\sin\xi\cos\varphi) - \pi/2] \operatorname{sign} \cos\xi + \pi/2 - \xi \} = (6c)$$
$$= \frac{\gamma^{3/2}}{R^{3/2}} \int_{0}^{6} \frac{dx(1 + R_{2}^{2}\sin^{2}x)}{[(2 + R_{2}^{2}\sin^{2}x)(1 - \sin^{2}\xi\cos^{2}x)]^{3/2}},$$

where  $\mathscr{L} = l/\lambda$ .

+

In the initial stage, when the field is weak and its effect on the motion of the beam insignificant, we obtain from (6) a linear growth of the field amplitude with time, due to the coherent addition of the fields of the spontaneous Cerenkov radiation of a sequence of beam bunches<sup>[12]</sup>. In the nonlinear stage, the reaction of the field on the particle motion can not be neglected, so that to explain the dynamics of beam-plasma interaction it is necessary to solve the system (6) numerically. The results of the field amplitude (Fig. 9b) and of the mean particle energy in the bunch (Fig. 6b) against the time (against the number of the electron bunch).<sup>5)</sup> Here the case  $\kappa = 0$  corresponds to strict synchronization of the beam with the plasma wave, and the case  $\kappa \neq 0$ 

<sup>5)</sup>In the calculation, the finiteness of the phase dimension of the bunch was taken into account by a special choice of the current form:

$$I(\xi) = \frac{1}{3} I_0 \sum_{k=-1}^{+1} \delta \left[ \xi - \frac{\pi}{4} + k \varphi_0 \right], \quad \varphi_0 = \frac{\pi}{9}$$

<sup>&</sup>lt;sup>4)</sup> Variation of the distribution function of the beam particles trapped by the wave in coordinate space as a consequence of their phase regrouping only stabilizes the growth of the field amplitude in the case of regular oscillations [ $^{6, 9-12, 20, 21}$ ]. For the irregular oscillations described by the quasilinear theory [ $^{3-5, 22, 23}$ ], the diffusion of the beam distribution function in momentum space proves to be more important.

takes into account the possibility of such synchronization being lost.

The regions of linear time increase of the field amplitude and of the beam particle energy losses correspond to a buildup of the energy of the field of spontaneous Cerenkov radiation of a coherent sequence of bunches. At  $\kappa = 0$ , saturation of the growth of the two quantities sets in as a consequence of the deceleration of the beam particles by the resonator field, as a result of which they enter the region of the accelerating phases. At  $\kappa \neq 0$ , the saturation of the growth of the field amplitude and energy loss results from the time variation of the field phase: at the point  $\zeta = -\pi/2$  the beam particles move through the plasma in zero field phase, so that the rate of amplitude growth becomes equal to zero. Further decrease of the field phase with time causes the beam particles to begin passing through the system in the accelerating field phase, and draws the energy buildup in the plasma at the expense of the collective coherent energy losses of the preceding bunches.

### 4. DISCUSSION OF RESULTS. CONCLUSIONS

Comparison of the experimental and theoretical results presented above shows that collective, coherent Cerenkov interaction of a modulated beam of relativistic electrons with a plasma was observed in the present experiment.

Agreement of the measured phase velocity of the induced wave with the velocity of the beam indicates that the observed interaction is based on the elementary Cerenkov effect. The relatively high beam-energy loss confirms the collective character of this interaction. In reality, the energy loss of an individual electron over the length of the whole apparatus, in agreement with (2) (see also<sup>[24]</sup>), do not exceed  $\Delta W_1 \approx eE_1 l$  $\approx 4 \times 10^{-6}$  eV, even in the absence of a density gradient. Allowance for coherence of the electron radiation within the limits of a bunch ( $N \approx 2 \times 10^{\circ}$ ) yields an upper limit of the energy loss of one electron,  $\Delta W_N$  $\sim$  N  $\Delta W_{1} \sim$  10 keV, i.e., one order of magnitude less than the measured value ( $\Delta W_{exp} \sim 200 \text{ keV}$ ). In this way, the experimentally observable beam energy losses can only be explained by the increase of the excited field at the expense of the coherent addition of the fields of the spontaneous Cerenkov radiation of a sequence of beam bunches entering the plasma.<sup>6)</sup> The presence of coherence is also confirmed by the linear time increase of the field amplitude and of the energy losses (Figs. 6 and 9). The important role of relativistic longitudinal mass increase of the beam electrons in this experiment manifested itself in a weakening of the deceleration of beam particles by the induced oscillation field. In fact, for energy losses  $\Delta W \equiv e E l$  $\sim$  200 keV, the displacement of the electron in the wave field according to (5) is at  $\gamma \sim 1$  one order of magnitude larger than the wavelength. In our experiment ( $\gamma = 5$ ) this effect proved to be negligible ( $\Delta z/\lambda$  $\sim$  0.2), and this ensured a high degree of radiation

conerence: the number of coherent bunches Ncoh  $\sim$  1500 (see Figs. 6 and 9) is almost two orders of magnitude greater than the number of bunches simultaneously present in the system. The actual energy losses at the field intensity maximum prove to be less than those calculated (see Figs. 6, 9, and 11) as a consequence of the spatial inhomogeneity of the plasma density and drift of the plasma resonant frequency (of the field phase) with time. The presence of plasma inhomogeneity causes the conditions for synchronization of the beam with the field to be satisfied over a limited portion of the interaction region<sup>[25]</sup>. As a result, the amplitude of the field induced by the beam is diminished, and the intensity of the energy loss from successive bunches of the beam in this field is weakened as well. It is exactly this plasma inhomogeneity which explains the experimentally observed smearing of the maximum on the plot of the field amplitude against the plasma density (see Figs. 8 and 10).

The presence of a decreasing section on the plots of the field amplitude and energy loss against time indicates that dissipative mechanisms that stabilize the field amplitude growth (see (4)) are not important under the conditions of this experiment. From the fact that the instant when accelerated particles start to appear occurs at the field-amplitude maximum (see Figs. 6 and 9) it follows that the field decay is due to change of the phase of this field with time, as a result of which the beam particles begin to transverse the plasma in the accelerating phase, and, in doing so, absorb the energy built up in the plasma at the expense of the coherent Cerenkov losses of the preceding bunches.

It follows from the theoretical curves in Figs. 6 and 9 that for a given spatial inhomogeneity of the plasma<sup>7</sup> this mechanism decreases the maximum value of the energy losses at the maximum by a factor of two. In the absence of a spatial plasma density gradient the maximum energy losses reach 1 MeV (see Fig. 11). Compensation for field-phase drift via control of the beam modulation frequency<sup>[26]</sup> increases that value by a factor of approximately one and a half.

Measurements of the field profile outside and inside the plasma waveguide (see Fig. 8) are found to agree both qualitatively and quantitatively with the theoretical estimates. When comparing theory and experiment qualitatively one should bear in mind that the field amplitude was measured inside the waveguide at a distance  $\Delta r = 2$  cm from the surface. The measured ratio of the field amplitudes on the waveguide axis and at its surface ( $R_{exp} \approx 13$ ) proves to be only one-third the theoretical value ( $R_{theor} \sim 40$ ).

The basic results of the experimental and theoretical studies detailed above can be formulated in the following manner.

1. We observed the effect of coherent energy losses from a relativistic electron beam and the excitation by the latter of collective oscillations in a plasma. The relative energy losses to collective interactions of an electron beam of comparatively small current  $I_0 \sim 1$  A

<sup>&</sup>lt;sup>6)</sup>Under the condition of this experiment, the characteristic growth time of the instability (stimulated emission, see [<sup>6</sup>]) is longer than the pulse duration by almost one order of magnitude.

<sup>&</sup>lt;sup>7)</sup>The spatial plasma inhomogeneity was taken into account in the calculation of the theoretical curves of Figs. 6 and 9 by choosing the beam-field coupling coefficient such as to reconcile the theoretical and experimental growth rate of the energy loss (field amplitude).

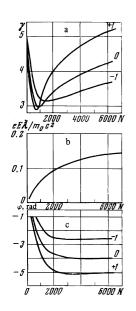


FIG. 11. Theoretical plots of the time variation of a) the energy loss, b) the field amplitude, c-the field phase; -1) beginning of bunch, 0) middle of bunch, +1) end of bunch.

is comparable with the losses of strong-current, unmodulated relativistic beams, thanks to the modulation and to the coherence effect. It is to be expected that further decrease of the degree of inhomogeneity of the plasma will permit an increase of these losses.

2. We have shown that the use of relativistic modulated beams makes it possible to excite oscillations with narrow frequency spectrum in plasma waveguides of great length  $(l \sim 2 \text{ cm})$ . The excited electric fields are concentrated for the most part inside the waveguide.

3. We have shown that the relativistic electronmass increase significantly weakens the reaction of the field on the motion of beam particles, thereby increasing the effectiveness of interaction of the relativistic beam with the plasma.

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