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THERMAL CONDUCTIVITY OF THE INTERMEDIATE STATE OF SUPERCONDUCTORS

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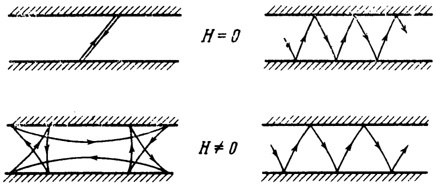
The tensor of the coefficients of thermal conductivity of electronic excitations in the normal layers in the intermediate state of a superconductor is calculated taking into account the special character, due to the Andreev reflection of electrons and holes from the boundaries with the superconducting layers, of the trajectories of the excitations in a magnetic field. A technique for averaging expressions containing the Green function of the electron system over the impurity distribution is developed by means of variational derivatives of the Green function with respect to the "external-source" field. The thermal conductivity of the normal layers is found to be substantially lower than that of the infinite normal metal and displays characteristic oscillations with variation of the normal-layer thickness.

A characteristic feature of the contact between a layer of the normal metal and a superconductor is, along with the proximity effect,[1] the distinctive reflection of normal electron excitations, with energy less than the magnitude of the gap Δ in the superconductor, from the interface of the layers. According to Andreev,[2] these excitations are reflected from the interface, being transformed from an electron to a hole and vice versa while the Fermi momentum is almost conserved and the sign of the velocity vector thus reversed. In other words, a coherent state is established, describing the correlations between the electrons and holes that are characteristic for a superconductor even deep into the normal layer, where the superconducting order parameter Δ is equal to zero.

In a previous paper by the authors,[3] it was shown formally that the described picture of reflection of excitations is conserved in the presence of a magnetic field. In this case, in the Schrödinger equation for the two-component wavefunction of the excitations, it is necessary to take into account the coherent phase difference of the superconducting order parameter that is established between two superconductors and compensates the increment of the vector potential of the magnetic field obtained on passing through the normal layer ($\Delta A_y = HL$; L is the layer thickness, and the y-axis is directed along the layer, perpendicular to the magnetic field). Taking this phase difference into account leads to non-conservation of the canonical momentum along the y-axis and, correspondingly, to a successive shift of the x-coordinate of the center of the orbit of a normal excitation by $\pm 2L$ for each reflection. At the same time, the ordinary (kinematic) momentum is conserved, and the velocity vector changes sign. As a whole, the motion of the excitations can be turned round into the successive passage, by an electron and a hole alternately, round a complete circle (see Fig. a). It is easily seen directly from the form of these trajectories that, as was shown earlier by one of the authors,[4] in the absence of a magnetic field the motion of an excitation along a straight-line segment confined between two superconductors is unstable to the switching on of an arbitrarily weak field. Unlike the case of ordinary specular reflection, in which a weak magnetic field perturbs the trajectory negligibly (see Fig. b), in the present case, the dynamics of the excitations becomes substantially more complicated, and this leads to a new spectrum[4,3] and, generally speaking, should be manifested in different kinetic characteristics of the normal-metal layer.[3]

The purpose of the present work is to calculate the thermal conductivity of the normal electron excitations in the most natural example of layers of a normal metal separated by superconducting layers, namely, in the intermediate state of a superconductor at temperatures $T \ll \Delta$, when the contribution of the superconducting layers to the thermal conductivity can be neglected. As is well known,[5] the magnetic field in the normal layers is parallel to the layer interfaces and is equal to the critical field H_c. The corresponding calculations of the thermal conductivity without allowance for the effect of a magnetic field were performed in [6].

In connection with the fact that the "isotopic spin" (cf. [3,4]) in the electron-hole space is an essentially quantum quantity, to calculate the conductivity tensor it is appropriate to make use of the general quantum linear-response methods,[7] although in the intervals between the reflections the motion of the excitations is semi-classical. A generalization of these methods to the case of thermal perturbations was proposed in [8,9]. Here we shall make use of the results of the work of Zubarev,[9] modifying them somewhat in such a way that, in the subsequent calculations, it is possible to apply the standard techniques of correlation functions and temperature Green functions.[10]



a b

According to [9], the nonequilibrium statistical operator of a system in the presence of a temperature gradient ∇T has the following form:[1)]

$$\rho = \exp\left\{-\int dV\left[\beta(\mathbf{r})\mathcal{H}(\mathbf{r}) - \int_{-\infty}^{0} dt\, e^{\nu t} q_\alpha(\mathbf{r}, t)\nabla_\alpha\beta(\mathbf{r})\right]\right\}, \qquad \nu \to +0. \quad (1)$$

Here $\beta(\mathbf{r}) = 1/T(\mathbf{r})$, and $T(\mathbf{r}) = T + \Delta T(\mathbf{r})$ is the local value of the temperature ($|\Delta T(\mathbf{r})| \ll T$, T = const); $\mathcal{H}(\mathbf{r})$ is the Hamiltonian density of the system. The heat flux operator $q_\alpha(\mathbf{r}, t)$ is considered in the Heisenberg picture:

$$q_\alpha(\mathbf{r}, t) = e^{i\mathcal{H}_0 t} q_\alpha(\mathbf{r}) e^{-i\mathcal{H}_0 t}, \quad \mathcal{H}_0 = \int dV\, \mathcal{H}(\mathbf{r}).$$

We shall describe the interaction of the electron excitations with nonmagnetic impurities randomly distributed in the normal layer by means of the scattering potential

$$U(\mathbf{r}) = \int dV'\, u(\mathbf{r} - \mathbf{r}')n(\mathbf{r}'),$$

where $n(\mathbf{r}) = \sum_i \delta(\mathbf{r} - \mathbf{r}_i)$ is the density of the distribution of impurities, $\mathbf{r}_i$ are their coordinates, and $u(\mathbf{r})$ is the potential of one impurity. Then the Hermitian Hamiltonian density $\mathcal{H}(\mathbf{r})$ of the normal excitations in the "isotopic-spin" representation has the following form:

$$\begin{gathered}\mathcal{H}(\mathbf{r}) = (\varphi^+(\mathbf{r}'), \quad {}^1\!/_2(\hat{\varepsilon}_\mathbf{r} + \hat{\varepsilon}_{\mathbf{r}'}{}^+) + \sigma_z U(\mathbf{r}), \quad \varphi(\mathbf{r}))_{\mathbf{r}'=\mathbf{r}},\\ \hat{\varepsilon}_\mathbf{r} = \sigma_z\xi(\hat{\mathbf{p}} - \sigma_z e\mathbf{A}(\mathbf{r})), \quad \hat{\varepsilon}_\mathbf{r}{}^+ = \sigma_z\xi(\hat{\mathbf{p}} + \sigma_z e\mathbf{A}(\mathbf{r})),\\ \xi(\mathbf{p}) = \frac{p^2}{2m} - \mu, \quad \hat{\mathbf{p}} = -i\nabla, \quad \hbar = c = 1, \quad \sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}. \end{gathered} \quad (2)$$

The operators $\varphi_s^+(\mathbf{r})$ and $\varphi_s(\mathbf{r})$ ($s = \pm 1$) are the canonical creation and destruction operators for electrons and holes and are related to the ordinary electron operators as follows:

$$\begin{pmatrix}\varphi_1\\ \varphi_{-1}\end{pmatrix} = \begin{pmatrix}\psi_\uparrow\\ \psi_\downarrow^+\end{pmatrix}, \qquad (\varphi_1{}^+, \varphi_{-1}{}^+) = (\psi_\uparrow{}^+, \psi_\downarrow)$$

and obey the Fermi anti-commutation relations

$$\begin{gathered}\{\varphi_s(\mathbf{r}), \varphi_{s'}{}^+(\mathbf{r}')\} \equiv \varphi_s(\mathbf{r})\varphi_{s'}{}^+(\mathbf{r}') + \varphi_{s'}{}^+(\mathbf{r}')\varphi_s(\mathbf{r}) = \delta_{ss'}\delta(\mathbf{r} - \mathbf{r}'),\\ \{\varphi_s(\mathbf{r}), \quad \varphi_{s'}(\mathbf{r}')\} = \{\varphi_s{}^+(\mathbf{r}), \quad \varphi_{s'}{}^+(\mathbf{r}')\} = 0.\end{gathered}$$

Using the expression (2) and the energy conservation law in operator form:

$$\partial\mathcal{H}(\mathbf{r})/\partial t + \operatorname{div}\mathbf{q}(\mathbf{r}) = 0, \quad i\partial\mathcal{H}(\mathbf{r})/\partial t = [\mathcal{H}(\mathbf{r}), \mathcal{H}_0],$$

we find the heat-flux operator $q_\alpha(\mathbf{r})$:

$$\begin{gathered}q_\alpha(\mathbf{r}) = {}^1\!/_2(\varphi^+(\mathbf{r}'), \quad \hat{v}_\alpha(\mathbf{r}, \mathbf{r}')(\varepsilon_\mathbf{r} + \sigma_z U(\mathbf{r}))\\ + (\hat{\varepsilon}_{\mathbf{r}'}{}^+ + \sigma_z U(\mathbf{r}'))\hat{v}_\alpha(\mathbf{r}, \mathbf{r}'), \; \varphi(\mathbf{r}))_{\mathbf{r}'=\mathbf{r}},\\ \hat{v}_\alpha(\mathbf{r}_1, \mathbf{r}_2) = \sigma_z[\hat{p}_1{}^\alpha - \hat{p}_2{}^\alpha - 2\sigma_z eA^\alpha(\mathbf{r}_1)]/2m.\end{gathered} \quad (3)$$

Confining ourselves to the study of the linear response of the system, we expand the operator ρ(1) in the perturbation ∇T, using formally the time representation and Liouville's equation for ρ:

$$i\frac{\partial\rho}{\partial t} = [\mathcal{H}_0 + \mathcal{H}_1, \rho], \quad \mathcal{H}_1 = \beta\int dV\int_{-\infty}^{0} dt\, e^{\nu t} q_\alpha(\mathbf{r}, t)\nabla_\alpha T(\mathbf{r}),$$

$$\begin{gathered}\rho = \rho_0 + \rho_1, \quad \rho_0 = \exp(-\beta\mathcal{H}_0),\\ \rho_1 = -i\int_0^\infty dt e^{-\eta t}[e^{-i\mathcal{H}_0 t}\mathcal{H}_1 e^{i\mathcal{H}_0 t}, e^{-\beta\mathcal{H}_0}], \quad \eta \to +0.\end{gathered} \quad (4)$$

By means of the expression found for ρ (cf. (4)) it is not difficult to obtain the connection between the average value of the heat flux and the corresponding correlation function:

$$\langle q_\alpha(\mathbf{r})\rangle = \frac{\operatorname{Sp}(\rho_1 q_\alpha(\mathbf{r}))}{\operatorname{Sp}\rho_0} = -\frac{1}{T}\int dV'\int_{-\infty}^{+\infty} t\, dt\,\langle q_\alpha; q_\beta\rangle(\mathbf{r}, \mathbf{r}'; t)\nabla_\beta T(\mathbf{r}').$$

The correlator $\langle q_\alpha, q_\beta\rangle(\mathbf{r}, \mathbf{r}'; t)$ is defined as follows:

$$\langle q_\alpha; q_\beta\rangle(\mathbf{r}_1, \mathbf{r}_2; t_1 - t_2) = i\theta(t_1 - t_2)\langle[q_\alpha(\mathbf{r}_1, t_1), \; q_\beta(\mathbf{r}_2, t_2)]\rangle, \quad (5)$$

where $\theta(t)$ is the Heaviside step function; the angular brackets denote averaging over the equilibrium state. Going over to the Fourier transform of the correlator

$$\langle q_\alpha; q_\beta\rangle(\omega) = \int_{-\infty}^{+\infty} dt\, e^{i\omega t}\langle q_\alpha; q_\beta\rangle(t)$$

and defining in a natural way the nonlocal thermal conductivity coefficient $\kappa_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2)$

$$\langle q_\alpha(\mathbf{r})\rangle = -\int dV'\,\varkappa_{\alpha\beta}(\mathbf{r}, \mathbf{r}')\nabla_\beta T(\mathbf{r}'), \quad (6)$$

we obtain for the latter the simple relation:

$$\varkappa_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{T}\frac{d}{id\omega}\langle q_\alpha; q_\beta\rangle(\mathbf{r}_1, \mathbf{r}_2; \omega)_{\omega\to 0}. \quad (7)$$

According to [10], the correlator (5) can be found in the temperature technique with a subsequent analytic continuation from the Matsubara frequencies to real frequencies in such a way that the function obtained be analytic in the upper half-plane of ω, in accordance with the definition (5). The calculation is conveniently performed by defining the temperature correlator in the form of a variational derivative (cf., e.g., [11])

$$\langle q_\alpha; q_\beta\rangle^T(x_1, x_2) = -\left(\frac{\delta\langle q_\alpha(x_1)\rangle}{\delta\chi_\beta(x_2)}\right)_{\chi=0}, \qquad x \equiv \mathbf{r}, \tau. \quad (8)$$

Here, the averaging is performed over a state which is described by a Hamiltonian with the "external sources" $\chi_\alpha(x)$:

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{int}, \quad \mathcal{H}_0 = \int dV\,\mathcal{H}(\mathbf{r}), \quad \mathcal{H}_{int} = \int dV\,\chi_\alpha(x)q_\alpha(\mathbf{r}). \quad (9)$$

Defining the temperature matrix Green function

$$\begin{gathered}K_{ss'}(x, x') = \langle T_\tau\tilde{\varphi}_s(x)\tilde{\tilde{\varphi}}_{s'}(x')\rangle,\\ \tilde{\varphi}(x) = e^{\tau\mathcal{H}}\varphi(\mathbf{r})e^{-\tau\mathcal{H}}, \; \tilde{\tilde{\varphi}}(x) = e^{\tau\mathcal{H}}\varphi^+(\mathbf{r})e^{-\tau\mathcal{H}}\end{gathered} \quad (10)$$

and using (3), we obtain an expression for the average heat flux appearing in (8):

$$\begin{gathered}\langle q_\alpha(x)\rangle_{\text{av}} = -\operatorname{Sp}(\hat{q}_\alpha(\mathbf{r})K(\tau - \tau'))_{\text{av}\;\tau'\to\tau+0},\\ \hat{q}_\alpha(\mathbf{r}) = {}^1\!/_2[\hat{v}_\alpha(\mathbf{r})(\hat{\varepsilon} + \hat{U}) + (\hat{\varepsilon} + \hat{U})\hat{v}_\alpha(\mathbf{r})].\end{gathered} \quad (11)$$

Here and below, the subscript "av" denotes averaging over all possible configurations of the impurities, and we have formally introduced the integral operators:

$$\begin{gathered}\hat{\varepsilon}(\mathbf{r}_1, \mathbf{r}_2) = \sigma_z\xi(\hat{\mathbf{p}}_1 - \sigma_z e\mathbf{A}(\mathbf{r}_1))\delta(\mathbf{r}_1 - \mathbf{r}_2),\\ \hat{U}(\mathbf{r}_1, \mathbf{r}_2) = \sigma_z U(\mathbf{r}_1)\delta(\mathbf{r}_1 - \mathbf{r}_2),\\ \hat{v}_\alpha(\mathbf{r}; \mathbf{r}_1, \mathbf{r}_2) = \hat{v}_\alpha(\mathbf{r}_1, \mathbf{r}_2)(\delta(\mathbf{r}_1 - \mathbf{r})\delta(\mathbf{r}_1 - \mathbf{r}_2)).\end{gathered}$$

In the following, we shall need an equation for the Green function (10); in the presence of the "sources" $\mathcal{H}_{int}$ (cf. (9)), this equation has the following appearance:

[1)] The complete system of equations for thermal perturbations (cf. [9]) contains, along with the energy conservation law, the particle-number conservation law corresponding to a perturbation $\nabla\mu$ of the chemical potential of the system. However, in metals, the electroneutrality condition ensures that the electron density and chemical potential μ are constant.

$$\left[\frac{\partial}{\partial\tau_1}+\hat{\varepsilon}+\hat{U}+\int dV\,\chi_\alpha(\mathbf{r},\tau_1)\,\hat{q}_\alpha(\mathbf{r})\right]K(\tau_1,\tau_2)=\delta(\tau_1-\tau_2). \tag{12}$$

The averaging over the distribution of impurities of the expressions which contain the Green function and appear in (11) can be performed, as is well known,[12] by averaging Dyson's equation. For small impurity concentrations, the averaging procedure reduces to a selective summation of a certain class of diagrams for the corresponding mass operator. In this paper, for this purpose we use the technique of variational derivatives of the Green function with respect to the field $\eta(\mathbf{r})$ of the fictitious "external sources" conjugate to the impurity density n(r). This approach makes it possible to develop general methods of averaging both the Green function itself and expressions of the type $\hat{\mathrm{U}}$K (cf. (11), (12)) and the different correlation functions. For this, we formally associate with the Green function $K(\tau_1, \tau_2)$ the functional

$$K(\tau_1,\tau_2;\eta)=\left(K(\tau_1,\tau_2)\exp\int dV\,n(\mathbf{r})\eta(\mathbf{r})\right)_{\text{av}}\Big/\left(\exp\int dV\,n(\mathbf{r})\eta(\mathbf{r})\right) \tag{13}$$

It is clear that the value of this functional for $\eta = 0$ is the Green function averaged over the position of the impurities:

$$K(\tau_1,\ \tau_2;\ \eta)_{\eta=0}=K_{\text{av}}(\tau_1,\ \tau_2).$$

By direct inspection, one may convince oneself of the validity of the following relation:

$$\left(n(\mathbf{r})K(\tau_1,\tau_2)\exp\int dV\,n(\mathbf{r})\eta(\mathbf{r})\right)_{\text{av}}\Big/\left(\exp\int dV\,n(\mathbf{r})\eta(\mathbf{r})\right)_{\text{av}}$$
$$=\frac{\delta K(\tau_1,\tau_2;\eta)}{\delta\eta(\mathbf{r})}+K(\tau_1,\tau_2;\eta)\,n(\mathbf{r};\eta). \tag{14}$$

Corresponding to Eq. (12) for the Green function is an equation for the functional (13), which, taking (14) into account, we can represent in the form:

$$\left\{\frac{\partial}{\partial\tau_1}+\hat{\varepsilon}+\hat{E}(\tau_1;\chi)+\int dV\,n(\mathbf{r};\eta)\,[\hat{u}(\mathbf{r})+\hat{v}(\mathbf{r},\tau_1;\chi)]\right\}$$
$$\times K(\tau_1,\tau_2;\eta)+\int dV\,[\hat{u}(\mathbf{r})+\hat{v}(\mathbf{r},\tau_1;\chi)]\frac{\delta K(\tau_1,\tau_2;\eta)}{\delta\eta(\mathbf{r})}=\delta(\tau_1-\tau_2), \tag{15}$$

where

$$\hat{E}(\tau;\chi)={}^1\!/_2\,[\varepsilon\hat{w}(\tau)+\hat{w}(\tau)\hat{\varepsilon}],\qquad \hat{w}(\tau)=\int dV\,\chi_\alpha(x)\hat{v}_\alpha(\mathbf{r}),$$

$$\hat{u}(\mathbf{r};\ \mathbf{r}_1,\ \mathbf{r}_2)=\sigma_z u(\mathbf{r}_1-\mathbf{r})\,\delta(\mathbf{r}_1-\mathbf{r}_2),\ \hat{v}(\mathbf{r},\ \tau;\ \chi)={}^1\!/_2[\hat{u}(\mathbf{r})\hat{w}(\tau)+\hat{w}(\tau)\hat{u}(\mathbf{r})].$$

Since the basic idea of the selective summation in the averaging over the impurities in [12] consists in the fact that the expansion in powers of the small concentration is performed in the mass operator and not in the Green function, which contains "dangerous" pole denominators, it is appropriate to introduce the mass operator $\hat{\mathrm{M}}$ into Eq. (15) in explicit form:

$$\left\{\frac{\partial}{\partial\tau_1}+\hat{\varepsilon}+\hat{E}(\tau_1;\chi)+\int dV\,n(\mathbf{r};\eta)\,[\hat{u}(\mathbf{r})+\hat{v}(\mathbf{r},\tau_1;\chi)]\right\}K(\tau_1,\tau_2;\eta)$$
$$+\int d\tau\,\hat{M}(\tau_1,\tau;\eta)K(\tau,\tau_2;\eta)=\delta(\tau_1-\tau_2). \tag{16}$$

We note immediately, however, that for $\eta = 0$ Eq. (16) goes over into the exact equation for the averaged Green function; therefore, it is sufficient to find the form of the operator $\hat{\mathrm{M}}$ for $\eta = 0$. Comparing (15) and (16) and using the known relation

$$\frac{\delta K(\tau_1,\tau_2;\eta)}{\delta\eta(\mathbf{r})}=-\iint d\tau_1'\,d\tau_2'\,K(\tau_1,\tau_1';\eta)\frac{\delta K^{-1}(\tau_1'\ \ \tau_2';\eta)}{\delta\eta(\mathbf{r})}K(\tau_2',\tau_2;\eta),$$

we find the equation which the mass operator $\hat{\mathrm{M}}$ satisfies:

$$\hat{M}(\tau_1,\tau_2;\eta)+\iint dV\,d\tau[\hat{u}(\mathbf{r})+\hat{v}(\mathbf{r},\tau_1;\chi)]K(\tau_1,\tau;\eta)$$
$$\times\frac{\delta\hat{M}(\tau,\tau_2;\eta)}{\delta\eta(\mathbf{r})}=-\iint dV_1\,dV_2\,[\hat{u}(\mathbf{r}_1)+\hat{v}(\mathbf{r}_1,\tau_1;\chi)]$$
$$\times K(\tau_1,\tau_2;\eta)\frac{\delta n(\mathbf{r}_2;\eta)}{\delta\eta(\mathbf{r}_1)}[\hat{u}(\mathbf{r}_2)+\hat{v}(\mathbf{r}_2,\tau_2;\chi)] \tag{17}$$

In Eq. (17), it is now possible to perform an expansion in the impurity concentration. It is not difficult to see that this expansion, generally speaking, corresponds to an expansion in the variational derivatives $\delta/\delta\eta$. With the aim of simplifying the calculations, we make use of the Born approximation for the electron-impurity scattering amplitude. Then in Eq. (17) we can confine ourselves to terms quadratic in the scattering potential.[2] In addition, for the subsequent calculation of the correlation function (8), it is sufficient to retain only terms linear in the perturbation $\chi_\alpha(x)$. Thus, from (17) we find

$$\hat{M}(\tau_1,\tau_2;\eta)=-\iint dV_1\,dV_2\,[\hat{u}(\mathbf{r}_1)+\hat{v}(\mathbf{r}_1,\tau_1;\chi)]\cdot$$
$$\times K(\tau_1,\tau_2;\eta)\frac{\delta n(\mathbf{r}_2;\eta)}{\delta\eta(\mathbf{r}_1)}[\hat{u}(\mathbf{r}_2)+\hat{v}(\mathbf{r}_2,\tau_2;\chi)].$$

Hence, taking into account the easily proved relations

$$n(\mathbf{r};\eta)_{\eta=0}=n_{\text{cp}}\equiv n,\ \left(\frac{\delta n(\mathbf{r}_1;\eta)}{\delta\eta(\mathbf{r}_2)}\right)_{\eta=0}=(n(\mathbf{r}_1)n(\mathbf{r}_2))_{\text{cp}}-n^2=n\delta(\mathbf{r}_1-\mathbf{r}_2);$$

we obtain the final expression for the operator $\hat{\mathrm{M}}$ for $\eta = 0$:

$$\hat{M}(\tau_1,\tau_2)\equiv\hat{M}(\tau_1,\tau_2;\eta)_{\eta=0}$$
$$=-n\int dV\,[\hat{u}(\mathbf{r})+\hat{v}(\mathbf{r},\tau_1;\chi)]K_{\text{av}}(\tau_1,\tau_2)\,[\hat{u}(\mathbf{r})+\hat{v}(\mathbf{r},\tau_2;\chi)]. \tag{18}$$

and the equation for the averaged Green function

$$\left\{\frac{\partial}{\partial\tau_1}+\hat{\varepsilon}+\hat{E}(\tau_1;\chi)+n\int dV\,[\hat{u}(\mathbf{r})+\hat{v}(\mathbf{r},\tau_1;\chi)]\right\}\cdot$$
$$\times K_{\text{av}}(\tau_1,\tau_2)+\int d\tau\,\hat{M}(\tau_1,\tau)K_{\text{av}}(\tau,\tau_2)=\delta(\tau_1-\tau_2). \tag{19}$$

The averaging over the impurity distribution in the formula (11) determining the average heat flux is performed analogously. Going over to the calculation of the correlator (8), we note that it follows from (11) and (14) that

$$\langle q_\alpha;q_\beta\rangle^T\,(x_1,x_2)=\frac{\delta}{\delta\chi_\beta(x_2)}\mathrm{Sp}\,\{{}^1\!/_2[\hat{v}_\alpha(\mathbf{r}_1)\,(\hat{\varepsilon}+\hat{U})+(\hat{\varepsilon}+\hat{U})\,\hat{v}_\alpha(\mathbf{r}_1)]$$
$$\times K(\tau_1,\tau_1+0)\}_{\text{av}\,\chi=0}=\frac{\delta}{\delta\chi_\beta(x_2)}\mathrm{Sp}\left\{q_\alpha^{(\varepsilon)}\,(\mathbf{r}_1)K_{\text{av}}(\tau_1,\tau_1+0)-\right.$$
$$\left.-\iiint dV\,d\tau_1'\,d\tau_2'\,\hat{Q}_\alpha(\mathbf{r},\mathbf{r}_1)K_{\text{av}}(\tau_1,\tau_1')\frac{\delta K^{-1}(\tau_1',\tau_2';\eta)}{\delta\eta(\mathbf{r})}K_{\text{av}}(\tau_2',\tau_1+0)\right\}_{\eta=0,\ \chi=0}, \tag{20}$$

where

$$\hat{q}_\alpha^{(\varepsilon)}(\mathbf{r})={}^1\!/_2[\hat{v}_\alpha(\mathbf{r})\,(\hat{\varepsilon}+n\hat{u}(0))+(\hat{\varepsilon}+n\hat{u}(0))\,\hat{v}_\alpha(\mathbf{r})],$$

$$\hat{u}(0)=\int dV\,\hat{u}(\mathbf{r}),\qquad \hat{Q}_\alpha(\mathbf{r}_1,\mathbf{r}_2)={}^1\!/_2[\hat{u}(\mathbf{r}_1)\hat{v}_\alpha(\mathbf{r}_2)+\hat{v}_\alpha(\mathbf{r}_2)\hat{u}(\mathbf{r}_1)].$$

The functional differentiation in (20) is performed taking into account the relations

$$\left(\frac{\delta K^{-1}(\tau_1,\tau_2;\eta)}{\delta\eta(\mathbf{r})}\right)_{\eta=0}=n[\hat{u}(\mathbf{r})+\hat{v}(\mathbf{r},\tau_1;\chi)]\delta(\tau_1-\tau_2),$$

$$\left(\frac{\delta\hat{w}(\tau_1)}{\delta\chi_\alpha(\mathbf{r},\tau_2)}\right)_{\chi=0}=\hat{v}_\alpha(\mathbf{r})\,\delta(\tau_1-\tau_2),\qquad \left(\frac{\delta\hat{v}(\mathbf{r}_1,\tau_1;\chi)}{\delta\chi_\alpha(\mathbf{r}_2,\tau_2)}\right)_{\chi=0}=\hat{Q}_\alpha(\mathbf{r}_1,\mathbf{r}_2)\,\delta(\tau_1-\tau_2).$$

Introducing for the variational derivative of the Green function the notation:

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[2] It can be shown, exactly as in [12], that taking account of the discarded terms in the approximation linear in the concentration gives the total scattering amplitude.

$$(\delta K_{cp}(\tau_1, \tau_2) / \delta\chi_\alpha(x))_{\chi=0} = L_\alpha(\tau_1, \tau_2; x), \qquad (21)$$

we obtain

$$\langle q_\alpha; q_\beta\rangle^T(x_1, x_2) = \mathrm{Sp}\Big\{ q_\alpha^{(s)}(\mathbf{r}_1) L_\beta(\tau_1, \tau_1+0; x_2) - n \iint dV\, d\tau\, \hat{Q}_\alpha(\mathbf{r}, \mathbf{r}_1)$$
$$\times [L_\beta(\tau_1, \tau; x_2)\hat{u}(\mathbf{r})K(\tau, \tau_1+0) + K(\tau_1, \tau)\hat{u}(\mathbf{r})L_\beta(\tau, \tau_1+0; x_2)]$$
$$- n\int d\bar{V}\, \hat{Q}_\alpha(\mathbf{r}, \mathbf{r}_1)K(\tau_1, \tau_2)\hat{Q}_\beta(\mathbf{r}, \mathbf{r}_2)K(\tau_2, \tau_1+0)\Big\} \qquad (22)$$

(here and below, for brevity, we have dropped the subscript "av" from the averaged Green function K_{av}). The variational derivative satisfies an integral equation which stems from Eq. (19):

$$L_\alpha(\tau_1, \tau_2; x) = -\iint d\tau_1'\, d\tau_2'\, K(\tau_1, \tau_1')\left(\frac{\delta K^{-1}(\tau_1', \tau_2')}{\delta\chi_\alpha(x)}\right)_{\chi=0}$$
$$\times K(\tau_2', \tau_2) = -K(\tau_1, \tau)\hat{q}_\alpha^{(s)}(\mathbf{r})K(\tau, \tau_2) + n\iint dV'\, d\tau'\cdot$$
$$\times [K(\tau_1, \tau)\hat{Q}_\alpha(\mathbf{r}', \mathbf{r})K(\tau, \tau')\hat{u}(\mathbf{r}')K(\tau', \tau_2) + K(\tau_1, \tau')\cdot$$
$$\times \hat{u}(\mathbf{r})K(\tau', \tau)\hat{Q}_\alpha(\mathbf{r}', \mathbf{r})K(\tau, \tau_2)] + n\iiint dV\, d\tau_1'\, d\tau_2'\cdot$$
$$\times K(\tau_1, \tau_1')\hat{u}(\mathbf{r}')L_\alpha(\tau_1', \tau_2'; x)\hat{u}(\mathbf{r}')K(\tau_2', \tau_2). \qquad (23)$$

Performing a Fourier transformation in τ in (22) and (23), we have

$$\langle q_\alpha; q_\beta\rangle^T(\mathbf{r}_1, \mathbf{r}_2; \nu_{n'}) = T\sum_{\omega_n}\mathrm{Sp}\Big\{\hat{q}_\alpha^{(s)}(\mathbf{r}_1)L_\beta(\omega_+, \omega_-; \mathbf{r}_2)$$
$$- n\int dV\, \hat{Q}_\alpha(\mathbf{r}, \mathbf{r}_1)[L_\beta(\omega_+, \omega_-; \mathbf{r}_2)\hat{u}(\mathbf{r})K(\omega_-) + K(\omega_+)$$
$$\times \hat{u}(\mathbf{r})L_\beta(\omega_+, \omega_-; \mathbf{r}_2) + K(\omega_+)\hat{Q}_\beta(\mathbf{r}, \mathbf{r}_2)K(\omega_-)]\Big\};$$
$$\nu_{n'} = 2\pi T n', \quad \omega_+ = \omega_n = \pi T(2n+1), \quad \omega_- = \omega_n - \nu_{n'}, \qquad (24)$$
$$n = 0, \pm 1, \pm 2, \ldots$$

and an equation for L_α:

$$L_\alpha(\omega_+, \omega_-; \mathbf{r}) = -K(\omega_+)\hat{q}_\alpha^{(s)}(\mathbf{r})K(\omega_-)$$
$$+ n\int dV'\, K(\omega_+)[\hat{Q}_\alpha(\mathbf{r}', \mathbf{r})K(\omega_-)\hat{u}(\mathbf{r}')K(\omega_-) + \hat{u}(\mathbf{r}')K(\omega_+)\hat{Q}_\alpha(\mathbf{r}', \mathbf{r})K(\omega_-)$$
$$+ \hat{u}(\mathbf{r}')L_\alpha(\omega_+, \omega_-; \mathbf{r})\hat{u}(\mathbf{r}')K(\omega_-)]. \qquad (25)$$

The Fourier transformation in (24) and (25) is defined, as usual,[10] as follows:

$$K(\tau_1 - \tau_2) = T\sum_{\omega_n} K(\omega_n)\exp[-i\omega_n(\tau_1 - \tau_2)],$$
$$L_\alpha(\tau_1, \tau_2; x) = T^2\sum_{\omega_1, \omega_2} L_\alpha(\omega_1, \omega_2; \mathbf{r})\exp[-i\omega_1(\tau_1 - \tau) - i\omega_2(\tau - \tau_2)],$$
$$\langle q_\alpha; q_\beta\rangle^T(\tau_1 - \tau_2) = T\sum_{\nu_n}\langle q_\alpha; q_\beta\rangle^T(\nu_n)\exp[-i\nu_n(\tau_1 - \tau_2)].$$

The resulting relations (24) and (25) can be simplified by substituting

$$L_\alpha(\omega_+, \omega_-; \mathbf{r}) = -{}^1\!/_2 i(\omega_+ + \omega_-)R_\alpha(\omega_+, \omega_-; \mathbf{r})$$
$$- {}^1\!/_2 K(\omega_+)\hat{v}_\alpha(\mathbf{r}) - {}^1\!/_2\hat{v}_\alpha(\mathbf{r})K(\omega_-)$$

and then using Eq. (19) for the Green function for $\chi = 0$:

$$[-i\omega + \hat{\varepsilon} + n\hat{u}(0)]K(\omega) - n\int dV\, \hat{u}(\mathbf{r})K(\omega)\hat{u}(\mathbf{r})K(\omega) = 1. \qquad (26)$$

The formula (24) for the correlator of the heat fluxes then acquires the form:

$$\langle q_\alpha; q_\beta\rangle^T(\mathbf{r}_1, \mathbf{r}_2; \nu_{n'}) = \frac{1}{2}T\sum_{\omega_n}\mathrm{Sp}\Big\{\hat{v}_\alpha(\mathbf{r}_1)\Big[\frac{(\omega_+ + \omega_-)^2}{2}R_\beta(\omega_+, \omega_-; \mathbf{r}_2)$$
$$- \frac{i(\omega_+ + \omega_-)}{2}(K(\omega_+)\hat{v}_\beta(\mathbf{r}_2) + \hat{v}_\beta(\mathbf{r}_2)K(\omega_-)) - K(\omega_+)\hat{q}_\beta^{(s)}(\mathbf{r}_2) - \hat{q}_\beta^{(s)}(\mathbf{r}_2)K(\omega_-)$$
$$+ n\int dV'(K(\omega_+)\hat{u}(\mathbf{r}')K(\omega_+)\hat{Q}_\beta(\mathbf{r}', \mathbf{r}) + \hat{Q}_\beta(\mathbf{r}', \mathbf{r})K(\omega_-)\hat{u}(\mathbf{r}')K(\omega_-))\Big]\Big\} \qquad (27)$$

where the quantity $R_\alpha(\omega_+, \omega_-; \mathbf{r})$ satisfies the equation

$$R_\alpha(\omega_+, \omega_-; \mathbf{r}) - n\int dV'\, K(\omega_+)\hat{u}(\mathbf{r}')R_\alpha(\omega_+, \omega_-; \mathbf{r})\cdot$$
$$\times \hat{u}(\mathbf{r}')K(\omega_-) = K(\omega_+)\hat{v}_\alpha(\mathbf{r})K(\omega_-). \qquad (28)$$

The following calculations will be performed with the assumption that the impurity potential is short-range: $u(\mathbf{r}) = u_0\delta(\mathbf{r})$. Then Eq. (26) for the Green function is simplified:

$$[-i\omega + \hat{\varepsilon} - nu_0^2\sigma_z K(\mathbf{r}, \mathbf{r}; \omega)\sigma_z]K(\omega) = 1 \qquad (29)$$

(here we have omitted the constant term nu_0, which leads to a renormalization of the chemical potential). The Green function $K(\mathbf{r}, \mathbf{r}; \omega)$ of coincident arguments can be calculated by making use of its bilinear expansion in the eigenfunctions of the impurity-free system:

$$K_{ss'}(\mathbf{r}, \mathbf{r}', \omega) = \sum_\lambda \frac{\psi_s^\lambda(\mathbf{r})\psi_{s'}^\lambda(\mathbf{r}')}{\varepsilon(\lambda) - i\tilde{\omega}}. \qquad (30)$$

Here s and s′ are the indices of the matrix elements of the Green function ($s = \pm 1$); $\psi_s^\lambda(\mathbf{r})$ and $\epsilon(\lambda)$ are the eigenfunctions and spectrum corresponding to the set of quantum numbers λ; $\tilde{\omega}$ is the "renormalized" frequency, the relation of which to the usual Matsubara frequency can be found from (29). The eigenfunctions of the system satisfy the Schrödinger equation:

$$\hat{\varepsilon}\psi^\lambda(\mathbf{r}) = \varepsilon(\lambda)\psi^\lambda(\mathbf{r}) \qquad (31)$$

and were found in papers [3,4]. According to these papers, the functions $\psi_s^\lambda(\mathbf{r})$ are the two-component wavefunctions of the electron-hole excitations, with ψ_1 corresponding to the electron and ψ_{-1} corresponding to a hole with charge and mass of the opposite sign.

In the case under consideration (the intermediate state of a superconductor), pair electron-hole excitations with energy $\epsilon(\lambda) < \Delta$ are trapped in the normal layers ($la < x < la + L$, $l = 1, 2, \ldots, N$ labels the layer, and a is the period of the structure; the x-axis is perpendicular to the layers) between the superconducting regions ($la + L < x < (l+1)a$) and move in a magnetic field H_c directed along the z-axis. On reflection from the boundary with the superconductor, the respective components ψ_1 and ψ_{-1} of the wavefunction, as observed above, acquire an extra phase factor $\exp(\pm 2i\Phi y)$ ($\Phi = eHL$) in the gauge $A_x = A_z = 0$, $A_y = Hx$. Therefore, Eq. (31) possesses translational symmetry along the y-axis: $y \to y + \pi/\Phi$, and, related to this, the spectrum of the excitations has a band character and the wavefunctions have the form of Bloch waves. The expressions found in [3,4] for the wavefunctions and spectrum can be generalized without difficulty to the case of 2N alternating normal and superconducting layers, when the latter are sufficiently thick ($N \gg 1$, $a - L \gg \xi_0$, ξ_0 is the coherence length):

$$\begin{pmatrix}\psi_1 \\ \psi_{-1}\end{pmatrix}(\mathbf{r}) = \sum_\nu \begin{pmatrix} u(k + 2\Phi\nu - \Phi l - \Phi(x - la)/L) \\ u(k + 2\Phi\nu + \Phi l + \Phi(x - la)/L)\end{pmatrix}$$
$$\times \exp[i(p_z z + ky + 2\Phi\nu y)], \qquad la < x < la + L;$$
$$u(\xi) = \begin{cases} 0, & |\xi| > p \\ \sqrt{2\Phi/\pi v}\,(p^2 - \xi^2)^{-1/4}\sin S(\xi), & |\xi| < p \end{cases} \qquad p = \sqrt{p_F^2 - p_z^2}, \qquad (32)$$

Here $[x]$ is the integer part of x: $[x] = x - \{x\}$, where the fractional part $\{x\}$ is defined such that $\{x\} > 0$; v is the volume of one normal layer: $v = L_y L_z L$; the role of the extra quantum number, arising from the periodicity of

$$S(\xi)=-\frac{L}{\Phi}\int_{-p}^{\xi}(-1)^{[(t-k)/\Phi]}\sqrt{p^2-t^2}\,dt+\frac{mL\varepsilon}{\Phi}\int_{-p}^{\xi}\frac{dt}{\sqrt{p^2-t^2}},$$

$$\varepsilon=\Omega n+\varepsilon_0(k,\ p_z),\quad \Omega=eH_c/m,\quad n=0,\ \pm1,\ \pm2,\ldots,$$

$$\varepsilon_0(k,p_z)=\frac{1}{\pi m}\int_{-p}^{p}(-1)^{[(t-k)/\Phi]}\sqrt{p^2-t^2}\,dt.$$

the structure in x, is played by the layer number l. Obviously, the spectrum has the character of a set of strongly overlapping bands and is periodic in the quasi-momentum with period 2Φ. Thus, a complete rearrangement of the ordinary space-quantization spectrum occurs in a magnetic field,[2] and special magnetic quantization arises, associated with the instability, noted above, of the trajectories of the excitations with respect to the switching on of an arbitrarily weak magnetic field. In real systems, as was pointed out in [3,4], magnetic quantization appears only in the presence of a sufficiently strong magnetic field, when the radius of the orbit of the electron-hole excitation is small compared with the dimensions of the normal layer along the y-axis: $r_H = p/eH \ll L_y$. It is obvious that for the appearance of important magnetic-quantization effects, the mean free path of the excitations must also satisfy this condition: $l \gg r_H$. These inequalities will be used in the course of the following calculations.

We return to Eq. (29) for the Green function. By means of formulas (30) and (32), we find (the calculations are given in the Appendix)

$$K_{ss'}(\mathbf{r},\mathbf{r};\omega)=i\delta_{ss'}\frac{mp_F}{2\pi}\operatorname{sign}\tilde{\omega};\qquad \tilde{\omega}=\omega+\frac{1}{2\tau}\operatorname{sign}\omega,\qquad \frac{1}{\tau}=nu_0^2\frac{mp_F}{\pi},$$

where τ has the meaning of the "mean free time" of the excitations.

To simplify the following calculations we turn to the determination of the thermal conductivity coefficient $\kappa_{\alpha\beta}$ (6), (7). The latter, generally speaking, is an integral operator in the coordinates; however, it can be shown that the characteristic radius of the kernel of this operator is of the order of the mean free path l. At the same time, the temperature gradient ∇T varies appreciably over distances much greater than 1; therefore, the integral relation (6) between q and ∇T can be replaced by the local relation:

$$q_\alpha(\mathbf{r})=-\varkappa_{\alpha\beta}(\mathbf{r})\nabla_\beta T(\mathbf{r}),\qquad \varkappa_{\alpha\beta}(\mathbf{r})=\int dV'\varkappa_{\alpha\beta}(\mathbf{r},\mathbf{r}').$$

Moreover, because of the macroscopic uniformity of the system ($l \gg L$), we can assume that ∇T = const and confine ourselves to calculating the average heat flux (V is the total volume of the sample):

$$\bar{q}_\alpha=-\bar{\varkappa}_{\alpha\beta}\nabla_\beta T,$$

$$\bar{\varkappa}_{\alpha\beta}=\frac{1}{V}\iint dV\,dV'\varkappa_{\alpha\beta}(\mathbf{r},\mathbf{r}')=\frac{1}{VT}\frac{d}{id\omega}\iint dV\,dV'\langle q_\alpha;q_\beta\rangle(\mathbf{r},\mathbf{r}';\omega)_{\omega\to0}. \tag{33}$$

Combining the formulas (27) and (33) and denoting by the symbol $i\nu_{n'} \to \omega + i0$ the analytic continuation in the sense indicated above, we have

$$\bar{\varkappa}_{\alpha\beta}=\frac{1}{T}\frac{d}{id\omega}(\Lambda_{\alpha\beta}(\nu_{n'})_{i\nu_{n'}\to\omega+i0})_{\omega\to0},$$

$$\Lambda_{\alpha\beta}(\nu_{n'})=\frac{1}{2V}T\sum_{\omega_n}\mathrm{Sp}\left\{\hat{v}_\alpha\left[\frac{(\omega_++\omega_-)^2}{2}R_\beta(\omega_+,\omega_-)\right.\right.$$
$$\left.\left.-\frac{i(\omega_++\omega_-)}{2}(K(\omega_+)\hat{v}_\beta+\hat{v}_\beta K(\omega_-))-K(\omega_+)\hat{q}_\beta^{(e)}-\hat{q}_\beta^{(e)}K(\omega_-)\right]\right\}. \tag{34}$$

In the expression for $\Lambda_{\alpha\beta}$ we have omitted the terms that do not depend on $\nu_{n'}$ (and which, consequently, go to zero on differentiation with respect to ω), and have introduced the notation:

$$\hat{v}_\alpha=\int dV\,\hat{v}_\alpha(\mathbf{r}),\quad \hat{q}_\alpha^{(e)}=\int dV\,\hat{q}_\alpha^{(e)}(\mathbf{r}),\quad R_\alpha(\omega_+,\omega_-)=\int dV\,R_\alpha(\omega_+,\omega_-;\mathbf{r}).$$

An equation for $R_\alpha(\omega_+, \omega_-)$ can be obtained from (28), and, if we take into account the short-range character of the scattering potential, has the following appearance:

$$\begin{gathered}R_\alpha(\omega_+,\ \omega_-)-nu_0^2K(\omega_+)\sigma_z\bar{R}_\alpha(\omega_+,\ \omega_-)\sigma_zK(\omega_-)=K(\omega_+)\hat{v}_\alpha K(\omega_-),\\ \bar{R}_\alpha(\mathbf{r}_1,\ \mathbf{r}_2)=\delta(\mathbf{r}_1-\mathbf{r}_2)R_\alpha(\mathbf{r}_1,\ \mathbf{r}_1)\equiv\delta(\mathbf{r}_1-\mathbf{r}_2)R_\alpha(\mathbf{r}_1).\end{gathered} \tag{35}$$

Putting here $\mathbf{r}_1 = \mathbf{r}_2$ and substituting the bilinear expansions (30) of the Green functions into the right-hand side of (35), we obtain the equation for $R_\alpha(\mathbf{r})$:

$$R_\alpha(\mathbf{r})-nu_0^2\int dV'\,K_+(\mathbf{r},\mathbf{r}')\sigma_zR_\alpha(\mathbf{r}')\sigma_zK_-(\mathbf{r}',\mathbf{r})$$
$$=\sum_{\lambda_+\lambda_-}\frac{\psi_+(\mathbf{r})\psi_-^*(\mathbf{r})}{(\varepsilon_+-i\tilde{\omega}_+)(\varepsilon_--i\omega_-)}\langle\lambda_+|\hat{v}_\alpha|\lambda_-\rangle. \tag{36}$$

Here $K_\pm = K(\omega_\pm)$, $\epsilon_\pm = \epsilon(\lambda_\pm)$, $\psi_\pm(\mathbf{r}) = \psi^{\lambda_\pm}(\mathbf{r})$, and $\langle\lambda_+|\hat{v}_\alpha|\lambda_-\rangle$ are the matrix elements of the velocity operator:

$$\langle\lambda_+|\hat{v}_\alpha|\lambda_-\rangle=\int dV\,\mathrm{Sp}(\psi_+^*(\mathbf{r}')\hat{v}_\alpha(\mathbf{r},\mathbf{r}')\psi_-(\mathbf{r}))_{\mathbf{r}'=\mathbf{r}}, \tag{37}$$

which, as is not difficult to check, have the following form:

$$\langle\lambda_+|\hat{v}_\alpha|\lambda_-\rangle=-\delta_{k_+k_-}\delta_{p_{z+}p_{z-}}\delta_{l_+l_-}v_\alpha(k_+,\ p_{z+},\ n_+-n_-). \tag{38}$$

Analytic expressions for v_α are given in the Appendix; their important property is their periodicity in the quasi-momentum k:

$$v_\alpha(k+2\Phi,\ p_z,\ n)=v_\alpha(k,\ p_z,\ n).$$

We turn to the solution of Eq. (36). Omitting the intermediate calculations connected with performing the summation over the quantum numbers, we give the final expression for the right-hand side of (36):

$$\begin{gathered}-\frac{m\delta_{ss'}}{4\pi}(\delta_+-\delta_-)\int_{-p_F}^{p_F}dp_z\sum_{n=-\infty}^{+\infty}\int_{-1}^{1}dy\,T_n(y)\frac{v_\alpha(py+s\Phi l+s\Phi(x-la)/L,p_z,n)}{\tilde{\omega}\quad+\tilde{\omega}_-+i\Omega n}\\ la<x<la+L,\end{gathered} \tag{39}$$

where s, s′ = ±1, are, as above, the indices of the matrix elements, $\delta_\pm = \operatorname{sign}\omega_\pm$, and

$$T_n(x)=\frac{1}{\pi\sqrt{1-x^2}}\cos\left(n\int_{-1}^{x}\frac{dt}{\sqrt{1-t^2}}\right)=\frac{(-1)^n}{\pi}\frac{\cos(n\arccos x)}{\sqrt{1-x^2}}.$$

In accordance with the form of (39), it is natural to seek the solution of Eq. (36) in the form of a periodic function

$$\begin{gathered}R_{ss'}^{\alpha}(\mathbf{r})=\delta_{ss'}R_\alpha\left(s\Phi l+s\Phi\frac{x-la}{L}\right),\qquad la<x<la+L,\\ R_\alpha(z+2\Phi)=R_\alpha(z).\end{gathered} \tag{40}$$

Substituting (40) into (36), and performing the necessary calculations, we obtain an equation for $R_\alpha(z)$:

$$R_\alpha(z)-\frac{\delta_+-\delta_-}{2\tau}\int\frac{dp_z}{2p_F}\int\int_{-1}^{1}dx\,dy\,Q_\omega(x,y)R_\alpha(z+px-py)$$
$$=-\frac{mp_F}{2\pi}(\delta_+-\delta_-)\int\frac{dp_z}{2p_F}\sum_n\int_{-1}^{1}dx\frac{T_n(x)v_\alpha(z+px,p_z,n)}{\tilde{\omega}_+-\tilde{\omega}_-+i\Omega n} \tag{41}$$

where

$$Q_\omega(x,y)=\sum_n\frac{T_n(x)T_n(y)}{\tilde{\omega}_+-\tilde{\omega}_-+i\Omega n}.$$

We expand the periodic functions occurring in (41) in

Fourier series: $R_\alpha(z)=\sum_\nu R_\alpha(\nu)e^{\pi i\nu z/\Phi}$,

$$v_\alpha(z,p_z,n)=\sum_\nu v_\alpha(\nu,p_z,n)e^{\pi i\nu z/\Phi}.$$

Then the integral equation (41) goes over into an algebraic equation for the Fourier components R_α; the solution of this has the following form:

$$R_\alpha(\nu)=-\frac{mp_F}{2\pi}\frac{\delta_+-\delta_-}{1-(\delta_+-\delta_-)Q_\omega(\nu)/2\tau}\int\frac{dp_z}{2p_F}\sum_n\frac{T_n(\nu,p_z)v_\alpha(\nu,p_z,n)}{\tilde{\omega}_+-\tilde{\omega}_-+i\Omega n},\tag{42}$$

where

$$T_n(\nu,p_z)=\int_{-1}^{1}dx\,e^{\pi i\nu px/\Phi}T_n(x),\qquad Q_\omega(\nu)=\int\frac{dp_z}{2p_F}Q_\omega(\nu,p_z),$$

$$Q_\omega(\nu,p_z)=\int_{-1}^{1}\int_{-1}^{1}dx\,dy\,Q_\omega(x,y)e^{\pi i\nu p(x-y)/\Phi}=\sum_n\frac{T_n(\nu,p_z)T_n^*(\nu,p_z)}{\tilde{\omega}_+-\tilde{\omega}_-+i\Omega n}.$$

After substitution of the resulting solution (42) into Eq. (35), the latter is transformed into an equality determining the quantity $R_\alpha(\omega_+,\omega_-)$. Using formula (34) for $\bar{\kappa}_{\alpha\beta}$, we obtain finally $\bar{\kappa}_{\alpha\beta}=\kappa^{(1)}_{\alpha\beta}+\kappa^{(2)}_{\alpha\beta}$, where

$$\varkappa^{(1)}_{\alpha\beta}=-\eta\frac{mp_F}{4\pi^2T}\int\frac{dp_z}{2p_F}\sum_\nu\sum_n v_\alpha^*(\nu,p_z,n)v_\beta(\nu,p_z,n)$$

$$\times\int_{-\infty}^{+\infty}d\varepsilon\frac{d}{id\omega}\Big(T\sum_{\omega_n}\Pi_1(\omega_+,\omega_-)\Big)_{\substack{i\nu_{n'}\to\omega+i0\\ \omega\to0}},$$

$$\varkappa^{(2)}_{\alpha\beta}=\eta\frac{mp_F}{4\pi^2T}\int\frac{dp_z}{2p_F}\int\frac{dp_z'}{2p_F}\sum_\nu\sum_{n,m}T_n^*(\nu,p_z)T_m(\nu,p_z')v_\alpha^*(\nu,p_z,n)$$

$$\times v_\beta(\nu,p_z',m)\int_{-\infty}^{+\infty}d\varepsilon\frac{d}{id\omega}\Big(T\sum_{\omega_n}\Pi_2(\omega_+,\omega_-)\Big)_{\substack{i\nu_{n'}\to\omega+i0,\\ \omega\to0}}\tag{43}$$

$$\Pi_1(\omega_+,\omega_-)=\frac{(\varepsilon_++\varepsilon_--i\delta_+'-i\delta_-')^2+i(\delta_+'-\delta_-')(\varepsilon_+-\varepsilon_--i\delta_+'+i\delta_-')}{(\varepsilon_+-i\tilde{\omega}_+)(\varepsilon_--i\tilde{\omega}_-)}$$

$$\Pi_2(\omega_+,\omega_-)=\frac{\delta_+'-\delta_-'}{1-(\delta_+'-\delta_-')Q_\omega(\nu)}\frac{1}{\tilde{\omega}_+-\tilde{\omega}_-+i\Omega m}\frac{(\omega_++\omega_-)^2}{(\varepsilon_+-i\tilde{\omega}_+)(\varepsilon_--i\tilde{\omega}_-)}$$

where $\delta'_\pm=\delta_\pm/2\tau$, $\epsilon_+=\epsilon$, $\epsilon_-=\epsilon-\Omega n$, and $\eta=L/a$ is the concentration of the normal phase. It is not difficult to convince oneself that, as ω_+, $\omega_-\to\infty$, the function $\Pi_1(\omega_+,\omega_-)$ falls off like $1/\omega_+$ and $1/\omega_-$ respectively; therefore, it can be represented in the form of an integral of the Cauchy type (cf. [11]), which is convenient for the summation over the frequencies and the analytic continuation:

$$\Pi_1(\omega_+,\omega_-)=-\int_{-\infty}^{+\infty}\int\frac{dz_+dz_-}{(2\pi)^2}\frac{\chi_1(z_+,z_-)}{(z_+-i\omega_+)(z_--i\omega_-)},$$

where $\chi_1(z_+,z_-)$ is the spectral function:

$$\chi_1(z_+,z_-)=\frac{4z_+z_-+(z_+-z_-)(\varepsilon_+-\varepsilon_-)}{[(\varepsilon_+-z_+)^2+1/4\tau^2][(\varepsilon_--z_-)^2+1/4\tau^2]}\frac{1}{\tau^2}.$$

The summation over the frequencies and the analytic continuation are performed taking into account the properties of the Cauchy integral and lead to the following result:

$$T\sum_{\omega_n}\Pi_1(\omega_+,\omega_-)_{i\nu_{n'}\to\omega+i0}=\frac{1}{2}\int_{-\infty}^{\pm\infty}\int\frac{dz_+dz_-}{(2\pi)^2}\frac{\operatorname{th}(z_+/2T)-\operatorname{th}(z_-/2T)}{z_+-z_--\omega-i0}\chi_1(z_+,z_-).$$

The summation and analytic continuation occurring in the formula for $\kappa^{(2)}_{\alpha\beta}$ are performed analogously; after integrating over the energy in (43), we obtain the following expressions for the components of the thermal conductivity tensor

$$\bar{\varkappa}_{\alpha\beta}=\varkappa^{(1)}_{\alpha\beta}+\varkappa^{(2)}_{\alpha\beta},$$

$$\varkappa^{(1)}_{\alpha\beta}=\eta\frac{mp_F}{3}T\tau\sum_\nu\sum_n\int\frac{dp_z}{2p_F}\frac{v_\alpha^*(\nu,p_z,n)v_\beta(\nu,p_z,n)}{1+i\Omega\tau n},$$

$$\varkappa^{(2)}_{\alpha\beta}=\eta\frac{mp_F}{3}T\tau\sum_\nu\sum_{n,m}\int\frac{dp_z}{2p_F}\int\frac{dp_z'}{2p_F}$$

$$\times\frac{T_n^*(\nu,p_z)T_m(\nu,p_z')}{1-Q(\nu)}\frac{v_\alpha^*(\nu,p_z,n)v_\beta(\nu,p_z',m)}{(1+i\Omega\tau n)(1+i\Omega\tau m)},\tag{44}$$

where

$$Q(\nu)=\int\frac{dp_z}{2p_F}\sum_n\frac{T_n(\nu,p_z)T_n^*(\nu,p_z)}{1+i\Omega\tau n}.$$

The further calculations are performed taking into account the inequalities $\Omega\tau\gg1$ and $\Phi\ll p_F$[3)] (or $L\ll r_H\ll l$). Then the components $\bar{\kappa}_{xz}$, $\bar{\kappa}_{zx}$, $\bar{\kappa}_{yz}$ and $\bar{\kappa}_{zy}$ vanish because of the chosen geometry of the Fermi surface (quadratic dispersion law). In addition, $\bar{\kappa}_{xx}=\bar{\kappa}_{xy}=\bar{\kappa}_{yx}=0$ (for these, $\kappa^{(1)}=-\kappa^{(2)}$), which reflects the impossibility of heat transport by the normal excitations in the direction perpendicular to the layers, in view of the total reflection of the excitations from the interface. Only the components $\bar{\kappa}_{yy}$ and $\bar{\kappa}_{zz}$ are nonzero:

$$\bar{\varkappa}_{yy}=\frac{21\zeta(3)}{2\pi^3}\frac{L}{d_H}\eta\varkappa_n\left[1-\frac{4\sqrt{2}\pi^2}{7\zeta(3)}\left(\frac{L}{d_H}\right)^{1/2}\Pi_{-2}\left(\frac{d_H}{L}\right)\right],$$

$$\bar{\varkappa}_{zz}=\frac{21\zeta(3)}{2\pi^3}\frac{L}{d_H}\eta\varkappa_n\left[1+\frac{4\sqrt{2}\pi^2}{7\zeta(3)}\left(\frac{L}{d_H}\right)^{3/2}\Pi_{-3}\left(\frac{d_H}{L}\right)\right].\tag{45}$$

Here $\kappa_n=\pi^2n_e\tau T/3m$ is the thermal conductivity coefficient of the normal metal, and $d_H=2r_H$; $\zeta(x)$ is the Riemann zeta-function ($\zeta(3)=1.202$); $\Pi_{-n}(x)$ is the primitive periodic function of order n considered in [3]:

$$\Pi(x)=\int_0^\infty(-1)^{[t-x]}\frac{dt}{\sqrt{t}}=\frac{4}{\pi}\sum_{s=0}^\infty\frac{\cos(\pi(2s+1)x+\pi/4)}{(2s+1)^{3/2}},$$

$$\Pi(x+2)=\Pi(x).$$

The formulas (45) show that the thermal conductivity of the normal layers of the intermediate state of a superconductor is substantially (because $L\ll d_H$) less than the thermal conductivity of the normal metal. In addition, because of the complicated dynamics of the motion of the excitations and the presence of correlation between the electron and the hole, oscillating terms appear in the components $\bar{\kappa}_{\alpha\beta}$. It can be seen from the Figure that these oscillations are connected with the discontinuous change in the number of reflections of the excitation on continuous variation of the thickness L of the normal layer (i.e., on change of the external magnetic field in which the sample is situated). Numerical estimates show that the largest relative magnitude of the oscillations (~20%) should be observed when the thermal conductivity is measured along the y-axis.

In conclusion, we point out again that, as we should expect, on decrease of the mean free path, the magnetic quantization effects disappear. In particular, the formulas (44) in the case $L\ll l\ll r_H$ give the result obtained earlier by Andreev[6] by the kinetic-equation method disregarding the magnetic field:

[3)] It is clear that it is in precisely this case that the effects of magnetic quantization appear most strongly; in the opposite limiting case ($r_H\ll L$), the effects of the special reflection of the excitations are small and we have the ordinary Landau quantization.

$$\bar{\varkappa}_{yy}=\bar{\varkappa}_{zz}=\frac{21\zeta(3)}{4\pi^2}\frac{L}{l}\eta\varkappa_n.$$

Finally, for $l \ll L \ll r_H$, the characteristics of the reflection of the excitations become unimportant, and the result is the trivial one: $\bar{\kappa}_{yy} = \bar{\kappa}_{zz} = \eta\kappa_n$.

APPENDIX

To calculate the Green function $K(\mathbf{r}, \mathbf{r}; \omega)$ of coincident arguments, we substitute the formulas (32) for the wavefunctions and spectrum into the bilinear expansion (30):

$$K_{ss'}(\mathbf{r},\mathbf{r};\omega)=\sum_\lambda\frac{\psi_s^\lambda(\mathbf{r})\psi_{s'}^{*\lambda}(\mathbf{r})}{\varepsilon(\lambda)-i\tilde{\omega}}=\frac{\Phi}{2\pi^3L}\int dp_z\int dk\sum_{l,l'}\sum_n\frac{1}{\varepsilon(\lambda)-i\tilde{\omega}}$$

$$\times\sum_{\nu,\nu'}e^{2\Phi i(\nu-\nu')y}\frac{\sin S(\xi)\sin S(\xi')}{(p^2-\xi^2)^{1/4}(p^2-\xi'^2)^{1/4}},$$

$$\xi=k+2\Phi\nu-s\Phi l-s\Phi\frac{x-la}{L},\qquad \xi'=k+2\Phi\nu'-s'\Phi l'-s'\Phi\frac{x-l'a}{L}.$$

Discarding in the latter expression all those terms oscillating over an electron wavelength, and going over from integration over k and summation over ν to integration over ξ, we have

$$K_{ss'}(\mathbf{r}\ \mathbf{r};\omega)=\delta_{ss'}\frac{\Phi}{4\pi^3L}\int dp_z\int_{-p}^{p}\frac{d\xi}{\sqrt{p^2-\xi^2}}$$

$$\times\sum_n\frac{1}{\Omega n+\varepsilon_0(\xi+s\Phi l+s\Phi(x-la)/L,p_z)-i\tilde{\omega}}.$$

We note that in this formula we can replace the summation over n by integration over the energy: $\Omega\sum_n\ldots\to\int\ldots d\epsilon$. This is connected with the strong overlap, noted above, of the bands in the spectrum ($\Omega \ll \epsilon_0$). The integral over ϵ is to be understood, as in [10], in the sense of the principal value:

$$\int_{-\infty}^{+\infty}\ldots d\varepsilon=\lim_{R\to\infty}\int_{-R}^{R}\ldots d\varepsilon.$$

Taking all this into account, we obtain

$$K_{ss'}(\mathbf{r},\mathbf{r};\omega)=i\delta_{ss'}\frac{mp_F}{2\pi}\operatorname{sign}\tilde{\omega}.$$

Substituting this expression into (39), we arrive at an equation for $\tilde{\omega}$:

$$\tilde{\omega}=\omega+\frac{1}{2\tau}\operatorname{sign}\tilde{\omega},\qquad\frac{1}{\tau}=nu_0^2\frac{mp_F}{\pi}.$$

Choosing a single-valued branch of the function $\tilde{\omega}(\omega)$, we have finally,

$$\tilde{\omega}=\omega+\frac{1}{2\tau}\operatorname{sign}\omega.$$

Calculation of the matrix elements from formula (37), taking into account the properties of the wavefunctions (32), gives the following result:

$$v_x(k,p_z,n)=\frac{i}{\pi m}\int_{-p}^{p}d\xi(-1)^{[(\xi-k)/\Phi]}\sin\left(n\int_{-p}^{\xi}\frac{dt}{\sqrt{p^2-t^2}}\right)$$

$$v_y(k,p_z,n)=\frac{1}{\pi m}\int_{-p}^{p}d\xi\frac{(-1)^{[(\xi-k)/\Phi]}\xi}{\sqrt{p^2-\xi^2}}\cos\left(n\int_{-p}^{\xi}\frac{dt}{\sqrt{p^2-t^2}}\right),$$

$$v_z(k,p_z,n)=\frac{v_z}{\pi}\int_{-p}^{p}d\xi\frac{(-1)^{[(\xi-k)/\Phi]}}{\sqrt{p^2-\xi^2}}\cos\left(n\int_{-p}^{\xi}\frac{dt}{\sqrt{p^2-t^2}}\right).$$

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