INTERACTION BETWEEN A VORTEX AND THE BOUNDARY BETWEEN TWO SUPERCONDUCTORS

G. S. MKRTCHYAN, F. R. SHAKIRZYANOVA, E. A. SHAPOVAL, and V. V. SHMIDT

Solid State Physics Institute, USSR Academy of Sciences

Submitted February 23, 1972

Zh. Eksp. Teor. Fiz. **63**, 667–669 (August, 1972)

The interaction energy for a superconducting vortex interacting with the boundary between two superconductors is studied using the London approximation as a function of the distance from the boundary to the vortex. It is shown that if $\kappa_1 > \kappa_2$ and $\lambda_1 > \lambda_2$, the energy will be a nonmonotonic function of this distance, and the vortex will be pinned by the boundary.

THE large critical currents of hard superconductors are due to the pinning of the vortices by the various inhomogeneities in the superconductor. In this paper we study the interaction of a vortex with the boundary between two superconductors. Assume that the boundary is located in the plane $X = 0$ and that the half-space $X > 0$ is filled with one superconductor having a penetration depth λ_1 and correlation length ξ_1 ($\lambda_1 \gg \xi_1$), while the region where $X < 0$ contains the other superconductor in which $\lambda_2 \gg \xi_2$. The vortex filament is parallel to the Z axis and passes through the point $\mathbf{r}_L$. When the penetration depth is variable we can obtain a modified London equation by varying the free energy (see[1] for example):

$$\mathbf{h} + \mathrm{rot}(\lambda^2 \,\mathrm{rot}\,\mathbf{h}) = \mathbf{\Phi}_0 \delta_2(\mathbf{r} - \mathbf{r}_L); \tag{1}$$

$\mathbf{\Phi}_0$ is a vector directed along the vortex thread and is equal to the magnetic flux quantum $\pi\hbar c/e$; h is the field created by the vortex. In our case, when λ is a piecewise-constant function of position, the following boundary conditions follow easily from Eq. (1):

$$h_{1t} = h_{2t}, \quad \lambda_1^2(\mathrm{rot}\,\mathbf{h}_1)_t = \lambda_2^2(\mathrm{rot}\,\mathbf{h}_2)_t. \tag{2}$$

The second of these two conditions can also be easily found from the stipulation that the tangential component of the vector potential **A** and the London equations $\mathbf{j} \sim \mathbf{A}/\lambda^2$ be continuous.

By solving Eq. (1) with the conditions (2) taken into account and then substituting the solution into the well-known expression for the free energy per unit vortex length $F = \Phi_0 h/8\pi|_{r=r_L}$), we arrive at this final result:

$$F = F_0 + \left(\frac{\Phi_0}{4\pi\lambda_1}\right)^2 \int_0^\infty \exp\left\{-\frac{2x_L}{\lambda_1}\mathrm{ch}\,\varphi\right\} \frac{\lambda_1^2 \,\mathrm{ch}\,\varphi - \lambda_2^2[\mathrm{sh}\,\varphi + (\lambda_1/\lambda_2)^2]^{1/2}}{\lambda_1^2\,\mathrm{ch}\,\varphi + \lambda_2^2[\mathrm{sh}\,\varphi + (\lambda_1/\lambda_2)^2]^{1/2}} d\varphi, \tag{3}$$

where x_L is the distance in the first superconductor from the vortex to the boundary ($x_L > 0$), $F_0 \approx (\Phi_0/4\pi\lambda_1)^2 \ln \kappa_1$ is the free energy of an isolated vortex in an infinite type-I superconductor of the first kind, and $\kappa_1 = \lambda_1/\xi_1$. The interaction with the boundary is contained in the second term.

We obtain the following limiting cases from Eq. (3):

$$F = F_0 + \left(\frac{\Phi_0}{4\pi\lambda_1}\right)^2 \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}\left(\frac{\pi\lambda_1}{4x_L}\right)^{1/2} \exp\left\{-\frac{2x_L}{\lambda_1}\right\}, \quad x_L \gg \lambda_1, \tag{4}$$

$$F = F_0 + \left(\frac{\Phi_0}{4\pi\lambda_1}\right)^2 K_0\left(\frac{2x_L}{\lambda_1}\right), \quad \lambda_2 \ll \lambda_1, x_L, \tag{5}$$

$$F = F_0 - \left(\frac{\Phi_0}{4\pi\lambda_1}\right)^2 K_0\left(\frac{2x_L}{\lambda_1}\right), \quad \lambda_2 \gg \lambda_1, \tag{6}$$

$$F = F_0 + \left(\frac{\Phi_0}{4\pi\lambda_1}\right)^2 \frac{\lambda_1^2 - \lambda_2^2}{\lambda_1^2 + \lambda_2^2} \ln\frac{\lambda_1}{x_L}, \quad x_L \ll \lambda_1, \lambda_2, \tag{7}$$

where K_0 is the Macdonald function. Equation (6) corresponds to a contact between a superconductor and a non-superconducting medium. It was derived earlier by Bean and Livingston.[2] Equation (7) is logarithmically accurate. The figure shows a plot of $F(x_L)$ under the assumption that $\lambda_1 > \lambda_2$, $\kappa_1 > \kappa_2$.

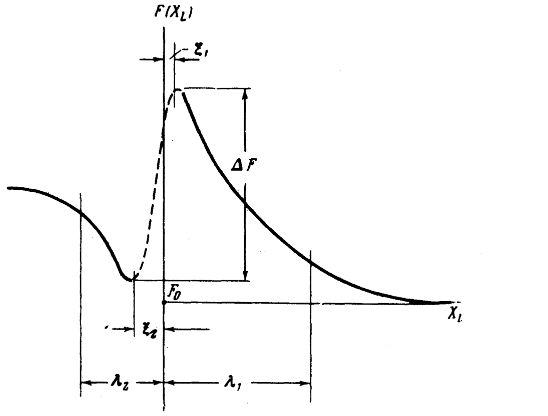
Although Eq. (7) holds only when $x_L \gg \xi_1$, it nevertheless enables us to estimate (with logarithmic accuracy) the energy of a vortex at a distance on the order of ξ_1 from the boundary:

$$F(\xi_1) \approx \frac{\Phi_0^2}{8\pi^2} \frac{\ln \varkappa_1}{\lambda_1^2 + \lambda_2^2}.$$

The change in energy of the vortex at the boundary (more precisely, over a distance on the order of $\xi_1 + \xi_2$) is

$$\Delta F = \frac{\Phi_0^2}{8\pi^2} \frac{\ln(\varkappa_1/\varkappa_2)}{\lambda_1^2 + \lambda_2^2}.$$

to logarithmic accuracy. It will be positive, i.e., the vortex will be pinned, if $\kappa_1 > \kappa_2$ (at $\lambda_1 > \lambda_2$). In the other case the change is negative and there is no potential well at the boundary. According to the microscopic theory, $\kappa \sim 1/l$, $\lambda \sim 1/\sqrt{l}$ (l is the mean free path of the electron). Therefore, if the superconductors differ neither in type of material nor in impurity concentra-



tion, the energy change will be positive and the vortex will be pinned.

If the vortex is in a potential well, it will experience a restoring force f_+ when it moves to the right, and a force f_- when it moves to the left:

$$f_+ \sim \frac{\Phi_0^2}{8\pi^2}\ln\frac{\varkappa_1}{\varkappa_2}\frac{1}{(\lambda_1^2+\lambda_2^2)(\xi_1+\xi_2)}$$

$$f \sim \left(\frac{\Phi_0}{4\pi\lambda_2}\right)^2\frac{\lambda_1^2-\lambda_2^2}{\lambda_1^2+\lambda_2^2}\frac{1}{\xi_2} \qquad (8)$$

We will assume that the superconductor is located in an external magnetic field parallel to the Z axis and such that both superconductors are in the mixed state. The pinning of one vortex row near the boundary makes it possible to pass a lossless current along the boundary in the Y direction; it will flow in a band $\sim\lambda_1+\lambda_2$ wide. The critical current can be estimated by finding the Lorentz force exerted on the vortices in this layer by the current, and equating this force to the restoring force of Eq. (8), which also acts on the trapped vortices. Thus, assuming $\lambda \sim 10^{-5}$ cm, $\kappa_1/\kappa_2 \sim 10$, $H_{cm} \sim 10^3$ Oe, and $H_{ext} \sim 10^4$ Oe, the critical current per unit length along the Z axis is $I_{cr} \sim 100$ A/cm. The corresponding current density is $j_c \sim I_c/(\lambda_1+\lambda_2) \sim 10^6$ A/cm^2. Since the restoring forces f_+ and f_- are different, the critical current in the positive and negative Y directions will also be different.

[1] P. deGennes, The Superconductivity of Metals and Alloys, Benjamin, 1965.

[2] C. P. Bean and J. D. Livingston, Phys. Rev. Lett. 12, 14 (1964).

Translated by D. L. Burdick
69