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DIFFUSION IN A HIGH FREQUENCY GAS DISCHARGE

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Under strong skin effect conditions an HF gas discharge is studied in which deviation from local thermodynamic equilibrium is caused by diffusion and among the various recombination processes the main role is played by electron attachment to atoms leading to negative ion formation. A situation of this type is encountered, for example, in a weakly ionized hydrogen discharge at approximately atmospheric pressure. The discharge structure is investigated and expressions are derived for the dependence of the discharge parameters on the input power. It is shown that with increase of the power supplied to the discharge, the plasma temperature, with allowance for diffusion, increases very weakly, being proportional to the logarithm of the logarithm of the power. In contrast to pulsed discharges, strong electron diffusion in a stationary HF discharge not only does not tend to conceal the influence of recombination processes on the discharge, but even enhances their effect. The formulas presented in the paper may be useful for determining the ratio of the diffusion coefficient to the gas recombination coefficient from the experimental data. The effect of a stationary magnetic field parallel to the discharge current on the HF discharge is investigated. It is shown that the presence of diffusion leads to a considerable increase of surface resistance of the plasma. One might be tempted to explain the effect of a magnetic field on the discharge observed by Kapitza[1] as being due to the influence of the field on the ambipolar diffusion coefficient. However, consistent account of diffusion greatly increases the discrepancy between the surface resistance values experimentally determined by Kapitza and the values calculated in the present work under the assumption of a cold plasma.

1. INTRODUCTION

WE construct in this paper a theory of high-frequency (HF) gas discharge under conditions of strong skin effect with allowance for the deviation, due to electron diffusion, from local thermodynamic equilibrium.

The structure of a high-frequency gas discharge in the presence of local thermodynamic equilibrium was investigated by Pitaevskiĭ and the author in[2]. Local thermodynamic equilibrium certainly takes place at sufficiently high pressure. At pressures on the order of atmospheric, however, the presence of electron diffusion can lead under certain conditions to an appreciable deviation of the electron concentration from the equilibrium value given by the Saha formula. The question of the influence of electron diffusion on the properties of an HF discharge is thus of considerable interest[1)].

We assume that the plasma in the HF discharge is a single-temperature one. According to estimate (2.1) of[2], the difference between the temperature of the electrons in the discharge and the temperatures of the ions and atoms is of the order of 10–15% at atmospheric pressure. We shall neglect this difference.

In the thermodynamic equilibrium state, the degree of ionization of the gas depends only on the temperature and does not depend on the concrete ionization and recombination processes. The number of electrons produced per unit time in a given volume element of the gas by ionization is exactly equal to the number of recombination acts per unit time in the same volume element. In accordance with the detailed-balancing principle, each direct process is exactly compensated for by a corresponding inverse process.

The presence of spatial inhomogeneity of the gas leads to diffusion of the electrons and ions to a colder region, where they ultimately recombine. The neutral atoms, to the contrary, move from the colder region into a hotter one, where they are ionized. The diffusion can thus greatly influence the ionization-recombination balance in the discharge.

In view of the tremendous variety of recombination and ionization processes in gases (few of which have been investigated so far), it is impossible to study the influence of the diffusion in general form. We confine ourselves here to a study of diffusion in an HF discharge in such a gas, in which foremost among the recombination processes is capture of electrons by neutral atoms to form negative ions; the latter, in turn, recombine rapidly with positive ions. Such a situation obtains, for example, in a hydrogen discharge at atmospheric pressure and a temperature on the order of 6000°K.

We obtain in this paper the dependence of the plasma temperature in the discharge and of the surface resistance on the power at an arbitrary ratio of the diffusion length (see the next section) and the depth of penetration of the field into the plasma at the maximal temperature. The analytic expressions obtained in the limiting case of strong diffusion make it possible to determine from the experimental data the ratio of the diffusion coefficient to the coefficient of trapping of the electron by the atom. We emphasize that in pulsed discharges the presence of diffusion makes it difficult to measure the trapping or recombination coefficients. In a stationary

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[1)] The effect of diffusion of HF breakdown in gases in the absence of skin effect is described in detail in MacDonald's book [3]. In this article we are investigating gas discharge in the stationary state and do not touch upon questions connected with gas breakdown and with non-stationary processes.

HF discharge the situation is exactly the reverse: the larger the diffusion, the larger the deviation from thermodynamic equilibrium and the larger the role played by the concrete electron-capture processes. Thus, in a stationary HF discharge the presence of electron diffusion uncovers additional possibilities of experimentally studying the elementary processes in a plasma.

In the limiting case of strong diffusion, the power needed to heat the plasma to a specified temperature depends very strongly (exponentially) on the diffusion coefficient. Consequently even a relatively small change of the diffusion coefficient in a weak magnetic field can exert a strong influence on the plasma parameters. For the same reason, the deviation of the electron temperature from its equilibrium value is somewhat larger than expected from the estimate (2.1) in[2] if the electron diffusion is taken into account.

Kapitza[1] has observed that a constant magnetic field influences a high-frequency plasma filament. This influence cannot be explained by assuming local thermodynamic equilibrium and a low degree of gas ionization. It will be shown below that allowance for the diffusion of the electrons in the "jacket" makes it possible to explain the influence of the magnetic field on the Kapitza discharge. On the other hand, this increases the disparity between the experimental values of the surface resistance of the plasma and the values calculated theoretically with allowance for electron diffusion assuming a cold plasma.

2. INITIAL EQUATIONS AND THEIR INVESTIGATION

We consider an HF discharge in the stationary state, when the Joule heat released in the plasma by the electromagnetic field is transferred by the thermal conductivity of the gas to the cooled walls. We neglect the influence of the radiation on the energy balance, and assume also that the plasma dimensions are large both in comparison with the depth of penetration of the field and in comparison with the diffusion length. The latter make it possible to reduce the problem to a planar one, so that all the quantities can be regarded as functions of one coordinate x. Eliminating the electric vector from the wave equation and from the heat-conduction equation (see[2]), we obtain an equation for the plasma temperature T:

$$\frac{d^3}{dx^3}\frac{1}{\sigma}\frac{d}{dx}\varkappa\frac{dT}{dx}-\frac{64\pi^2\omega^2\sigma}{c^4}\varkappa\frac{dT}{dx}=0. \tag{2.1}$$

Here κ is the thermal conductivity of the gas, and ω is the frequency of the electromagnetic field. The conductivity of the gas σ can be expressed in terms of the electron concentration N_e and the effective number ν_{eff} of the collisions ($\omega \ll \nu_{eff}$):

$$\sigma = e^2N_e / m\nu_{eff}. \tag{2.2}$$

We confine ourselves to temperatures at which collisions between the electrons and neutral atoms are most important. Then ν_{eff} does not depend on the electron concentration.

The ionization-recombination balance equation with allowance for the diffusion of the electrons and their adhesion to the atoms is

$$\frac{\partial N_e}{\partial t}=\operatorname{div}D_e\left(\operatorname{grad}N_e+\frac{e\mathbf{E}_0}{kT}N_e\right)-\beta N_e+\alpha, \tag{2.3}$$

where α and β are the coefficients of ionization and trapping of the electrons by the atoms, and $\mathbf{E}_0$ is the electric field due to the separation of the charges in the diffusion process. The change of the electron concentration per unit time consists of the processes of diffusion, ionization, and formation of negative ions. In the stationary state $\partial N_e/\partial t = 0$. Taking into account the one-dimensional character of the problem, eliminating E_0, and neglecting small changes of the coefficient of ambipolar diffusion D over the diffusion length $d = (D/\beta)^{1/2}$:

$$dD / dx \ll D / d, \tag{2.4}$$

we rewrite (2.3) in the form

$$Dd^2N_e / dx^2 = \beta N_e - \alpha. \tag{2.5}$$

In the absence of diffusion, the electron concentration should be in equilibrium and satisfy the Saha formula: $N_e = N_{e.eq}(T)$, $D = 0$. Putting $D = 0$ in (2.5) we obtain $\alpha = \beta N_{e.eq}(T)$. Thus, in the presence of diffusion, the electron concentration satisfies the equation

$$d^2N_e / dx^2 = (N_e - N_{e.eq.}(T)) / d^2. \tag{2.6}$$

The number of negative ions α decaying per unit time is proportional to their concentration N_-. The diffusion equation takes the form (2.6) under the condition that in the discharge the concentration of the negative ions differs little from its equilibrium value. A detailed derivation of (2.6) is given in the Appendix.

Equations (2.1) and (2.6) constitute a complete system describing an HF discharge in the stationary state with allowance for diffusion and the formation of negative ions.

It follows from (2.6) that the electron concentration N_e changes significantly over distances on the order of $d = (D/\beta)^{1/2}$. This is of the same order as the distance over which the electron diffuses from the instant of atom ionization to the instant of formation of the negative ion. We shall call d the diffusion length. We assume that besides the diffusion coefficient D, other quantities that vary little over the diffusion length are the thermal conductivity of the gas κ and the coefficient β of trapping of the electron by the atom:

$$d\varkappa / dx \ll \varkappa / d, \quad d\beta / dx \ll \beta / d. \tag{2.7}$$

The conditions (2.4) and (2.7) allow us to regard the coefficients D, κ, and β as independent of x. These coefficients are functions of the temperature and vary significantly, generally speaking, over the same distances as the temperature itself. It can be shown (see[4]) that the temperature changes significantly over distances on the order of $\delta_m I/T_m$. Here T_m is the maximum temperature in the discharge, $\delta_m = c(8\pi\omega\sigma(T_m))^{-1/2}$ is the depth, corresponding to the temperature T_m, of penetration of the field in the plasma, and I is the gas ionization potential. The conditions (2.4) and (2.7) are thus equivalent, generally speaking, to the condition

$$\nu \equiv \delta_m / d \gg T_m / I, \tag{2.8}$$

which will be assumed satisfied from now on. The parameter introduced in (2.8) is the ratio of the depth of penetration of the field into the plasma at the maximum temperature T_m to the diffusion length, and constitutes the main parameter of the theory. γ decreases with increasing diffusion coefficient.

In (2.1) and (2.6) it is convenient to change over to the dimensionless variables Θ, n, and ζ ($I \gg T_m$):

$$\zeta=\frac{x}{\delta},\quad \Theta=\frac{I(T_m-T)}{2T_m^2},\quad n=\frac{N_e}{N_{\text{e. eq.}}(T_m)},\quad \frac{N_{\text{e. eq.}}(T)}{N_{\text{e. eq.}}(T_m)}=e^{-\Theta}. \tag{2.9}$$

In terms of the variables Θ, n, and ζ, Eqs. (2.1) and (2.6) take the form

$$\frac{d^3}{d\zeta^3}\frac{1}{n}\frac{d^2\Theta}{d\zeta^2}-n\frac{d\Theta}{d\zeta}=0, \tag{2.10}$$

$$d^2n/d\zeta^2=\gamma^2(n-e^{-\Theta}). \tag{2.11}$$

Outside the plasma, as $x \to -\infty$, the electron concentration N_e tends to zero, and the heat flux dT/dx tends to a specified value S_0. In terms of the dimensionless variables (2.9) these conditions take the form

$$n=0,\quad \zeta\to-\infty, \tag{2.12}$$

$$d\Theta/d\zeta=-S_0I\delta_m/2\varkappa_mT_m^2,\quad \zeta\to-\infty. \tag{2.13}$$

Inside the plasma, as $x \to +\infty$, the ratio of the electron concentration to its maximum value at infinity tends to unity:

$$n=1,\quad \zeta\to+\infty. \tag{2.14}$$

The dimensionless temperature Θ tends exponentially to zero as $\zeta \to +\infty$[2]:

$$\Theta=e^{-(\zeta-\zeta_0)},\quad \zeta\to+\infty. \tag{2.15}$$

The boundary conditions (2.12), (2.14) and (2.15) determine uniquely the sought functions Θ and n that satisfy Eqs. (2.10) and (2.11). These boundary conditions contain no parameters, and the equations themselves depend on only one parameter γ. Since furthermore ζ does not enter explicitly in (2.10) and (2.11), the sought functions Θ and n are functions of $\zeta - \zeta_0$ and depend on γ as a parameter:

$$n=n_\gamma(\zeta-\zeta_0),\quad \Theta=\Theta_\gamma(\zeta-\zeta_0), \tag{2.16}$$

and outside the plasma, as $\zeta \to -\infty$, the dimensionless heat flux $d\Theta/d\zeta$ does not depend on ζ and is a function of γ only:

$$d\Theta/d\zeta=-f(\gamma),\quad \zeta\to-\infty. \tag{2.17}$$

Comparing (2.13) and (2.17), we obtain the energy flux density needed to heat the plasma to the temperature T_m:

$$S_0(T_m)=2f(\gamma)\varkappa_mT_m^2/I\delta_m. \tag{2.18}$$

In the limiting case of low diffusion, $d \ll \delta_m$, as it should be, we have

$$d\Theta/d\zeta=-1.57,\quad \gamma\gg1, \tag{2.19}$$

so that formula (2.18) goes over into formula (4.17) of[2]:

$$S_0(T_m)=3.14\varkappa_mT_m^2/I\delta_m,\quad \gamma\gg1.$$

The function $f(\gamma)$ is determined during the course of the solution of Eqs. (2.10) and (2.11) with boundary conditions (2.12), (2.14), and (2.15). The electron concentration n is specified in our problems on both ends of the integration region. This complicates the numerical solution of the equations. For convenience in the numerical integration, we obtain the next term of the asymptotic behavior of the concentration n as $\zeta \to +\infty$. Substituting (2.15) in (2.11) and recognizing that Θ tends to zero as $\zeta \to +\infty$, we get

$$d^2n/d\zeta^2=\gamma^2(n-1+e^{-(\zeta-\zeta_0)}),\quad \zeta\to+\infty. \tag{2.20}$$

Equation (2.20) has a solution that tends to unity as $\zeta \to +\infty$, in the form

$$n=1-\frac{\gamma^2}{\gamma^2-1}e^{-(\zeta-\zeta_0)}+Ce^{-\gamma(\zeta-\zeta_0)},\quad \zeta\to+\infty. \tag{2.21}$$

This solution contains the arbitrary constant C, the value of which is determined from the condition that as $\zeta \to -\infty$ the electron concentration tends to zero in accordance with (2.12). We integrate numerically from $\zeta \to +\infty$ to $\zeta \to -\infty$, starting from the asymptotic expressions (2.15) and (2.21). We choose the constant C to satisfy the condition (2.12). This makes it possible to obtain relatively simply the solution of (2.10) and (2.11) at $\gamma \sim 1$. The results of the numerical calculations are shown in Figs. 1 and 2. Figure 1 shows a plot of the function $f(\gamma)$ that enters in (2.18). Figures 2a and 2b show plots of the temperature Θ and the concentration n for different values of the parameter γ^2. Figure 2c shows plots of the heat release $d^2\Theta/d\zeta^2$ at different values of γ^2.

With increasing diffusion length (with decreasing γ), the concentration of the electrons n decreases more and moves slowly in the direction $\zeta \to -\infty$. At the same time the temperature Θ (and its second derivative) is independent of γ as $\zeta \to +\infty$. As a result the maximum heat release shifts to the left in proportion to the diffusion length. The diffusion of the electrons leads also to an increase of the electron concentration outside the discharge, as a result of which the heat release increases with decreasing γ at given value of T_m.

3. CASE OF LARGE DIFFUSION LENGTH

When the diffusion length d is large in comparison with the depth of penetration of the field in the plasma, $d \gg \delta_m$, there appears another possibility of analytically investigating Eqs. (2.10) and (2.11). Remaining

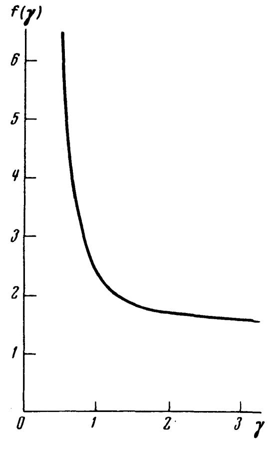

FIG. 1. Plot of the function f(γ) that determines the influence of the diffusion on the power dependence of the temperature.

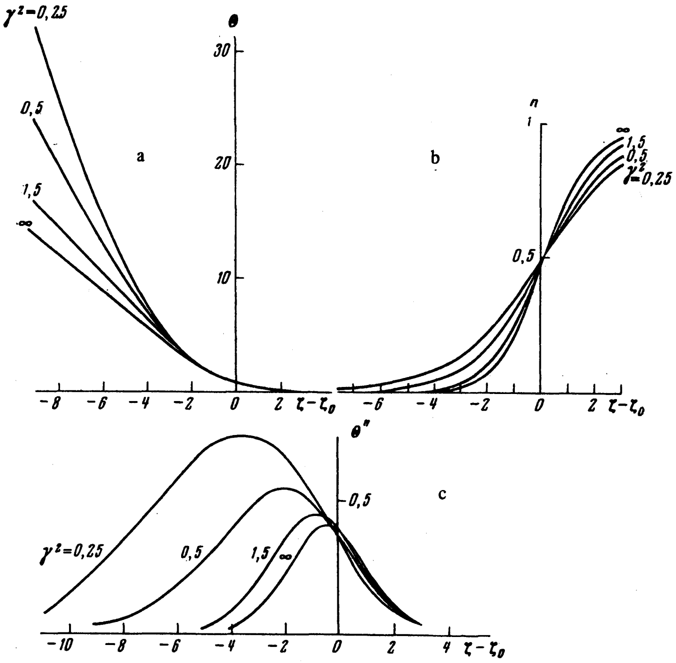

FIG. 2. The dimensionless temperature (a), dimensionless electron concentration, and heat release Θ″ (c) vs. the dimensionless coordinate at various values of the parameter γ^2.

within the framework of condition (2.8), we assume the parameter γ to be small:

$$1 \gg \gamma \gg T_m / I. \qquad (3.1)$$

It follows from (2.10) and (2.11) that the electron concentration n varies significantly over distances of the order of $1/\gamma$ (measured in the scale of ζ), whereas the temperature Θ varies over distances on the order of $n^{-1/2}$. It follows from condition (3.1) that in a wide range of electron concentrations, so long as

$$n \gg \gamma^2, \qquad (3.2)$$

$n(\zeta)$ is a slowly varying function in comparison with the temperature Θ. This circumstance enables us, when solving (2.10) within the framework of the condition (3.2) to assume $n(\zeta)$ to be a slowly varying function.

We seek the solution of (2.11) separately in three overlapping regions.

1. Inside the plasma, in the region $\zeta - \zeta_0 \gg 1$, we can put $e^{-\Theta} \approx 1$, and (2.11) takes the form

$$d^2n / d\zeta^2 = \gamma^2 (n - 1).$$

Its solution, which satisfies condition (2.14), can be written in the form

$$n = 1 - C_1 e^{-\gamma(\zeta-\zeta_0)}, \quad \zeta - \zeta_0 \gg 1. \qquad (3.3)$$

2. In the intermediate region $|\zeta - \zeta_0| \ll 1/\gamma$, neglecting terms of order γ^2 in (2.11), we have $d^2n/d\zeta^2 = 0$, whence

$$n = C_2 + C_3(\zeta - \zeta_0), \quad |\zeta - \zeta_0| \ll 1/\gamma. \qquad (3.4)$$

3. Finally, outside the discharge, in the region $\zeta_0 - \zeta \gg 1$, we can neglect the term $e^{-\Theta}$ in (2.11), and its solution that satisfies the condition (2.12) can be written in the form

$$n = C_4 e^{\gamma(\zeta-\zeta_0)}, \quad \zeta_0 - \zeta \gg 1. \qquad (3.5)$$

From the conditions that expressions (3.3) and (3.4) must coincide in the region $1 \ll \zeta - \zeta_0 \ll 1/\gamma$ and expressions (3.4) and (3.5) must coincide in the region $1 \ll \zeta_0 - \zeta \ll 1/\gamma$ we obtain $C_1 = C_2 = C_4 = 1/2$, $C_3 = \gamma/2$. Thus, under condition (3.1) the electron concentration is

$$n(\zeta - \zeta_0) = \begin{cases} \frac{1}{2} e^{\gamma(\zeta-\zeta_0)}, & -\infty < \zeta - \zeta_0 \ll 1/\gamma, \\ 1 - \frac{1}{2} e^{-\gamma(\zeta-\zeta_0)}, & -\infty < \zeta_0 - \zeta \ll 1/\gamma. \end{cases} \qquad (3.6)$$

When solving Eq. (2.10), the concentration of the electrons can be assumed known and equal to (3.6), and slowly varying in the region (3.2). Putting

$$d\Theta / d\zeta = g \qquad (3.7)$$

and confining ourselves to terms of order n'/n, we obtain from (2.10)

$$\frac{d^4 g}{d\zeta^4} - 3 \frac{n'}{n} \frac{d^3 g}{d\zeta^3} - n^2 g = 0. \qquad (3.8)$$

We seek the solution of (3.8) in the form

$$g = -A e^{\varphi}. \qquad (3.9)$$

Substituting (3.9) in (3.8) we obtain $(\varphi')^4 = n^2$ in the first approximation in n'/n, whence

$$\varphi = \pm \int n^{1/2} d\zeta. \qquad (3.10)$$

In the next approximation, taking (3.10) into account, we have

$$\frac{dA}{d\zeta} = 3A \left(\frac{n'}{n} - 2 \frac{\varphi''}{\varphi'} \right) = 0,$$

and thus, accurate to terms of higher order in the parameter n'/n, the factor A is a constant. Choosing the sign in formula (3.10) from the condition that $\Theta' \to 0$ as $\zeta \to \infty$, we get

$$\Theta'(\zeta - \zeta_0) = -A \exp\left\{ - \int_{-\infty}^{\zeta-\zeta_0} (n(\zeta))^{1/2} d\zeta \right\}, \quad n \gg \gamma^2. \qquad (3.11)$$

Since the temperature Θ itself tends to zero as $\zeta \to \infty$, we have

$$\Theta(\zeta - \zeta_0) = A \int_{\zeta-\zeta_0}^{\infty} d\zeta' \exp\left\{ - \int_{-\infty}^{\zeta'} (n(\zeta''))^{1/2} d\zeta'' \right\}, \quad n \gg \gamma^2. \qquad (3.12)$$

To calculate the constant A we integrate (2.11) from $-\infty$ to $+\infty$. Noting that $n'(-\infty) = n'(+\infty) = 0$, we obtain the relation

$$\int_{-\infty}^{+\infty} (n - e^{-\Theta}) d\zeta = 0, \qquad (3.13)$$

the physical meaning of which is that the diffusion does not alter the total number of particles. Substituting expression (3.6) for the electron concentration in (3.13), we obtain

$$\int_{-\infty}^{\zeta_0} e^{-\Theta} d\zeta = \int_{\zeta_0}^{+\infty} (1 - e^{-\Theta}) d\zeta. \qquad (3.14)$$

The integrands in (3.14) differ significantly from zero only in the region $|\zeta - \zeta_0| \sim 1 \ll 1/\gamma$. To determine the constant A it suffices therefore to know $\Theta(\zeta - \zeta_0)$ only in the region $|\zeta - \zeta_0| \ll 1/\gamma$. Substituting (3.6) in (3.12), we get

$$\Theta = A \int_{\zeta-\zeta_0}^{\infty} \exp\left(-\frac{\sqrt{2}}{\gamma} \exp\left(\frac{\gamma\zeta'}{2} \right) \right) d\zeta' \approx A \sqrt{2} \exp\left(-\frac{\sqrt{2}}{\gamma} - \frac{\zeta - \zeta_0}{\sqrt{2}} \right), \quad |\zeta - \zeta_0| \ll 1/\gamma. \qquad (3.15)$$

Putting $x = Be^{-\zeta/\sqrt{2}}$, $B = A\sqrt{2}e^{-\sqrt{2}/\gamma}$, we transform (3.14) into

$$\lim_{N\to\infty}\left(\sqrt{2}\int_{Be^{-N/\sqrt{2}}}^{\infty} e^{-x}\frac{dx}{x}-N\right)=0.$$

After integrating by parts we obtain

$$-\ln B+\int_{0}^{\infty}\ln x e^{-x}\,dx=0. \tag{3.16}$$

The integral in (3.16) is equal to −C, where C = 0.577... is the Euler constant. Thus, we obtain ultimately

$$A=\frac{1}{\sqrt{2}}\exp\left(\frac{\sqrt{2}}{\gamma}-C\right). \tag{3.17}$$

Thus, in the region (3.2), i.e., at $\zeta_0-\zeta \ll (2\gamma)^{-1}\ln(2\gamma)^{-1}$, the dimensionless temperature Θ is determined completely by formulas (3.12) and (3.17). To find the function $f(\gamma)=-\Theta'(-\infty)$, we must know $\Theta(\zeta-\zeta_0)$ also in the region $n \lesssim \gamma^2$. In this region, the electron concentration is

$$n={}^1\!/_2 e^{\gamma(\zeta-\zeta_0)},\quad n\ll 1. \tag{3.18}$$

Substituting (3.18) in (2.10), we otbtain for the function (3.17) a linear homogeneous equation

$$\frac{d^3}{d\zeta^3}\left(2e^{-\gamma(\zeta-\zeta_0)}\frac{dg}{d\zeta}\right)-\frac{1}{2}e^{\gamma(\zeta-\zeta_0)}g=0. \tag{3.19}$$

Introducing in lieu of ζ the new variable

$$z=\frac{\sqrt{2}}{\gamma}\exp\left(\frac{\gamma(\zeta-\zeta_0)}{2}\right), \tag{3.20}$$

We reduce Eq. (3.19) for the function g(z) to the dimensionless form

$$\frac{d^4g}{dz^4}+\frac{1}{z^2}\frac{d^2g}{dz^2}-\frac{1}{z^3}\frac{dg}{dz}-g=0. \tag{3.21}$$

In the region $n \gg \gamma^2$, i.e., at $z \gg 1$, we obtain with the aid of (3.11)

$$g=-Ae^{-z},\quad z\gg 1. \tag{3.22}$$

The function $g_0(z)$, which satisfies Eq. (3.21) and has the asymptotic form $g_0(z)=e^{-z}$, $z \gg 1$, is a universal function that does not depend on the parameters. Its plot, obtained by numerical integration, is shown in Fig. 3. In particular, $g_0(0) = 0.725...$ By virtue of the linearity and homogeneity of (3.21), we conclude that the function $f(\gamma)$ in (2.18) has the following form when $\gamma \ll 1$:

$$f(\gamma)=Ag_0'(0)=0{,}725\frac{e^{-C}}{\sqrt{2}}e^{\sqrt{2}/\gamma}\approx 0{,}288e^{\sqrt{2}/\gamma},\ \gamma\ll 1. \tag{3.23}$$

With increasing diffusion length, the function $f(\gamma)$, and hence also the power needed to heat the plasma to a given temperature, increases exponentially. Owing to the large argument of the exponential, even small changes of the diffusion coefficient can greatly influence the discharge parameters.

4. SURFACE RESISTANCE OF THE PLASMA

To calculate the surface resistance ρ of the plasma, we start from the relation

$$\rho=\frac{1}{2}\left(S_0\Big/\int_{-\infty}^{+\infty}\sigma S dx\right), \tag{4.1}$$

which follows directly from formulas (4.1) and (4.20) of[2]. Changing to the dimensionless variables of (2.9) and noting that $\sigma_m = c^2/8\pi\omega\delta_m$, we obtain

$$\rho=4\pi\omega c^{-2}\delta_m\varphi(\gamma), \tag{4.2}$$

where

$$\varphi(\gamma)=\left|\Theta'(-\infty)\Big/\int_{-\infty}^{+\infty}n\Theta' d\zeta\right|. \tag{4.3}$$

The plot of the function $\varphi(\gamma)$ at $\gamma \sim 1$ obtained by numerically integrating (2.1) and (2.11), is shown in Fig. 4. If the diffusion length is small in comparison with the depth of penetration of the field into the plasma ($\gamma \gg 1$), then the electron concentration is determined by the Saha formula. The results should then go over into the results of[2]. In terms of the dimensionless variables we have $n = e^{-\Theta}$, and the integral in (4.3) is equal to unity. Taking (2.19) into account, we have[2)]

$$\rho=6.28\pi\omega\delta_m/c^2=6.28\pi\cdot 10^{-9}\omega\delta_m\,[\text{ohm}],\quad \gamma\gg 1. \tag{4.4}$$

To calculate the function $\varphi(\gamma)$ in the opposite limiting case of large diffusion length, $\gamma \ll 1$, we note that it follows from (2.10) that

$$\int_{-\infty}^{+\infty}n\Theta' d\zeta=-\frac{d^2}{d\zeta^2}\frac{1}{n}\frac{d^2\Theta}{d\zeta^2}\Big|_{\zeta=-\infty}. \tag{4.5}$$

Changing to the variables g and z in accord with (3.7) and (3.20), we obtain from (4.5)

$$\int_{-\infty}^{+\infty}n\Theta' d\zeta=-\frac{\gamma}{2}\left(z\frac{d}{dz}\right)^2\frac{1}{z}\frac{dg}{dz}\Big|_{z=0},\quad \gamma\ll 1,$$

and consequently

$$\varphi(\gamma)=\frac{2}{\gamma}\left\{g(0)\Big/\left[\left(z\frac{d}{dz}\right)^2\frac{1}{z}\frac{dg}{dz}\right]_{z=0}\right\}\quad \gamma\ll 1. \tag{4.6}$$

Since Eq. (3.21), which defines the function g(z), does not contain any parameters, the expression in the curly

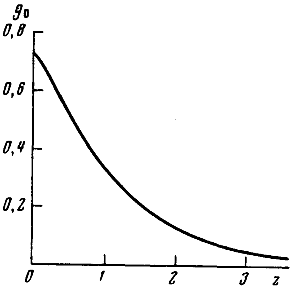

FIG. 3. Plot of the function $g_0(z)$.

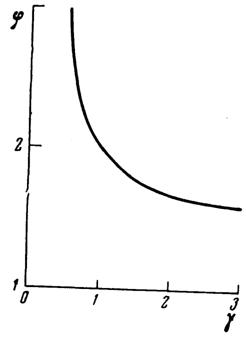

FIG. 4. Plot of the function $\varphi(\gamma)$, which determines the dependence of the surface resistance on the diffusion length.

[2)] The surface resistance given in formulas (4.21) and (4.22) of [2] and on p. 1896 of [4] is only one-half of the correct value.

brackets in (4.6) is a constant of the order of unity. A numerical solution yields

$$\varphi(\gamma)=1.57/\gamma,\quad \gamma\ll 1. \tag{4.7}$$

Substituting (4.7) in (4.2) we obtain the following expression for the surface resistance of the plasma in the case of a large diffusion length

$$\rho=\frac{6.28\pi}{\gamma}\frac{\omega\delta_m}{c^2}=6{,}28\pi\frac{\omega d}{c^2}=6.28\pi\cdot 10^{-9}\,\omega d[\mathrm{ohm}],\quad \gamma\ll 1. \tag{4.8}$$

Thus, if the diffusion length d is large in comparison with the depth of penetration δ_m of the field in the plasma, δ_m drops out of the expression for the surface resistance, and its place is taken by the diffusion length d. The electromagnetic field in this case penetrates into the discharge by an amount on the order of the diffusion length, regardless of the frequency. The surface resistance of the plasma is then appreciably larger (by $1/\gamma$ times) than the corresponding equilibrium value.

5. HIGH-FREQUENCY DISCHARGE IN A LONGITUDINAL MAGNETIC FIELD

We consider an HF gas discharge placed in a constant and homogeneous magnetic field whose vector **H** is parallel to the vector **E** of the electromagnetic field that maintains the discharge.

We note first that at a low degree of gas ionization and in the presence of local thermodynamic equilibrium, the characteristics of the discharge are practically independent of a magnetic field parallel to the current. Indeed, in actually realized fields the magnetic field does not influence the collision processes, and consequently does not upset the equilibrium distribution of the electron concentration. At a low degree of gas ionization, the main contribution to the thermal conductivity is made by neutral atoms and molecules, whose motion is not affected by a magnetic field. The Joule heat released in the plasma in the case of the ordinary (not anomalous) skin effect, or more accurately under the condition

$$\omega_0 v_T/c\nu_{\text{eff}}\ll 1, \tag{5.1}$$

is practically independent of the magnetic field, just as the longitudinal conductivity. Here ω_0 is the plasma frequency, v_T is the average thermal velocity of the electrons. Indeed, a weak field has little effect on the conductivity, by virtue of the condition $\Omega\ll\nu_{\text{eff}}$ ($\Omega = eH/mc$ is the Larmor frequency), and in a strong field the effect is small because $\omega_0 v_T/c\Omega\ll 1$. For hydrogen at atmospheric pressure and a temperature ~6000°K, the parameter (5.1) is of the order of 10^{-2}.

Allowance for the influence of electron diffusion, particularly if the diffusion length is large in comparison with the depth of penetration of the field in the plasma, changes the situation qualitatively. By decreasing the transverse-diffusion coefficient, the magnetic field can strongly influence the ionization-recombination balance. At atmospheric pressure, for electron concentrations of real interest, the diffusion is ambipolar. The dependence of the ambipolar diffusion on the magnetic field is given by[3)]

$$D(H)=2D_i(0)\left\{1+\sqrt{\frac{m}{M}}\left(1+\left(\frac{\Omega_e}{\nu_{\text{eff. e}}}\right)^2\right)+\frac{m}{M}\left(\frac{\Omega_e}{\nu_{\text{eff. e}}}\right)^2\right\}^{-1}$$

$$\approx 2D_i(0)\left\{1+\sqrt{\frac{m}{M}}\left(\frac{\Omega_e}{\nu_{\text{eff. e}}}\right)^2\right\}^{-1},\quad m\ll M. \tag{5.2}$$

The coefficient of ambipolar diffusion is equal to double the diffusion coefficient of the particles having a lower mobility. In a weak magnetic field, if $h\equiv(m/M)^{1/2}(\Omega_e/\nu_{\text{eff.e}})^2\ll 1$, we have $D(H) = 2D_i$, since the ions, being the heavier particles, have a lower mobility without a magnetic field. In a strong magnetic field, $h\gg 1$, the mobility of the electrons becomes smaller than the mobility of the ions, so that the ambipolar diffusion coefficient becomes equal to double the electron diffusion coefficient:

$$D(H)=2D_i(0)\sqrt{M/m}\,(\nu_{\text{eff. e}}/\Omega_e)^2=2D_e(H),\quad h\gg 1.$$

The parameter h for hydrogen at atmospheric pressure and ~6000°K becomes of the order of unity in fields $H_0\approx 80$ kOe.

If the diffusion length is large in comparison with the depth of penetration of the field into the plasma, the discharge parameters depend very strongly, exponentially, on the diffusion coefficient. Under these conditions a dependence on the magnetic field can appear in fields much weaker than H_0, in which the ambipolar-diffusion coefficient decreases by a relatively small amount. This circumstance might explain the influence of the magnetic field on the Kapitza discharge[1] through its effect on the ambipolar-diffusion coefficient in a cold plasma.

6. EFFECT OF DIFFUSION ON A DISCHARGE IN HYDROGEN

By way of a concrete example, we consider an HF discharge in hydrogen at atmospheric pressure and a field frequency $\omega = 10^{10}$, as is the case in Kapitza's experiments[1].

The energy dependence of the cross section for the capture of an electron by a hydrogen atom, calculated by Massey[6], is shown in Fig. 5. The coefficient β of trapping of an electron by an atom, which enters in the ionization-recombination balance equation (2.3), is expressed in the following manner in terms of the capture cross section $\sigma(E)$:

$$\beta(T)=N_0\int f_0(v)\,\sigma\left(\frac{mv^2}{2}\right)v\,d^3v, \tag{6.1}$$

where $f_0(v)$ is the Maxwellian electron velocity distribution function. The temperature dependence of the trap-

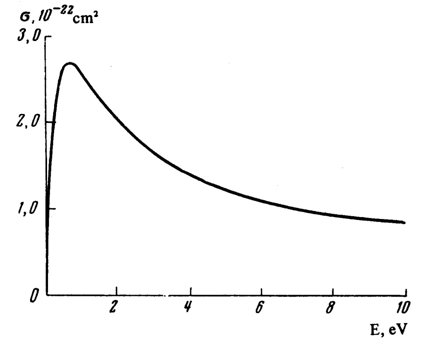

FIG. 5. Cross section for the trapping of an electron by a hydrogen atom vs the electron energy [5].

[3)] A one-temperature plasma is assumed in (5.2). More general expressions for the diffusion coefficients are given, for example, in the review of Gurevich and Tsedilina [5].

ping coefficient for hydrogen at atmospheric pressure is shown in Fig. 6.

Using the values of the impact-radiative recombination, given by Bates and Dalgarno[7], we can show that in hydrogen at atmospheric pressure, up to temperature on the order of 10 000°K, the main contribution to the ionization-recombination balance is made by trapping of electrons by neutral atoms with formation of negative ions.

The coefficient of particle diffusion in a gas, in the mean free path approximation ([8], p. 61), is equal to

$$D = v_T l/3, \tag{6.2}$$

where v_T is the average thermal velocity of the diffusing particle and l is the mean free path.

The ratio of the depth of penetration of the field into a hydrogen plasma to the diffusion length $\gamma = \delta_m/d$ is shown in Fig. 7 as a function of the temperature T_m. By determining the values of $f(\gamma)$ at $\gamma \sim 1$ from Fig. 1, and from formula (3.23) at $\gamma \ll 1$, and by using the temperature dependence of the thermal conductivity[9] (see also[4], Fig. 11), we find the dependence of the plasma temperature in a hydrogen discharge on the supplied power, shown in Fig. 8. We call attention to the fact that the parameter γ decreases with increasing temperature (see Fig. 7). According to (3.23), $f(\gamma)$ depends exponentially on δ_m. Since the depth of penetration itself depends exponentially on the temperature, we arrive at the conclusion that the plasma temperature in the discharge is proportional to the logarithm of the logarithm of the power S_0, i.e., it depends extremely weakly on S_0.

Figure 9 shows the temperature dependence of the surface resistance of the plasma with allowance for the electron diffusion (curve 1), calculated from formula (4.2). For comparison, the figure shows the temperature dependence of the surface resistance, as calculated from formula (4.4) without allowance for diffusion (curve 2). With increasing temperature, the depth of penetration of the field into the plasma decreases and ultimately becomes smaller than the diffusion length, which increases slightly. The depth of penetration of the field into the plasma then drops out of the expression for the surface resistance, and the latter becomes proportional to the diffusion length. As a result, the surface resistance has a characteristic minimum. The minimum value of the surface resistance (≈ 60 Ω) is several dozen times larger than the resistance measured by Kapitza and Filimonov.[10] Thus, a consistent allowance for the diffusion aggravates the contradiction encountered when an attempt is made to explain the Kapitza discharge from the point of view of a low temperature plasma.

For experimental applications, it is of interest to calculate the parameters of an infinite cylindrical discharge with allowance for diffusion. Let R be the radius of a cooled vessel (or resonator). In accordance with[2], the radius r_0 of the discharge satisfies the relation

$$r_0 = R \exp\left(-\frac{1}{S_0 r_0}\int_{T_0}^{T_m} \varkappa \, dT\right). \tag{6.3}$$

As the initial variable it is convenient to use instead of S_0 the power s_0 per unit length of the cylinder:

$$s_0 = 2\pi r_0 S_0. \tag{6.4}$$

For R = 10 cm, the dependence of the temperature at the discharge axis on the power input per unit length of the cylinder, is shown in Fig. 1a (solid curve). The discharge radius as a function of the power $r_0(s_0)$ is shown in Fig. 10b. For comparison, Fig. 10 shows the corresponding curves (dashed) without allowance for the diffusion. Allowance for the diffusion leads to a certain decrease of the discharge temperature and to an increase of its radius.

Remaining in the framework of the strong skin effect ($r_0 \gg \delta_m$), we are unable to investigate the influence of diffusion on the diffuse state of the discharge (see[4]).

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APPENDIX

AMBIPOLAR DIFFUSION IN THE PRESENCE OF NEGATIVE IONS

The particle balance equations are derived from the kinetic equations by the Chapman-Enskog method[11]. Let the degree of ionization of the gas be low, and let the

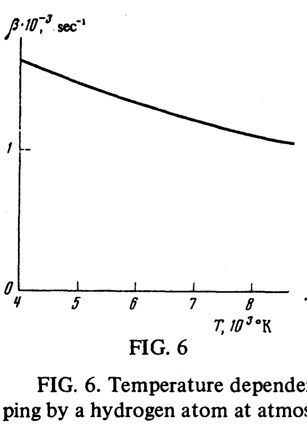

FIG. 6

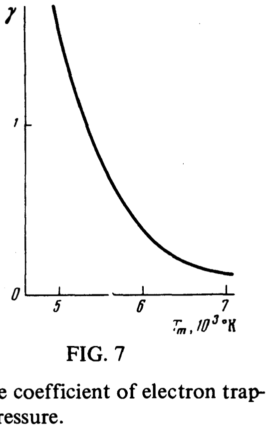

FIG. 7

FIG. 6. Temperature dependence of the coefficient of electron trapping by a hydrogen atom at atmospheric pressure.

FIG. 7. Temperature dependence of the ratio γ of the depth of penetration of the field to the diffusion length, for hydrogen at atmospheric pressure.

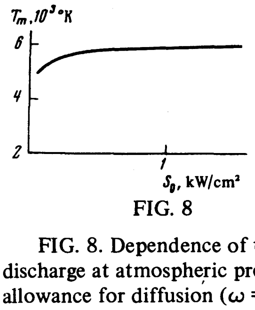

FIG. 8

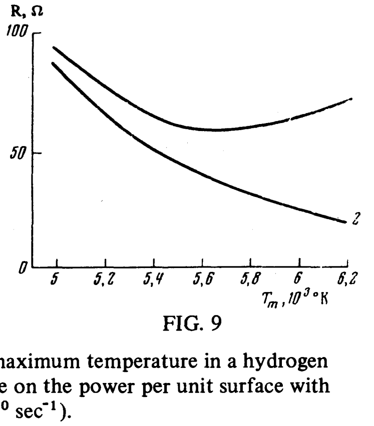

FIG. 9

FIG. 8. Dependence of the maximum temperature in a hydrogen discharge at atmospheric pressure on the power per unit surface with allowance for diffusion ($\omega = 10^{10}$ sec⁻¹).

FIG. 9. Temperature dependence of the surface resistance of a hydrogen plasma at atmospheric pressure with allowance (curve 1) and without allowance (curve 2) for diffusion ($\omega = 10^{10}$ sec⁻¹).

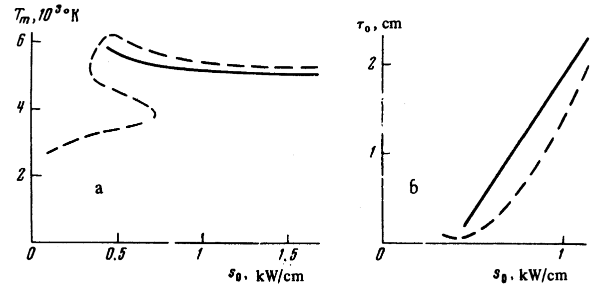

FIG. 10. Maximum plasma temperature (a) and radius (b) of a hydrogen discharge at atmospheric pressure vs. the power per unit cylinder length with (solid) and without (dashed) allowance for diffusion ($\omega = 10^{10}$ sec^{-1}, R = 10 cm).

problem be one-dimensional. We denote by E_y the HF electric field, and by E_0 the electric field due to separation of the charges in the plasma. We write the kinetic equation for the electron distribution function f_e in the form

$$\frac{\partial f_e}{\partial t} + v_x \frac{\partial f_e}{\partial x} - \frac{e}{m} E_y \frac{\partial f_e}{\partial v_y} - \frac{e}{m} E_0 \frac{\partial f_e}{\partial v_x} = - \frac{\partial \mathbf{j}_e}{\partial \mathbf{v}} - S_e^n. \quad \text{(A.1)}$$

In (A.1), the collision integral is represented in the form of a sum of two terms describing collisions with and without conservation of the number of electrons, respectively. We are interested in a case when the electron distribution function differs little from Maxwellian: $f_e = f_0 + f_1$, $|f_1| \ll f_0$, and the increment f_1 can be conveniently divided into an alternating part $\tilde{f}_1$ proportional to E_y, and a constant part $\bar{f}_1$ connected with the electron concentration gradient (in an HF discharge, the temperature gradient is smaller than the concentration gradient by a factor I/T_m), $f_1 = \tilde{f}_1 + \bar{f}_1$. Noting that $\partial \mathbf{j}_e\{f_0\}/\partial \mathbf{v} = 0$, we have

$$\frac{\partial \tilde{f}_1}{\partial t} + \frac{\partial}{\partial \mathbf{v}} \mathbf{j}_e \{\tilde{f}_1\} = \frac{e}{m} E_y \frac{\partial f_0}{\partial v_y}, \quad \text{(A.2)}$$

$$\frac{\partial}{\partial \mathbf{v}} \mathbf{j}_e \{\bar{f}_1\} = - v_x \left(\frac{\partial f_0}{\partial x} + \frac{eE_0}{kT} f_0 \right). \quad \text{(A.3)}$$

The terms $S_e^n\{f_0\}$, $v_x \partial \tilde{f}_1/\partial x$, and $(e/m)E_0 \partial \tilde{f}_1/\partial v_x$ can be neglected because the electron mean free path is small compared with the diffusion length. In the approximation of effective collision number, we get from (A.2) expression (2.2) for the conductivity of the gas. Writing the elastic-collision integral in (A.3) in the form[12] $\partial \mathbf{j}_e\{\bar{f}_1\}/\partial \mathbf{v} = \nu(v)\bar{f}_1$, we obtain

$$\bar{f}_1 = - \frac{v_x}{\nu(v)} f_0 \left\{ \frac{1}{N_e} \frac{\partial N_e}{\partial x} + \frac{eE_0}{kT} \right\}.$$

Integrating (A.1) over the velocities, averaging over time intervals that are long in comparison with the period of the HF oscillations, and regarding as the process that does not conserve the number of electron only trapping of electrons by neutral atoms (and, of course, the inverse process), we have

$$\frac{\partial N_e}{\partial t} - \frac{\partial}{\partial x} D_e \left(\frac{\partial N_e}{\partial x} + \frac{eE_0}{kT} N_e \right) = - \beta \left(N_e - N_{e\,\text{eq.}}(T) \frac{N_-}{N_{-\text{eq.}}} \right) \quad \text{(A.4)}$$

Here

$$D_e = \frac{1}{N_e} \int \frac{v_x^2}{\nu(v)} f_0 d^3 v$$

is the electron diffusion coefficient, β is the probability of electron trapping by neutral atoms, $N_{e.\text{eq}}$ and $N_{-\text{eq}}$ are respectively the numbers of electrons and negative ions in equilibrium at the given temperature. It is assumed here that during the trapping process the momentum is transferred to a third particle, a neutral atom.

We derive analogously equations for the concentrations of the negative and positive ions. Taking into account their mutual recombination, we have

$$\frac{\partial N_i}{\partial t} - \frac{\partial}{\partial x} D_i \left(\frac{\partial N_i}{\partial x} - \frac{eE_0}{kT} N_i \right) = - \beta_1 (N_i N_- - N_{i\,\text{eq.}} N_{-\text{eq.}}), \quad \text{(A.5)}$$

$$\frac{\partial N_-}{\partial t} - \frac{\partial}{\partial x} D_- \left(\frac{\partial N_-}{\partial x} + \frac{eE_0}{kT} N_- \right)$$
$$= - \beta_1 (N_i N_- - N_{i\,\text{eq.}} N_{-\text{eq.}}) + \beta \left(N_e - N_{e\,\text{eq.}} \frac{N_-}{N_{-\text{eq.}}} \right). \quad \text{(A.6)}$$

Here β_1 is the coefficient of recombination of positive and negative ions with one another, and D_i and D_- are the diffusion coefficients of the positive and negative ions. The electric field E_0 satisfies the Poisson equation

$$dE_0 / dx = 4\pi e (N_i - N_e - N_-). \quad \text{(A.7)}$$

Equations (A.4)—(A.7) constitute a complete system of equations describing the diffusion in a weakly ionized gas in the presence of negative ions. Adding Eqs. (A.4) and (A.6) and subtracting (A.5), we verify that Eqs. (A.4)—(A.6) do not violate the charge-conservation law. In the stationary state, with allowance for the smallness of the variation of the diffusion coefficients over the diffusion length, we have

$$\frac{d}{dx} \left(\frac{dN_e}{dx} + \frac{eE_0}{kT} N_e \right) = \frac{\beta}{D_e} \left(N_e - N_{e\,\text{eq.}} \frac{N_-}{N_{-\text{eq.}}} \right), \quad \text{(A.8)}$$

$$\frac{d}{dx} \left(\frac{dN_i}{dx} - \frac{eE_0}{kT} N_i \right) = \frac{\beta_1}{D_i} (N_i N_- - N_{-\text{eq.}} N_{i\,\text{eq.}}), \quad \text{(A.9)}$$

$$\frac{d}{dx} \left(\frac{dN_-}{dx} + \frac{eE_0}{kT} N_- \right) = \frac{\beta_1}{D_-} (N_- N_i - N_{-\text{eq.}} N_{i\,\text{eq.}})$$
$$- \frac{\beta}{D_-} \left(N_e - N_{e\,\text{eq.}} \frac{N_-}{N_{-\text{eq.}}} \right). \quad \text{(A.10)}$$

The large value of the coefficient β_1 of recombination of positive and negative ions causes the negative ions produced by trapping of electrons by neutral atoms to recombine more rapidly than to diffuse, so that their concentration is smaller than the electron concentration. Neglecting the left-hand side of Eq. (A.10), we have

$$\beta_1 (N_- N_i - N_{-\text{eq.}} N_{i\,\text{eq.}}) = \beta (N_e - N_{e\,\text{eq.}} N_- / N_{-\text{eq.}}), \quad \text{(A.11)}$$

whence

$$N_- = N_{-\,\text{eq.}} \frac{\beta N_e + \beta_1 N_{i\,\text{eq.}} N_{-\text{eq.}}}{\beta N_{e\,\text{eq.}} + \beta_1 N_i N_{-\text{eq.}}}. \quad \text{(A.12)}$$

When the concentrations of the electrons and ions coincide with the corresponding equilibrium values (inside the plasma), the concentration of the negative ions likewise does not differ from the equilibrium value. In the case of strong diffusion, the concentrations of the electrons and positive ions outside the discharge decrease much more slowly than the equilibrium concentrations, and we obtain from (A.12) for the concentration of the negative ions

$$N_- = \beta / \beta_1. \quad \text{(A.13)}$$

Under the considered conditions, expression (A.13) for the negative-ion concentration yields the same order of magnitude as the equilibrium expression. Taking this circumstance into account and noting also that the use of the exact expression for $N_-/N_{-\,eq}$ can influence only the pre-exponential factor in (3.23), we neglect the difference between the concentration of the negative ions and the equilibrium concentration.

Adding (A.8) and (A.9), neglecting the small difference between the electron and positive-ion concentrations, and taking (A.11) into account, we obtain Eq. (2.6), in which D is the ambipolar-diffusion coefficient (5.2).

Let us also calculate the electric field E_0 produced when the charges are separated in the diffusion process. Subtracting (A.9) from (A.8) and then eliminating $N_e - N_{e.eq}$ with the aid of (2.6), we get

$$\frac{1}{2}\frac{d}{dx}\left(\frac{d}{dx}(N_e-N_i)+\frac{eE_0}{kT}(N_e+N_i)\right)=-\frac{D_e-D_i}{D_e+D_i}\frac{d^2N_e}{dx^2}. \quad (A.14)$$

Since the plasma in the HF discharge is far from any charged surface, the electric field varies appreciably over the same distances as the electron concentration. $N_e - N_i$ is therefore of the same order as N_- or $(r_D/d)^2N_e$, where r_D is the Debye radius. Neglecting in (A.14) the difference between N_e and N_i, we obtain the well-known relation[13]

$$E_0=-\frac{kT}{e}\frac{D_e-D_i}{D_e+D_i}\frac{d}{dx}\ln N_e. \quad (A.15)$$

In particular, in the transition layer and in the region outside the plasma, in which the diffusion is ambipolar, the electric field in the case of a large diffusion length is equal to

$$E_0=-\frac{kT}{ed}\frac{D_e-D_i}{D_e+D_i}.$$

It can be shown that the derived formulas retain their form also in the presence of a magnetic field parallel to the y axis, if D_e and D_i are taken to mean the corresponding transverse-diffucion coefficients.

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[1] P. L. Kapitza, Zh. Eksp. Teor. Fiz. **57**, 1801 (1969) [Sov. Phys.-JETP **30**, 973 (1970)].

[2] B. E. Meĭerovich and L. P. Pitaevskiĭ, ibid. **61**, 235 (1971) [**34**, 121 (1972)].

[3] A. D. MacDonald, Microwave Breakdown in Gases, Wiley, 1966.

[4] B. E. Meĭerovich, Zh. Eksp. Teor. Fiz. **61**, 1891 (1971) [Sov. Phys.-JETP **34**, 1006 (1972)].

[5] A. V. Gurevich and E. E. Tsedilina, Usp. Fiz. Nauk **91**, 609 (1967) [Sov. Phys.-Uspekhi **10**, 214 (1967)].

[6] D. R. Bates and A. Dalgarno, in: Atomic and Molecular Processes, Chap. 7, Academic Press.

[7] H. S. W. Massey, Negative Ions, 2d ed., Cambridge, 1950.

[8] E. W. McDaniel, Collision Phenomena in Ionized Gases, Wiley, 1964.

[9] K. Behringer, W. Kollmar and J. Mentel, Zs. Physik, **215**, 127, 1968.

[10] P. L. Kapitza and S. I. Filimonov, Zh. Eksp. Teor. Fiz. **61**, 1016 (1971) [Sov. Phys.-JETP **34**, 542 (1972)].

[11] S. Chapman and T. Cowling, Mathematical Theory of Nonuniform Gases, Cambridge Univ. Press, 1962.

[12] V. L. Ginzburg and A. V. Gurevich, Usp. Fiz. Nauk **70**, 201 (1960) [Sov. Phys.-Uspekhi **3**, 115 (1960)].

[13] V. L. Granovskiĭ, Elektricheskiĭ tok v gaze (Electric Current in Gas), Gostekhizdat, 1952, Sec. 60.

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