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MAGNETIC PULSATION SPECTRA IN A NONISOTHERMAL PLASMA

M. A. LIVSHITZ and V. N. TSYTOVICH

P. N. Lebedev Physics Institute;

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Nonlinear interactions between magnetic pulsations and ion-sound pulsations in a nonisothermal magneto-active plasma are investigated. Interactions between collisionless Alfven or magneto-hydrodynamic waves and magnetized ion-sound waves in the low frequency region ($\omega > \omega_{Hi}$) are considered. Interactions between helicons (whistlers) and unmagnetized ion sound in the $\omega_{Hi} < \omega < \omega_{He}$ frequency range are considered. The nonlinear interaction equations obtained are used for estimating the characteristic interaction times and for estimating the stationary turbulence spectra.

INTRODUCTION

THE nonlinear interactions and the spectra of the magnetic pulsations of an isothermal ($T_e = T_i$) turbulent plasma were considered in[1,2]. The magnetic pulsations can have low frequencies $\omega < \omega_{Hi}$ and correspond to collisionless Alfven and magnetohydrodynamic waves at $\omega > \nu_i$ and high frequencies $\omega_{He} > \omega > \omega_{Hi}$ corresponding to helical waves—helicons (whistlers). Here $\omega_{H\alpha} = |e_\alpha| H_0/m_\alpha c$, e_α, m_α are respectively the Larmor frequency, the charge and mass of particles of type α, and H_0 is the magnetic field intensity.

Under conditions of a non-isothermal plasma ($T_e > T_i$), ion-acoustic oscillations are produced in both frequency regions: magnetized ones with spectrum $\omega_s = |k_z| v_s$ in the first frequency region and nonmagnetized $\omega_s = kv_s$ in the second. Here ω is the wave frequency, $\mathbf{k}$ is the wave vector, k_z is its projection on the external magnetic field H_0, and v_s is the speed of the ion sound; we assume henceforth $v_A \gg v_s$, where $v_A = H_0/\sqrt{4\pi n_0 m_i}$ is the Alfven velocity.

The turbulence spectra of the indicated magnetic pulsations, which determine the redistribution of their energy density with respect to $\mathbf{k}$ or ω, will depend not only in the interaction of the magnetic pulsations with one another, but also on their interaction with ion sound. Such interactions can serve also for the diagnostics of ion-acoustic turbulence, namely, from the intensity and distribution of the magnetic-oscillations one can assess the spectrum of the ion-acoustic oscillations. The magnetic oscillations are much easier to detect than the electric ones. This diagnostic method was proposed and used in a number of studies[3,4].

The analysis of the interaction between the magnetic pulsations and ion sound is apparently of importance in the problem of solar wind, where the magnetic oscillations can be directly excited by the anisotropic or hose instability. Their interaction with the ion acoustic oscillations could change the distribution of the magnetic oscillations and by the same token affect the spectrum of the latter. Finally, if intense ion-acoustic oscillations exist in the earth's magnetosphere as a result of the anomalous resistance of the plasma to the resultant strong electric field, then the conditions for helicon propagation, which is frequently detected near the earth, can be radically altered.

In spite of the importance of the aforementioned problems, no attempts have been made so far to investigate in detail the interaction between the magnetic and ion-acoustic pulsations, although a general procedure for the calculation of nonlinear interactions has by now been sufficiently well developed[5]. Thus, only estimating formulas were used in[3] for two ion-acoustic waves propagating along a magnetic field.

The purpose of the present paper is to study in detail interactions between magnetic and ion-acoustic pulsations and to obtain simple expressions that are convenient in use.

Since the procedure for calculating the probabilities of various nonlinear interactions is known, we do not present here the calculations connected with expansion of the rigorous expressions in powers of small parameters (such as m_e/m_i, v_s/v_A, $\omega_{H\alpha}$, $kv_{T\alpha}/\omega$), estimates of the relative contributions of the electrons and ions to the nonlinear currents, etc., but confine ourselves only to final results and indicate the limits of their applicability. When expressing the nonlinear interactions with the aid of the probabilities, we shall frequently use the differential forms of the nonlinear interactions, obtained under conditions when the intervals of the spectral restructuring are much smaller than the characteristic frequency. Although these intervals are finite, the approximate equations describing the differential restructuring correspond to an approximation in which the intervals are regarded as infinitesimally small physically. In spite of the approximate character of the equations obtained thereby, they are convenient for use both to estimate the characteristic times of the nonlinear interactions and to estimate the turbulence spectra. An approximation of this type corresponds to the approximation given in[6].

1. INTERACTIONS OF HELICONS WITH ION SOUND

An important type of interaction between helicons (whistlers) (3) and ion-sound waves (s) is induced scattering by ions. The matrix element of this scattering is determined by the contribution of the electron to the nonlinear current, if

$$\omega^w > \frac{\omega_{He}(1-x_1^2)}{(1-x^2)|x|}\left(\frac{v_A}{v_{Te}}\right)^2 = \frac{\omega_{Hi}(1-x_1^2)}{|x|(1-x^2)}\frac{H_0^2}{4\pi n_0 T_e} = \omega_*, \qquad (1)$$

ω^W is the helicon frequency, $v_{Te} = \sqrt{T_e/m_e}$, $x = \cos\theta$,

$x_1 = \cos\theta_1$, while θ and θ_1 are respectively the angles between the magnetic field on the propagation directions of the helicons and ion-acoustic waves, respectively.

Under these conditions, calculation of the scattering probability yields

$$w^{ws}(\mathbf{k}^w, \mathbf{k}_1^s) = 2\pi \frac{k^2|x|\omega_1^3(1-x^2)}{k_1^2 n_0^2 \omega_{Hi}(1+T_e/T_i)^2} \delta(\omega^w - \omega_1^s - (\mathbf{k}-\mathbf{k}_1)\mathbf{v}), \quad (2)$$

Physically, the infinitesimally small interval $\Delta\omega$ over which energy is transferred in one interaction act, is estimated at $\Delta\omega/\omega \sim \sqrt{T_i/T_e}$, i.e., it is sufficiently small at a small ratio T_i/T_e. The interaction described by the probability (2) has the following approximate differential form:

$$\frac{\partial W_{\omega x}^s}{\partial t} = \frac{4\pi^2\omega^3}{n_0 m_i v_A^2} \frac{W_{\omega x}^s}{(1+T_e/T_i)^2} \int dx_1 (1-x_1^2) \frac{\partial W_{\omega x_1}}{\partial \omega}, \quad (3)$$

$$\frac{\partial W_{\omega x}^w}{\partial t} = \frac{4\pi^2\omega^2(1-x^2)}{n_0 m_i v_A^2} \frac{W_{\omega x}^w}{(1+T_e/T_i)^2} \int dx_1 \left(\omega \frac{\partial W_{\omega x_1}}{\partial \omega} + 2W_{\omega x_1}^s\right). \quad (4)$$

Here $W_{\omega x}$ is the energy density of the turbulent oscillations, normalized in accordance with the relation

$$W = \int W_{\omega x}\, d\omega\, dx\, d\varphi. \quad (5)$$

From (3) we get an estimate of the characteristic time of interaction between the ion-acoustic oscillations and the helicons

$$\gamma \sim \omega W^w / n_0 m_i v_A^2 \sim \omega \tilde{H}_1^2 / H_0^2, \quad (6)$$

where the helicon energy is $W^w \approx \tilde{H}_1^2/4\pi$.

We see therefore that the helicon energy level that can influence the spectra of the ion-acoustic turbulence corresponds to the condition

$$\tilde{H}_1^2 / H_0^2 > \sqrt{m_e/m_i}, \quad (7)$$

for in this case the interaction with the helicons turns out to be stronger than the ion damping for the ion-acoustic waves. Under the conditions when the s-waves are excited by the drift of the electrons relative to the ions, the level of the helicons, which greatly overlaps the buildup of the s-waves by the electron drift, corresponds to

$$\tilde{H}_1^2 / H_0^2 > u / v_{Te}, \quad (8)$$

where u is the translational drift velocity of the electrons relative to the ions.

As seen from (3), a helicon energy spectrum that drops with frequency leads to damping of the ion-acoustic oscillations, while a growing spectrum leads to their buildup. The experimentally measured spectrum of the magnetic oscillations under conditions of collisionless shock waves produced when the solar wind flows around the earth's magnetosphere and when this phenomenon is simulated under laboratory conditions[7], corresponds to $W_\omega \sim 1/\omega^\nu$, where $2 < \nu < 3$, and $\tilde{H}_1^2$ is not too small in comparison with H_0^2, i.e., (3) always leads to damping of ion sound. This damping is estimated at

$$\gamma \sim \omega \frac{\tilde{H}_1^2}{H_0^2} \left(\frac{\omega_{min}}{\omega}\right)^{\nu-1}, \quad (9)$$

which can lead to stabilization of ion sound at $\nu = 2$ only if ω_{pi} is not too large in comparison with ω_{Hi}, i.e., at $v_A \sim c$, whereas in the solar wind we have $v_A \sim 10^{-3}$ c, i.e., the ion-sound instability should not be suppressed under such conditions.

Let us analyze now the results that follow qualitatively from (4). We consider first the case when the helicon energy is low enough, and the spectrum of the ion sound is regulated by the interaction of the s-oscillations with one another. According to the theory of[8], where correlation effects are taken into account (and incidentally also according to[6], where these effects are not taken into account), the dependence of the s-wave energy density on the frequency can be approximately written in the form

$$W_{\omega x}^s \sim \omega^{-1} \ln \lambda(\omega), \quad (10)$$

where $\lambda(\omega)$ is a slowly varying function of ω (to be sure, the numerical coefficients and the dependences on the angles are different in these cases). Thus, far from the points at which $\lambda(\omega)$ becomes equal to unity, formula (4) points to a buildup of helicons by ion-sound oscillations. Such a buildup will continue until the onset of helicon-helicon interaction, the characteristic time of which is estimated at[2]

$$1/\tau \sim \omega \tilde{H}_1^2 / H_0^2. \quad (11)$$

It also follows directly from this that the helicon energy becomes of the same order as the s-wave energy. Such a helicon level can influence the spectrum of the s-waves only under the conditions $H_0^2 \lesssim n_0 T_e$, which can not be satisfied when $v_s \ll v_A$. Thus, in the presence of ion-sound turbulence and when $v_A \gg v_s$ we obtain a stationary spectrum of helicons having an energy of the order of the ion-sound energy, but these helicons have little influence on the ion-sound turbulence.

It follows from dimensionality considerations that under conditions when the buildup of the helicons by ion sound is offset by their nonlinear interaction connected with the decay $w \rightleftarrows w' + w''$; the helicon energy density varies with frequency like $1/\omega^\nu$ with $\nu = 1$.

Condition (1) at $v_A > v_{Te}$ cannot be satisfied, since $\omega^W < \omega_{He}|x|$. Under solar-wind conditions v_A is frequently somewhat smaller than v_{Te}, i.e., at frequencies on the order of ω_{He}, where nonlinear interactions of the helicons with the s-waves are the most effectives, it already becomes possible to violate (1). If $v_A \gg v_{Te}$, then the inequality (1) is violated in a wide range of helicon existence. If the inequality opposite to (1) holds, then the contribution of the ions to the nonlinear current, which determines the scattering matrix element, predominates. The probability of scattering by ions takes the form

$$w^{ws}(\mathbf{k}^w, \mathbf{k}_1^s) = \frac{\pi}{4} \frac{\omega_1^3 \omega_{Hi}(1-x_1^2)(1+x^2)}{n_0^2\omega^2|x|(1+T_e/T_i)^2} \delta(\omega^w - \omega_1^s - (\mathbf{k}-\mathbf{k}_1)\mathbf{v}), \quad (12)$$

and the analogos of Eqs. (3) and (4) become

$$\frac{\partial W_{\omega x}^s}{\partial t} = \frac{\pi^2}{2} \frac{1-x^2}{(1+T_e/T_i)^2} \frac{\omega\omega_{Hi}}{n_0 T_e} W_{\omega x}^s \int dx_1 \frac{1+x_1^2}{|x_1|} \left(\omega \frac{\partial W_{\omega x_1}}{\partial \omega} - 3W_{\omega x_1}^w\right), \quad (13)$$

$$\frac{\partial W_{\omega x}^w}{\partial t} = \frac{\pi^2}{2} \frac{1+x^2}{|x|} \frac{\omega\omega_{Hi}}{n_0 T_e} \frac{W_{\omega x}^w}{(1+T_e/T_i)^2} \int dx_1 (1-x_1^2) \left(\omega \frac{\partial W_{\omega x_1}^s}{\partial \omega} + 4W_{\omega x_1}^s\right). \quad (14)$$

It is easily seen that under these conditions the interactions are more effective in the ratio $\omega_*/\omega \gg 1$ (see (1)). This is also accompanied by excitation of helicons by s-waves; saturation sets in when the energy contained in the helicons exceeds the energy of the ion-sound oscillations by a factor ω_*/ω (in the case of greatest interest when $\omega \sim \omega_{pi}$, we have $\omega_*/\omega = v_A^3/v_s^2 c$). This level of the helicon oscillation energy influences the s-wave spectrum if

$$v_A^2 > v_s c. \tag{15}$$

Condition (15) is stronger than the foregoing condition $\omega_{pi} < \omega_*$. Therefore if (15) is not satisfied but $\omega_* > \omega_{pi}$, the resultant helicon level has no effect on the ion sound and controlled by the excitation of the helicons from the ion sound and by the decay interactions between the helicons. From dimensionality considerations the corresponding helicon spectrum is given by $W_\omega \sim 1/\omega^\nu$ with $\nu = 2$. This value can be compared with the observed spectrum of the magnetic oscillations produced in a collisionless shock wave flowing around the earth's magnetosphere, viz., $W_\omega \sim 1/\omega^\nu$ with $2 < \nu < 3$ [7].

Another important nonlinear interaction used for diagnostics of ion-sound turbulence is the merging of two s-waves into a helicon. Calculation yields for this process the probability

$$w^{wss}(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2) = \frac{2(2\pi)^4 \delta(\mathbf{k} - \mathbf{k}_1 - \mathbf{k}_2)\delta(\omega^w - \omega_1^s - \omega_2^s)(1 - x^2)\omega\omega_1\omega_2}{n_0 m_i v_A^2 (1 + k_1^2 r_{De}^2)(1 + k_2^2 r_{De}^2)} \tag{16}$$

for isotropic s-turbulence, recognizing that $k_1^S \gg k^W$, we obtain, neglecting reabsorption of the helicons as a result of the inverse process $w \to s + s'$.

$$\frac{\partial W_{\omega x}^w}{\partial t} = \frac{\pi^2 v_s \omega_{pi}^3 \sqrt{\omega}(1 - x^2)}{n_0 m_i v_A^2 \omega_{Hi}^{5/2} |x|^{3/2} c^3} (W_{\omega/2v_s}^s)^2, \tag{17}$$

and for anisotropic turbulence

$$\frac{\partial W_{\omega x}^w}{\partial t} = \frac{2\pi(1 - x^2)\omega^2 \omega_{Hi}}{n_0 m_i v_A^2 |x|} \left(\frac{v_s}{v_A}\right)^2 \times \int \frac{W_{\omega_1 x_1} W_{\omega_2 x_2}^s}{\omega_1 \omega_2} \left[(1 - x^2)(1 - x_1^2) - \left(\frac{k^2 + k_1^2 - k_2^2 - 2kk_1 x x_1}{2kk_1}\right)^2\right]^{-1/2} \times d\omega_1\, d\omega_2\, dx_1\, \delta(\omega - \omega_1 - \omega_2). \tag{18}$$

It is seen from (17) that if we neglect the reaction of the helicons on the sound and integrate (17) along the "line of sight" L, we obtain a connection between the helicon and sound energy densities, i.e., by measuring the helicon energy density we obtain directly the s-wave energy density:

$$W_{\omega x}^w \approx \frac{\pi^2}{2} \frac{\omega_{Hi}^2 (1 - x^2) v_s}{n_0 m_i v_A^6} L (W_{\omega/2v_s}^s)^2. \tag{19}$$

2. SPECTRA OF ALFVENAND MAGNETOHYDRODYNAMIC PULSATIONS IN A NONISOTHERMAL PLASMA

Proceeding to lower frequencies of the magnetic perturbations, we note that they can be caused by direct excitation of Alfven (A) and magnetohydrodynamic (M) waves as a result of hose and anisotropic instability and of the interaction of these waves with each other. The turbulence spectra of such waves in an isothermal plasma in the absence of ion sound were considered in[1]. We indicate here some of the results obtained there.

Owing to the nonlinear interaction of the A and M waves, induced scattering by ions gives rise to a stationary A- and M-wave spectrum: $W_\omega \sim 1/\omega^\nu$ with $\nu = 1$. The angular distribution of the A waves is arbitrary here, and the spectrum of the M waves is one-dimensional; the M waves propagate along the field $\mathbf{H}_0$ or in the opposite direction.

In a nonisothermal plasma, the A and M waves can be excited not only directly, but also through nonlinear conversion from magnetized ion-sound (MS) waves. In addition, the MS waves themselves can be excited by magnetic pulsations in nonlinear interactions.

We start with the process of scattering of A and M waves into MS waves. We consider only effects of scattering by ions. Recognizing that the ions are magnetized when $\omega < \omega_{Hi}$, we obtain, by standard methods, the following expressions for the probabilities:

$$w^{M,MS}(\mathbf{k}^M, \mathbf{k}_1^{MS}) = \frac{\pi}{4} \frac{1 - x_1^2}{x_1^2} \frac{\omega\omega_1^3}{\omega_{Hi}^2} \left(\frac{v_s}{v_A}\right)^2 \times \frac{\delta(\omega^M - \omega_1^{MS} - (k_z - k_{1z})v_z)}{n_0^2 (1 + T_e/T_i)^2} \left[\frac{\omega x}{\omega_1 x_1} |x_1| \left(1 - \frac{v_z}{v_A} x\right) + \frac{v_s}{v_A}(1 - x^2)\right]^2,$$

$$w^{A,MS}(\mathbf{k}^A, \mathbf{k}_1^{MS}) = \frac{\pi}{2} \frac{\omega^3 v_s^4}{\omega_{Hi}^2 x^2} \left(1 - \frac{x v_z}{|x| v_A}\right)^2 \tag{20}$$

$$\times \frac{\delta(\omega^A - \omega_1^{MS} - (k_z - k_{1z})v_z)}{\omega_1 n_0^2 (1 + T_e/T_i)^2 v_A^2} \left[\sqrt{1 - x^2}\left(\frac{\omega x}{v_A |x|} - \frac{\omega_1 x_1}{v_s |x_1|}\right) - \sqrt{1 - x_1^2} \cos(\varphi - \varphi_1) \frac{\omega_1}{v_s |x_1|} x\right]^2. \tag{21}$$

The physically infinitesimal $\Delta\omega$ has the same estimated value as for the interaction of helicons with sound, $\Delta\omega \sim \omega\sqrt{T_i/T_e}$. Under these conditions, the interactions can be represented in differential form (we use the notation $C = (\pi^2\omega^4/2\omega_{Hi}^2 n_0 m_i v_A^2)(1 + T_e/T_i)^{-2}$):

$$\frac{\partial W_{\omega x}^M}{\partial t} = C W_{\omega x}^M x^2 \int \frac{1 - x_1^2}{x_1^2} dx_1 \left(\omega \frac{\partial W_{\omega x_1}^{MS}}{\partial \omega} + 2W_{\omega x_1}^{MS}\right), \tag{22}$$

$$\frac{\partial W_{\omega x}^A}{\partial t} = C W_{\omega x}^A \frac{1}{x^2} \int \frac{dx_1}{x_1^2}(2x_1^2 + x^2 - 3x^2 x_1^2)\left(\omega \frac{\partial W_{\omega x_1}^{MS}}{\partial \omega} - 2W_{\omega x_1}^{MS}\right), \tag{23}$$

$$\frac{\partial W_{\omega x}^{MS}}{\partial t} = C W_{\omega x}^{MS} \frac{1 - x^2}{x^2} \int x_1^2 dx_1 \left(\omega \frac{\partial W_{\omega x_1}^M}{\partial \omega} + 2W_{\omega x_1}^M\right), \tag{24}$$

$$\frac{\partial W_{\omega x}^{MS}}{\partial t} = C W_{\omega x}^{MS} \frac{1}{x^2} \int \frac{dx_1}{x_1^2}(2x^2 + x_1^2 - 3x^2 x_1^2)\left(\omega \frac{\partial W_{\omega x_1}^A}{\partial \omega} + 4W_{\omega x_1}^A\right). \tag{25}$$

Formulas (22)—(25) give an idea of what happens to A and M waves in the presence of intense ion-sound turbulence. In particular, if the MS-wave spectrum is given by $W_\omega \sim \omega^{-1} \ln \lambda(\omega)$, where $\lambda(\omega) \gtrsim 1$, then we see that the M waves build up and the A waves attenuate. The critical MS-wave level at which the buildup of the M waves exceeds their Landau damping

$$\gamma_L^M = \frac{m_e}{m_i} \frac{v_{Te}}{v_A} \frac{1 - x^2}{|x|} \omega \exp\left\{-\frac{v_A^2}{2x^2 v_{Te}^2}\right\}, \tag{26}$$

depends on the M-wave propagation angle and is the smallest for M-waves propagating in a direction close to that of the magnetic field

$$\frac{W_{\text{crit}}^{MS}}{n_0 T_e} \sim \sqrt{\frac{m_e}{m_i} \frac{v_A}{v_s}} \left(\frac{\omega_{Hi}}{\omega}\right)^2 \left(\frac{T_e}{T_i}\right)^2 \frac{1 - x^2}{|x|}. \tag{27}$$

We see therefore that if we accept the theoretical arguments of[9], where it is shown that $W^S/n_0 T_e$ is only somewhat larger than $\sqrt{m_e/m_i}$, and assume also that the remaining factors on the right-hand side of (27) are large, we find that if $v_A < v_{Te}$ the M waves can be excited only in a direction close to that of the magnetic field. The conditions for the excitation of the

M waves are greatly improved if $v_A > v_{Te}$, for then the Landau damping is exponentially small.

Furthermore, it is seen from (24) and (25) that intense A and M waves can excite ion-sound MS waves. On the basis of the results of[1], where it is shown that the spectrum of the A and M waves is given by $W_\omega \sim 1/\omega^\nu$ with $\nu = 1$, we see that nonlinear excitation of MS waves by A and M waves can take place. To be realizable, this excitation must exceed the Landau damping of the MS waves. In order of magnitude we have

$$\frac{\tilde{H}_1^{M^2}}{H_0^2} > \sqrt{\frac{m_e}{m_i}}\frac{x^2}{1-x^2}\left(\frac{\omega_{Hi}}{\omega}\right)^2\left(\frac{T_e}{T_i}\right)^2, \tag{28}$$

$$\frac{\tilde{H}_1^{A^2}}{H_0^2} > \sqrt{\frac{m_e}{m_i}}\left(\frac{\omega_{Hi}}{\omega}\right)^2\left(\frac{T_e}{T_i}\right)^2 x^2, \tag{29}$$

$x = \cos\theta$, where θ is the angle between k^{MS} and the magnetic field. Thus, the A and M waves, in the main, excite MS waves perpendicularly to the magnetic field.

In addition, noticeable excitation is possible when the main energy of the magnetic pulsations is concentrated at frequencies that are not too low in comparison with ω_{Hi}. We note that the hose instability yields $\tilde{H}_1^2 \sim H_0^2$ and $\omega \sim \omega_{Hi}$[10]. The presence of MS oscillations under these conditions can lead to interesting effects in the magnetosphere. The excitation of ion sound may be due, according to the foregoing, not to the presence of current in the plasma, but to the presence of magnetic pulsations excited by the hose instability. The electrons become scattered by the MS of oscillations, and this leads to an anomalous resistance to the currents under conditions when the drift velocity is lower than the sound velocity, $u < u_s$, The anomalous conductivity can be estimated in the following manner. The equation for steady-state the electron distribution in the presence of an electric field is

$$e\mathbf{E}\frac{\partial f}{\partial \mathbf{p}} = \frac{\partial}{\partial p_i}D_{ij}\frac{\partial f}{\partial p_j}. \tag{30}$$

The quasilinear term in the right-hand part describes the diffusion of the electrons on the MS oscillations, determined by a diffusion coefficient D that depends on the energy of the MS waves: $D \sim W^{MS}{}_\omega m_e/n_0$. The distribution function can be represented in the form

$$f = f_0 + f_1, \tag{31}$$

where f_0 is the isotropic part and f_1 is a small anisotropic part, which determines in fact the conductivity of the plasma. Under conditions when the buildup of MS waves by magnetic pulsations is offset by the nonlinear decay interactions of the MS waves with one another, with an increment $\gamma \sim \omega^{MS}/n_0T_e$, the energy level of the MS waves is expressed in terms of the energy level of the magnetic pulsations (i.e., in terms of $\tilde{H}_1^2/H_0^2$) in the following manner:

$$W^{MS} \sim n_0T_e\frac{\omega^2}{\omega_{Hi}^2}\left(\frac{T_i}{T_e}\right)^2\frac{\tilde{H}_1^2}{H_0^2}. \tag{32}$$

Expressing f_1 in terms of f_0 with the aid of (30) and recognizing that D is determined by the upper frequencies in the MS-wave spectrum ($\omega \sim \omega_{Hi}$), we obtain the following dependence of the anomalous conductivity on the level of the magnetic pulsations:

$$\sigma \approx \omega_{pe}\frac{c}{v_A}\left(\frac{H_0}{\tilde{H}_1}\right)^2\left(\frac{T_e}{T_i}\right)^2. \tag{33}$$

These estimates are suitable if one can neglect the influence of the MS waves on the A and M waves. The criterion of this condition is determined from (27).

In the opposite case, the M-wave spectrum becomes restructured in such a way that their influence on the MS waves becomes small. Then the spectrum of either the MS or the M waves takes the form $W_\omega \sim 1/\omega^\nu$ with $\nu = 2$.

We now turn to decay interactions in which A, M, and MS waves take part. The decays MS ⇄ M + M, MS ⇄ A + A, MS ⇄ A + M, MS ⇄ MS + M, MS ⇄ MS + A, are rigorously forbidden, as shown by an analysis of the conservation laws, if $v_s \ll v_A$. The processes M ⇄ M + MS, M ⇄ A + MS, A ⇄ A + MS, A ⇄ M + MS, are allowed only for a narrow interval of MS-wave frequencies, $\omega^{MS} < 2\omega_{Hi}v_s/v_A$. In the entire remaining frequency interval $2\omega_{Hi}v_s/v_A < \omega < \omega_{Hi}$, only M ⇄ MS + MS and A ⇄ MS + MS decays can play any role; the probabilities of these decays are given by

$$w^{M,MS,MS}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2) = \frac{(2\pi)^4\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\delta(\omega^M-\omega_1^{MS}-\omega_2^{MS})\omega\omega_1^2\omega_2^2x^2}{2n_0m_i\omega_{Hi}^2v_A^2|x_1x_2|}$$
$$\times|\sin(\varphi-\varphi_1)\sin(\varphi-\varphi_2)|\sqrt{1-x_1^2}\sqrt{1-x_2^2}; \tag{34}$$

$$w^{A,MS,MS}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2) = \frac{(2\pi)^4\delta(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\delta(\omega^A-\omega_1^{MS}-\omega_2^{MS})\omega_1\omega_2}{2n_0T_e\omega\omega_{Hi}^2}$$
$$\times[\omega_1^2\operatorname{tg}\theta_1\cos(\varphi-\varphi_1)-\omega_2^2\operatorname{tg}\theta_2\cos(\varphi-\varphi_2)]^2. \tag{35}$$

From the conservation laws it follows here that these interactions must be taken into account only when the MS-wave spectrum is isotropic, and in the opposite case the phase volume of the MS waves that take part in the decay is small. These interactions can be used for the diagnostics of MS turbulence. Neglecting the absorption of A and M waves as a result of the inverse processes, we obtain the connection between the energies of the MS and A and M waves:

$$\frac{\partial W_{\omega\Omega}^A}{\partial t} = \frac{\omega^3}{(2\pi)^3|x|^3v_A^3}\int w^{A,MS,MS}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)\frac{W^{MS}_{\omega_1\Omega_1}W^{MS}_{\omega_2\Omega_2}}{\omega_1\omega_2}d\omega_1\,d\omega_2\,d\Omega_1\,d\Omega_2, \tag{36}$$

$$\frac{\partial W_{\omega\Omega}^M}{\partial t} = \frac{\omega^3}{(2\pi)^3v_A^3}\int w^{M,MS,MS}(\mathbf{k},\mathbf{k}_1,\mathbf{k}_2)\frac{W^{MS}_{\omega_1\Omega_1}W^{MS}_{\omega_2\Omega_2}}{\omega_1\omega_2}d\omega_1\,d\omega_2\,d\Omega_1\,d\Omega_2. \tag{37}$$

The foregoing formulas can be used to estimate various effects in the interaction of low-frequency waves in a magnetoactive plasma and to estimate qualitatively the turbulence spectra. If it is necessary to obtain a more detailed picture of not only the frequency distribution but also the angular distribution of the turbulent pulsations, it is advantageous to solve the derived equations for the nonlinear interactions numerically; this will also cast light on manner whereby the stationary spectrum is established.

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