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STRUCTURE OF THREE-COMPONENT VECTOR FIELDS IN SELF-FOCUSING WAVEGUIDES

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It is shown that when the real vector nature of the electromagnetic field is taken into account the equations of nonlinear electrodynamics lead to new self-focusing waveguide solutions. The possibility appears of existence of self-focusing waveguides in which the transverse and longitudinal fields are comparable in magnitude. The set of self-focusing waveguides discovered, which is characterized by a unique polarization structure of the electric field, includes the previously studied TE and TM modes as particular cases. A qualitative analysis carried out for the equations of nonlinear electrodynamics has permitted the states of vector self-focusing waveguides to be classified in the case of plane geometry. It is shown that under certain conditions in a nonlinear medium a peculiar phenomenon arises—spatial stratification of the electromagnetic field into almost self-focusing regions of TE and TM fields. It is characteristic that the change in space of the almost self-focusing field mode is due to nonlinear interaction in the weak-field region.

1. The unflagging interest in problems of electromagnetic wave propagation in nonlinear media, in particular in the problem of excitation of self-focusing waveguides, leads to the necessity of investigating the structure of the electromagnetic field in such waveguides. For this purpose it is necessary to take into account the real vector (three-component) nature of the electric field of self-focusing waveguides, and also the fact that the nonlinear dielectric permittivity of the medium leads to coupling of the transverse and longitudinal degrees of freedom of the electromagnetic field.

Up to the present time only the simplest one-component[1-4] and two-component[5,6] self-focusing waveguides have been studied. However, it has been completely unclear whether or not self-focusing waveguides exist in the general (three-component) case if the transverse and longitudinal degrees of freedom of the electromagnetic field are taken into account. As the analysis carried out below of the solutions of the equations of nonlinear electrodynamics has shown, a previously unstudied sequence of three-component self-focusing waveguides exists, which is characterized by a unique polarization structure and which includes as special cases the previously known types of self-focusing waveguides[1-6]. It should be noted that the transverse and longitudinal fields are comparable in magnitude for the three-component self-focusing waveguides which have been found. The region of localization of the electric field of the solutions found is several times larger than the region of localization of the fields for the one- and two-component solutions. A qualitative analysis of the equations of nonlinear electrodynamics has permitted classification of the states of three-component self-focusing waveguides in the case of plane geometry. An unusual phenomenon has appeared—the stratification of the electromagnetic field in space into almost self-focusing regions of TE- and TM-mode fields which arise under certain conditions in a nonlinear medium. It is characteristic that the change in space of the almost self-focusing type of field is due to nonlinear interaction in the weak-field region. We will point out that, if the longitudinal fields are neglected, the problem of self-focusing electromagnetic waves with two circular polarizations has been discussed by Berkhoer and Zakharov[7]. However, the equations of nonlinear electrodynamics considered by us, in contrast to those used by Berkhoer and Zakharov, correspond to inclusion of both transverse and longitudinal fields.

In the case of cylindrical geometry the three-component self-focusing solution found also is characterized by transverse and longitudinal fields comparable in magnitude.

2. The equations of nonlinear electrodynamics (see ref. 6) permit solutions of the form

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}(x)\exp(ik_z z) \tag{2.1}$$

and lead to the equations

$$k_z E_x' = \frac{[k_z^2\varepsilon + 2(k^2\varepsilon - k_z^2)E_x^2\, d\varepsilon/dE^2]E_z - 2k_z E_x E_y p\, d\varepsilon/dE^2}{\varepsilon + 2E_x^2\, d\varepsilon/dE^2}; \tag{2.2}$$

$$E_y' = p, \quad k_z E_z' = (k_z^2 - k^2\varepsilon)E_x, \quad p' = (k_z^2 - k^2\varepsilon)E_y.$$

Here

$$E^2 = E_x^2 + E_y^2 + E_z^2, \quad k = \omega / c,$$

and $\epsilon(E^2) > 0$ is the real dielectric permittivity of the medium.

Equations (2.2) permit separation of two exact types of solutions with one-component $(0, E_y, 0)$ and two-component $(E_x, 0, E_z)$ electric vectors, and also the more general case of three-component solutions. A feature of the three-component solutions is that the coupling between the projections E_y and (E_x, E_z) of the electric vector is accomplished only through the nonlinearity—the dielectric permittivity.

The system of Eqs. (2.2) shows that the phase space is four-dimensional (E_x, E_y, E_z, p). The equations of nonlinear electrodynamics (2.2) lead to the following conservation law—the first integral

$$\mathcal{H} = p^2 - k_z^2 E_y^2 + \frac{(k_z^2 - k^2\varepsilon)^2 - k_z^4}{k_z^2}E_x^2 + k^2\int_0^{E^2} dq\,\varepsilon(q), \tag{2.3}$$

which permits p to be expressed as a function of the electric vector **E** and the parameter $\mathcal{H}$. An obvious consequence of (2.3) is the inequality

$$p^2 = \mathscr{H} + k_z^2E_y^2 - \frac{(k_z^2-k^2\varepsilon)^2-k_z^4}{k_z^2}E_x^2 - k^2\int_0^{E^2} dq\,\varepsilon(q) \geqslant 0, \quad (2.4)$$

which determines in the space of the electric vector **E** the region in which all integral curves are located—solutions of the equations of nonlinear electrodynamics (2.2) with a given value of the parameter $\mathscr{H}$.

Two "separatrix" planes pass through the zero point of the electric-vector space:

$$k_zE_z = \pm(k_z^2 - k^2\varepsilon(0))^{1/2}E_x. \quad (2.5)$$

For $\mathscr{H} = 0$, which is necessary for self-channeling of the electromagnetic field, the integral curves enter and leave the zero point of the electric-vector space along straight lines determined by the intersection of the plane

$$E_y = CE_z \quad (2.6)$$

with the separatrix planes (2.5). The limiting or boundary surface $p(E_xE_yE_z;\ 0) = 0$, inside which are located all self-focusing solutions for the case of a positive dielectric permittivity $\epsilon(E^2)$ increasing with increasing E^2, is similar in its topological structure to the surface of a degenerate torus (Fig. 1). The section of the boundary surface by the plane $E_y = 0$ is a separatrix loop corresponding to the self-focusing two-component solution (a plane waveguide layer of the TM type). The one-component self-focusing solution—a plane waveguide layer of the TE type—corresponds in the space of the electric vector **E** to motion along the portion of the axis E_y from the point of zero field to the intersection of the E_y axis with the boundary surface, and back (see Fig. 1).

We note that the surface p = const in the electric-vector space is a toroidal surface enclosing the circle

$$k^2\varepsilon(E_x^2+E_y^2) = k_z^2, \quad E_z = 0$$

and collapsing to this circle as the constant (const) increases.

A qualitative analysis shows that an integral curve leaving the zero point of electric-vector space along one of the separatrix planes (2.5) in a direction characterized by the parameter C subsequently goes out unavoidably to the boundary surface p = 0. After touching the boundary surface, the integral curve goes into the interior of the allowed region and subsequently comes out again to the boundary surface. Thus, the motion of a representative point in the space of the electric vector **E** is a succession of motions connecting the motions of the representative point out to the boundary surface and characterized by a constant sign of p for motion inside the allowed region. We should expect (and this is confirmed by the results of numerical calculations) that for definite values of the parameter C an integral curve leaving the region of zero field, after some finite number of contacts with the boundary surface, will again return to the region of zero field. This situation corresponds to a self-focusing three-component distribution of the electromagnetic field in a nonlinear medium.

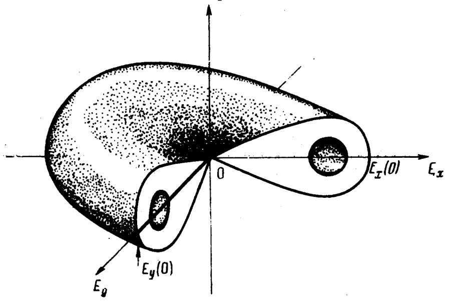

FIG. 1. Boundary surface p = 0. Motion along the line 0–E_y(0)–0 corresponds to a self-focusing waveguide of the TE type; motion along the loop 0–E_x(0)–0 formed by intersection of the boundary surface by the plane E_y = 0 corresponds to a waveguide of the TM type.

3. Numerical integration of the equations of nonlinear electrodynamics (2.3) was carried out for a dielectric permittivity of the form

$$\varepsilon = \varepsilon_0 + \varepsilon_2E^2/E_N^2, \quad (3.1)$$

where E_N is the characteristic field of the nonlinearity, and for the boundary conditions

$$\lim_{E\to 0}\frac{E_z}{E_x} = \pm\frac{(k_z^2-k^2\varepsilon(0))^{1/2}}{k_z}, \quad \lim_{E\to 0}\frac{E_y}{E_z} = C. \quad (3.2)$$

The numerical calculations were carried out by leading the integral curve from the zero point of electric-vector space along a straight line located in the separatrix plane and characterized by the parameter C, i.e., taking account of the asymptotic behavior of the solution in the vicinity of zero field.

As a result of the qualitative analysis of the solutions and the numerical calculations, the existence has been established of a sequence of spatially localized states of three-component fields. For example, for $k_z^2 = 2k^2\epsilon_0$ one of the eigenvalues of the parameter lies between the limits

$$1.13 < C < 1.14. \quad (3.3)$$

The distribution of the electric field in space, and also the projection of the motion of the electric vector in the planes (E_z, E_x) and (E_y, E_x) are shown in Fig. 2. The region of localization of the three-component solution found is larger than the region of localization of the one- and two-component self-focusing solutions by several times. However, the greatest values of the projections E_y and (E_x, E_z) of the electric vector for the three-

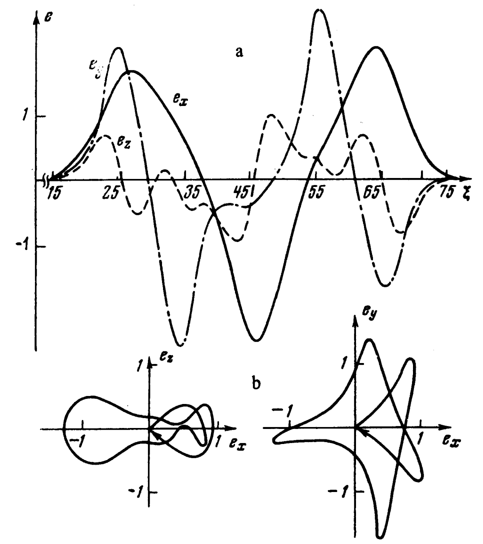

FIG. 2. a–Distribution of self-focusing electric field in space for $C \approx 1.1$; $\xi = kx\sqrt{\epsilon_0}$, $e = (\epsilon_2/\epsilon_0)^{1/2}E/E_N$. b–Electric-vector projections on the planes (E_z, E_x) and (E_y, E_x), which characterize the polarization structure of the self-focusing field.

component self-focusing states do not exceed the greatest values of the corresponding projections for one- and two-component self-focusing states. This is due to the fact that the characteristic dimensions of the boundary surface (see Fig. 1) are determined essentially by the parameters of the one- and two-component self-focusing states. We note that the longitudinal and transverse fields for three-component self-focusing waveguides are comparable in magnitude, which results in a unique polarization structure of the electric field in such waveguides.

We note that the three-component self-focusing states can be arranged in order of the number of contacts of the electric vector, describing a closed curve in (E_x, E_y, E_z) space, with boundary surface p = 0. In particular, the solution with the eigenvalue (3.3) corresponds to six contacts with the boundary surface (between the departure and return of the integral curve at the zero-field region). We recall that the one-component self-focusing solution corresponds to one contact, and the two-component solution to motion along a curve lying on the boundary surface.

Numerical integration of Eqs. (2.2) for initial values of the electric vector lying on the boundary surface and close to the vector (0, $E_y(0)$, 0) or ($E_x(0)$, 0, 0), where $E_y(0)$ and $E_x(0)$ are the values of the electric-field projections on the symmetry plane of the one- and two-component self-focusing states, leads to the solution shown in Fig. 3. This solution corresponds to a layering of the electromagnetic field in a nonlinear medium into "almost completely" self-focusing states of one- and two-component fields (TE and TM modes). The uniqueness of the phenomenon lies in the fact that the change in space of the one- and two-component almost self-focusing states is due to the nonlinear action in the weak-field region. As the electric vector approaches its value on the symmetry plane of the one- and two-component self-focusing waveguides, the electric field in the region where the nonlinear interaction is important becomes steadily weaker, but the space interval separating the almost self-focusing fields of the TE and TM modes increases.

4. In the case of cylindrical geometry the equations of nonlinear electrodynamics permit solutions of the form

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}(\rho)\exp(ik_z z + im\varphi) \tag{4.1}$$

and lead to the system of equations

$$\begin{gathered}
-\frac{d^2E_z}{d\rho^2} - \frac{1}{\rho}\frac{dE_z}{d\rho} - \frac{k_z}{\rho}\frac{d}{d\rho}(\rho E_z) - \frac{mk_z}{\rho}E_\varphi = \left(k^2\varepsilon - \frac{m^2}{\rho^2}\right)E_z,\\
k_z\frac{dE_z}{d\rho} + \frac{m}{\rho^2}\frac{d}{d\rho}(\rho E_\varphi) = \left(k^2\varepsilon - k_z^2 - \frac{m^2}{\rho^2}\right)E_\rho,\\
-\frac{d^2E_\varphi}{d\rho^2} - \frac{1}{\rho}\frac{dE_\varphi}{d\rho} - \frac{mk_z}{\rho}E_z - m\frac{d}{d\rho}\left(\frac{E_\rho}{\rho}\right) = \left(k^2\varepsilon - k_z^2 - \frac{1}{\rho^2}\right)E_\varphi.
\end{gathered} \tag{4.2}$$

For m = 0 it is possible to separate not only the two types of exact solutions with one- and two-component vectors—(0, E_φ, 0) and (E_ρ, 0, E), but also the more general three-component solution. The feature of this solution is that for m = 0 the coupling between the electric-vector projections E_φ and (E_ρ, E_z) occurs only through the nonlinearity—the dielectric permittivity.

For m = 0 the boundary conditions have the form

$$\lim_{\rho\to 0} E_z = E_z(0), \quad \lim_{\rho\to 0}\frac{dE_\varphi}{d\rho} = kH_z(0). \tag{4.3}$$

Here $E_z(0)$ and $H_z(0)$ are the projections of the electric and magnetic fields on the axis of the self-focusing cylindrical waveguide. For this case $E_\varphi(0) = E_\rho(0) = 0$. Numerical integration carried out for the dielectric permittivity (3.1) for $k_z^2 = 2k^2\epsilon_0$ showed that the self-focusing three-component state of the electromagnetic field is realized, for example, for

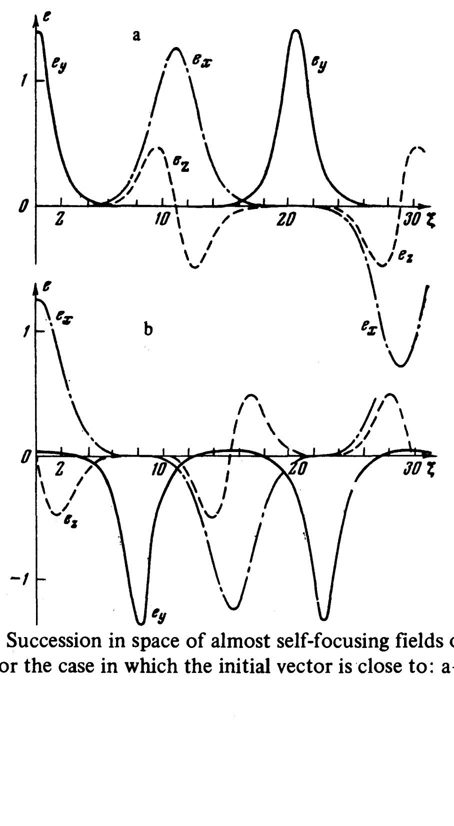

FIG. 3. Succession in space of almost self-focusing fields of TE and TM types for the case in which the initial vector is close to: a–$E_y(0)$, b–$E_x(0)$.

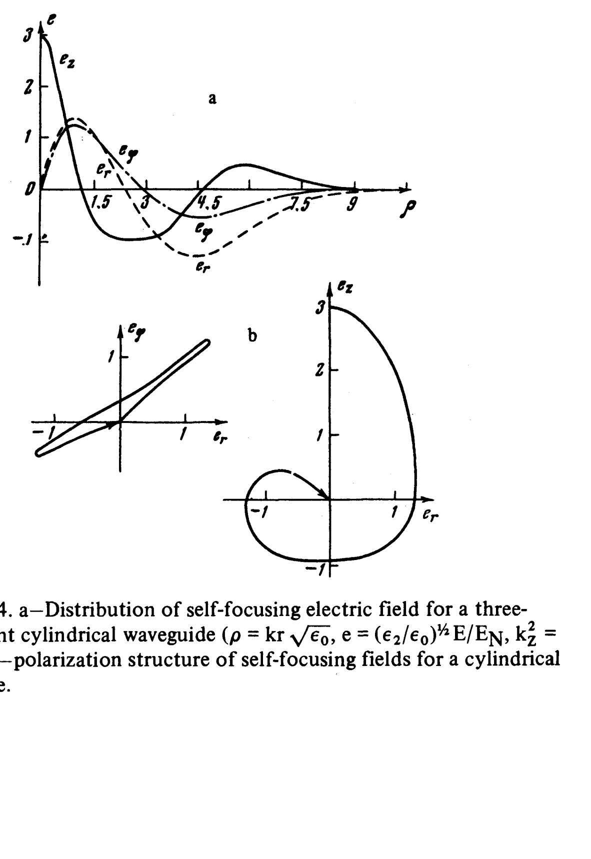

FIG. 4. a–Distribution of self-focusing electric field for a three-component cylindrical waveguide ($\rho = kr\sqrt{\epsilon_0}$, $e = (\epsilon_2/\epsilon_0)^{1/2}E/E_N$, $k_z^2 = 2k^2\epsilon_0$); b–polarization structure of self-focusing fields for a cylindrical waveguide.

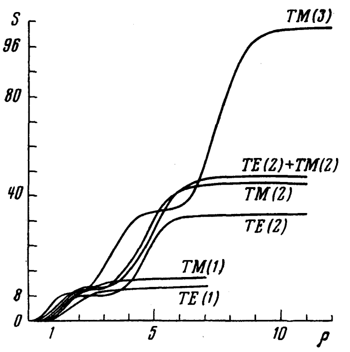

FIG. 5. Distribution of energy flux in the cross section of self-focusing cylindrical waveguides: TE(1); TE(2)—ground and first excited states of one-component waveguide; TM(1), TM(2), TM(3)—ground and first two excited states of two-component waveguide; TE(2) + TM(2)—three-component waveguide state found.

$$E_z(0)\approx 3\left(\frac{\varepsilon_0}{\varepsilon_2}\right)^{1/2}E_N,\qquad H_z(0)\approx 2.49\frac{\varepsilon_0}{\varepsilon_2^{1/2}}E_N. \tag{4.4}$$

The distribution of fields, and also the electric-vector projections on the planes (E_φ, E_ρ) and (E_z, E_ρ), which characterize the polarization structure of the electromagnetic field, are shown in Fig. 4. The behavior of the electric-field projections E_φ and (E_ρ, E_z) in the self-focusing waveguide mode found is similar in its characteristic features to the behavior of the corresponding projections of the first higher states of one- and two-component self-focusing cylindrical waveguides[6]. In particular, both the field-localization region and the energy flux are of the same order as for the self-focusing waveguides shown above (Fig. 5). We point out that in the transition to highly excited states of self-focusing waveguides a rapid rise in energy flux occurs. For example, for the second excited state of a cylindrical waveguide of the TM type the energy flux rises by an order of magnitude.

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