THEORY OF MULTI-PULSED EXCITATION OF SIGNALS OF THE LIGHT-ECHO TYPE

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Some principally novel effects arising on multi-pulsed excitation of signals of the light-echo type are considered theoretically. An exact expression for the power of the echo response from a particle system with a spin of arbitrary magnitude is found for an arbitrary number of exciting pulses of arbitrary physical nature. It is assumed that the energy spectrum of a single particle is nondegenerate and equidistant and that the operator for momentum interaction with the particles is linear with respect to the effective spin. The operator of the observable quantity, however, may be an arbitrary analytic function of the effective spin components. The following features of quantum systems excited by multi-pulses are shown to exist: the possibility of obtaining effective wave vectors or arbitrary magnitude, increase of time resolution of the crystal for investigation of a dense sequence of pulses, increase of information on the dynamics of the quantum systems and on external generators in the responses of the light-echo type and increase of effective duration of phase memory. As examples the excitation of sound by electro-magnetic pulses and the excitation of an electro-magnetic field by sound pulses are considered. It is shown that even a small increase of the number of exciting pulses removes in certain cases insurmountable difficulties of the two-pulse excitation scheme. Some possible technical applications of multi-pulsed excitation of an echo-signal set are discussed.

INTRODUCTION

LIGHT echo (LE), theoretically predicted and experimentally observed in[1] and [2], respectively, is a sharply-directional coherent light beam containing valuable information on the dynamics of optical quantum systems and on the external generators that excite the medium. Even the first theoretical and experimental investigation of light echo in gases [3] and crystals [2,4] have confirmed the point of view [1] that pulsed coherent optics makes it possible to investigate and employ for practical purposes dynamic processes that occur in a medium at nanosecond time intervals. In cases when the wavelengths λ of the exciting external generators or of the field radiated by the medium is much smaller than the dimensions of the quantum system, the number of possible different coherent states of the medium is very large. When a quantum system is excited by a series of n successive light pulses, each of these coherent states $\Psi(\mathbf{k}_1, \ldots, \mathbf{k}_n; \Delta t_1, \ldots, \Delta t_n; \tau_1, \ldots, \tau_{n-1})$ is determined by the wave vectors $\mathbf{k}_1, \ldots, \mathbf{k}_n$ of the exciting pulses, by the durations $\Delta t_1, \ldots, \Delta t_n$ of these pulses, and by the time intervals $\tau_1, \ldots, \tau_{n-1}$ between these pulses. Therefore the quantum system stores in "its memory" all these parameters of the excitation process and combines them in a definite manner. Moreover, at definite time intervals the system again issues this information in the form of light pulses of duration $T_r \sim T_{1r}(\lambda^2 N/S)^{-1}$, where T_{1r} is the spontaneous emission time of a single particle, N is the number of excited particles, and S is the area of the end face of the sample. A similar situation arises in the case of excitation of quantum systems by hypersound and terasound ($\nu \sim 10^{12}$ sec^{-1})[5], and also when optical and acoustic exciting pulses are combined[6]. Some of these processes can apparently be used in optical computers[7].

In the present paper we develop a general theory of multipulse optical and acoustic excitation of quantum systems with nondegenerate equidistant discrete spectrum in the case when the operators of the interaction of the exciting pulses with the particles are linear in the effective spin. The obtained formulas determine the instants of time, the directions, the wave vectors, and the intensities of the emitted responses after an n-pulse optical and acoustical excitation of the system. From the obtained general formulas it is also seen that a change of both the sequence (in time) and the magnitude of all the parameters of the exciting pulses leads to the formation of macroscopically distinguishable states $\Psi(\mathbf{k}_1, \ldots, \mathbf{k}_n; (\Delta t_1, \ldots, \Delta t_n; \tau_1, \ldots, \tau_{n-1})$ of the quantum system. Each of these states can yield up to $(3^n + 2n - 1)/4$ responses that differ in the emission direction, the times of appearance, and the radiation intensity. Thus, optical quantum systems do indeed possess a dynamic phase memory with large volume and are promising from the point of view of their use as memory elements in optical computers.

Particular interest attaches to two sequences of optical and acoustic exciting pulses, which are generalized variants of the sequences of Carr and Purcell (CP)[8] and of Waugh and Wang (WW)[9] and are well known in nuclear spin echo. As particular cases of the general theory we discuss here in greater detail effects that arise in the case when these sequences are used in optics and acoustics.

In a CP sequence, the first 90° pulse is followed by (n − 1) 180° pulses, and the time intervals between the pulses as well as the wave vectors of the pulses are arbitrary. The main advantages of a CP sequence is the possibility of obtaining the following effective wave vector

$$k_{eff} = (-1)^n\left[-k_1 + \sum_{m=2}^{n} (-1)^m 2k_m\right].$$

In particular, if the wave vectors of all the odd pulses in the sequence are equal to the wave vector of the first pulse, and the wave vectors of all the even pulses are equal to the wave vector of the first pulse taken with a minus sign, i.e., $k_m = (-1)^{m+1}k_1$, $m = 1,2,3,\ldots,n$, then we obtain $k_{eff} = (-1)^{n+1}(2n - 1)k_1$. By the same token, we can use a coherent field with a wave vector k_1 to generate another coherent field with wave vector $(2n - 1)k_1$ of the same frequency. By way of an example we can consider the generation of terasound pulses with the aid of optical pulses of the infrared band. Since the ratio of the electromagnetic and acoustic wave velocities is approximately 10^5, a series of $n \sim 10^5$ optical pulses is needed for this purpose.

The WW sequence consists of n 90° pulses. The time interval between the first and second pulses is equal to τ, and each of the remaining time intervals between pulses is equal to 2τ. The wave vectors of the first two pulses are arbitrary, and the wave vectors of all the remaining pulses are equal to the wave vector of the second pulse.

The main reason why the WW sequence is of interest is that when $n \gg 2$, $T_1 > T_2$ and $\tau < T_2$, where T_2 is the time of irreversible relaxation of the mean values of the transverse components of the effective spin at $n = 2$, and T_1 is the relaxation time of the average value of the longitudinal component of the effective spin, which is the same for all n; the effective time T_{2eff} of the irreversible transverse relaxation becomes longer and tends to T_1 with decreasing τ [9].

Simple considerations show that the process of lengthening of the phase memory of the transverse spin components in the case of multipulse excitation of spin systems, observed in[9], should take place also for the transverse components of the electric dipole moment in the case of optical excitation. Indeed, the theory of irreversible damping of signals such as light echo differs from the theory of irreversible damping of spin echo, in principle, only in that the different lattice sums that describe the two-particle interactions contain factors of the type $\exp(ik_m \cdot r_j)$, where r_j are the radius vectors of the ions. However, as a rule, the lattice sums converge at distances on the order of $(10^{-7}-10^{-6})$ cm, where $\exp(ik_m \cdot r_j) \sim 1$. Thus, there is no essential difference between the laws of irreversible relaxation in experiments aimed at observing spin and light echo, and our statement is justified. Similar considerations hold true also for acoustic excitation at frequency $\nu \leq 10^{11}$ sec^{-1}.

Finally, we present a scheme for the excitation of a coherent electromagnetic signal with the aid of three-acoustic pulses. This example shows that a phenomenon that does not occur in two-pulse excitation can be observed already when the number of exciting pulses is increased by one.

1. RESPONSES OF A SPIN SYSTEM TO n-PULSE EXCITATION

We consider a certain physical system made up of N identical particles having an arbitrary effective spin R, with a static Hamiltonian in the form

$$\mathscr{H}_R = \mathscr{H}_0 + \mathscr{H}', \mathscr{H}_0 = \sum_j \hbar\omega_0 R_{j3},$$
$$\mathscr{H}' = \sum_j \hbar\Delta\omega_j R_{j3};\ \omega_0 > 0;\ j = 1, 2, \ldots, N. \tag{1}$$

Here ω_0 is the resonant angular frequency of the unperturbed particles, $\omega_0 + \Delta\omega_j$ is the resonant angular frequency of the perturbed j-th particle, and R_{j1}, R_{j2}, and R_{j3} are the operators of the Cartesian components of the effective spin of the j-th particle. The commutation relations of these operators are of the form

$$[R_{j\sigma}, R_{j'\sigma'}]_- = i\delta_{jj'}e_{\sigma\sigma'\sigma''}R_{j\sigma''}, \tag{2}$$

where $\sigma_{jj'}$ is the Kronecker symbol, $e_{\sigma\sigma'\sigma''}$ is an antisymmetrical unit tensor of third rank, each of the indices σ, σ', and σ'' takes on the values 1, 2, and 3, and summation with respect to the index σ'' is implied. We shall henceforth use the notation

$$R_{j(\pm1)} = R_{j1} \pm iR_{j2};\quad R_{j(0)} = R_{j3}. \tag{3}$$

Assume that, starting with the instant of time $t = 0$, n periodic coherent resonant pulses from several generators of different physical nature (optical, acoustic, etc.) act on the system at certain time intervals. The Hamiltonian of the interaction of the field of the m-th pulse with the system is of the form (at $R = 1/2$ and in particular, $n = 2$ see[2] and[6] for light and sound, respectively)

$$\mathscr{H}_m(t) = -{}^1/_4\hbar[U_-(t - t_{0m}) - U_+(t - t_m)]\cdot$$
$$\times \sum_j [a_{j(m)}\exp(-i\omega_0 t)R_{j(1)} + a^*_{j(m)}\exp(i\omega_0 t)R_{j(-1)}];$$
$$m = 1, 2, \ldots, n. \tag{4}$$

Here $a_{j(m)}$ are complex numerical quantities that do not depend on the time t and take into account the amplitude of the field of the pulse and are constants characterizing the interaction of the field with the particles, t_{0m} and t_m are the instants of time of the start and end of the action of the pulse on the system, respectively, with $t_{01} = 0$, while $U_-(x)$ and $U_+(x)$ are asymmetrical unit step functions[10] of the real variable x

$$U_-(x) = \begin{cases} 0 \text{ for } x < 0, \\ 1 \text{ for } x \geqslant 0; \end{cases}\qquad U_+(x) = \begin{cases} 0 \text{ for } x \leqslant 0, \\ 1 \text{ for } x > 0. \end{cases} \tag{5}$$

The simplest model Hamiltonian corresponding to the process of n-pulse excitation of a system of particles can be written in the form

$$\mathscr{H}(t) = \mathscr{H}_R + \sum_{m=1}^{n} \{\mathscr{H}_m(t) - [U_-(t - t_{0m}) - U_+(t - t_m)]\mathscr{H}'\}. \tag{6}$$

In many concrete problems[2,6], the scatter of the resonant frequencies of the particles during the time that the pulses act on the system does not play an important role, and therefore the term with $\Delta\omega_j$ is missing from $\mathscr{H}(t)$ at the corresponding instance of time.

Assume that at $t = 0$ the system (1) is in a state described by the density matrix

$$\rho = \prod_j \rho_j,\quad \rho_j = \exp[\zeta R_{j(0)}][\mathrm{Sp}\exp\zeta R_{j(0)}]^{-1}, \tag{7}$$

where $\zeta = -\hbar\omega_0/k_B T$, k_B is Boltzmann's constant, and T is the temperature. At an arbitrary instant of time $t \geq 0$, the mean value $\langle f \rangle$ of the observed physical quantity characterizing the system (1) and described in the Schrödinger representation by the analytic operator function

$$f \equiv f[R_{1(1)}, R_{2(1)}, \ldots, R_{N(1)}; R_{1(0)}, R_{2(0)}, \ldots, R_{N(0)}; R_{1(-1)}, R_{2(-1)}, \ldots, R_{N(-1)}],$$

can be calculated from the formula

$$\begin{aligned}\langle f \rangle &= \operatorname{Sp} \mathscr{L}(t)\rho[\mathscr{L}(t)]^{-1}f = \operatorname{Sp}\rho[\mathscr{L}(t)]^{-1}f\mathscr{L}(t)\\ &= \operatorname{Sp}\rho f[R_{1(1)}(t), R_{2(1)}(t), \ldots, R_{N(1)}(t); R_{1(0)}(t), R_{2(0)}(t), \ldots\\ &\ldots, R_{1(-1)}(t), R_{2(-1)}(t), \ldots, R_{N(-1)}(t)],\end{aligned} \tag{8}$$

where $R_{j(\gamma)}(t)$ ($j = 1, 2, \ldots, N$; $\gamma = 0, \pm 1$ is an operator describing the corresponding combination of the Cartesian components of the effective spin of the j-th particle, written in the Heisenberg representation

$$R_{j(\gamma)}(t) = [\mathscr{L}(t)]^{-1}R_{j(\gamma)}\mathscr{L}(t),$$
$$\mathscr{L}(t) = P\left\{\exp\left[-i\hbar^{-1}\int_0^t \mathscr{H}(t')dt'\right]\right\}. \tag{9}$$

The operator $\mathscr{L}(t)$ is the matrix evolution function[11] written in the form of the universally accepted symbolic exponential with the aid of the Dyson chronological operator P,[12] and $\mathscr{H}(t')$ is the Hamiltonian indicated in (6). Using the commutation relations (2), we can show that at $t \geq t_n$, where t_n is the instant of time when the action of the last (n-th) pulse on the system (1) terminates, the following equation holds true:

$$R_{j(\gamma)}(t) = R_{j(\gamma)}(t, n), \tag{10}$$

where the operator $R_{j(\gamma)}(t, n)$ is a linear combination of the operators (3) in the form

$$R_{j(\gamma_n)}(t, n) = \exp(i\gamma_n\omega_0 t) \sum_{\gamma_0, \gamma_1, \ldots, \gamma_{n-1} = 0, \pm 1} i^{\gamma_0 - \gamma_n}$$
$$\times \prod_{m=1}^{n} \exp[-i\gamma_m \varkappa_{j(m)}]\, \alpha_{j(m)}^{\gamma_{m-1}-\gamma_m} B^{(m)}_{\gamma_{m-1}, \gamma_m} R_{j(\gamma_0)}; \quad \gamma_n = 0, \pm 1. \tag{11}$$

The quantities B are defined here as follows:

$$\begin{gathered}2B^{(m)}_{1,1} = \cos\theta_{j(m)} + 1, \quad 2B^{(m)}_{1,-1} = \cos\theta_{j(m)} - 1, \quad B^{(m)}_{0,0} = \cos\theta_{j(m)},\\ 2B^{(m)}_{1,0} = -B^{(m)}_{0,1} = \sin\theta_{j(m)}; \qquad B^{(m)}_{-\gamma_{m-1}, -\gamma_m} = B^{(m)}_{\gamma_{m-1}, \gamma_m};\\ \gamma_{m-1}, \gamma_m = 0, \pm 1; \qquad m = 1, 2, \ldots, n.\end{gathered} \tag{12}$$

In (11) and (12) we have used the following notation [cf. (4)]:

$$\begin{gathered}\varkappa_{j(m)} = -\tau_m \Delta\omega_j, \quad \tau_m = t_{0m+1} - t_m \text{ for } m < n; \quad \varkappa_{j(n)} = -(t - t_n)\Delta\omega_j;\\ \theta_{j(m)} = {}^1\!/_2 |a_{j(m)}| \Delta t_m; \quad \alpha_{j(m)} |a_{j(m)}| = a_{j(m)}; \quad \Delta t_m = t_m - t_{0m};\\ m = 1, 2, \ldots, n.\end{gathered} \tag{13}$$

After investigating (9) with the aid of the properties of the step functions[10] and the properties of the evolution operator[11] for the instants of time t satisfying the condition $t_n \geq t \geq 0$, we can show that relation (10) for the instants of time t satisfying the condition $t_\nu \leq t \leq t_{0\nu+1}$ ($\nu = 1, 2, \ldots, n - 1$), is valid if we replace n by ν in (10)—(13). Consequently, this method can be used to analyze accurately and rigorously the responses of the system (1) in the process of n-pulse excitation (6), which appear at instants between the times of the exciting pulses, as will be done in the exposition that follows.

2. POWER OF ECHO SIGNALS

We consider the case when the pulses (4) acting on the system (1) are traveling plane waves. In this case the quantities $\theta_{j(m)}$(13) are the same for all the particles[6], $\theta_{j(m)} = \theta_m$, and the quantities $\alpha_{j(m)}$(13) take the form [6]

$$\alpha_{j(m)} = \beta_m \exp(i\mathbf{k}_m \mathbf{r}_j), \tag{14}$$

where $\mathbf{k}_m$ is the wave vector of the field of the m-th pulse in the sample, $\mathbf{r}_j$ is the radius vector of the mass center of the j-th particles, and β_m is a complex constant that takes into account the initial phase of the m-th pulse. We consider radiation of the system (1), due to spontaneous transition only between neighboring levels of the particle. The power of this radiation per unit solid angle in the direction of the wave vector **k** at the instant of time $t \geq t_n$ can be obtained, taking (7), (8), and (11) into account, from the following formulas (at $R = 1/2$ and $n = 2$, see[2,6]):

$$\begin{gathered}I(\mathbf{k}, t) = I_0(\mathbf{k}) \operatorname{Sp}\rho[\mathscr{L}(t)]^{-1}R_{k+}R_{k-}\mathscr{L}(t) = I_1(\mathbf{k}) + I_2(\mathbf{k}, t);\\ I_1(\mathbf{k}) = I_0(\mathbf{k}) \sum \operatorname{Sp}\rho_j R_{j(1)}(t, n) R_{j(-1)}(t, n);\\ I_2(\mathbf{k}, t) = I_0(\mathbf{k}) \sum_j \sum_{l \neq j} \exp[i\mathbf{k}(\mathbf{r}_j - \mathbf{r}_l)] \operatorname{Sp}\rho_j R_{j(1)}(t, n) \operatorname{Sp}\rho_l R_{l(-1)}(t, n);\\ R_{k\pm} = \sum \exp(\pm i\mathbf{k}\mathbf{r}_j) R_{j(\pm)}; \qquad j, l = 1, 2, \ldots, N.\end{gathered} \tag{15}$$

Here $I_0(\mathbf{k})$ is the radiation power of an isolated particle per unit solid angle in the direction of **k**, $I_1(\mathbf{k})$ is the incoherent part of the power, i.e., it is proportional to the number of particles n in the system (1) and does not depend on the time t, and $I_2(\mathbf{k}, t)$ is the coherent part of the power, i.e., it is proportional to $N^2 - N$ and depends on the time t.

With the aid of (11) we can easily analyze the relations (15) even if they are not written out in detail. It is obvious that in the general case, in order for the coherent part of the radiation part to reach macroscopic values, it is necessary that some terms in the formula for $I_2(\mathbf{k}, t)$, in the expression following the two signs of summation over the particles, be independent of the indices j and l. This takes place if the following two conditions are simultaneously satisfied:

$$\begin{gathered}t = t_n + \gamma_1'\tau_1 + \gamma_2'\tau_2 + \ldots + \gamma'_{n-1}\tau_{n-1};\\ \mathbf{k} = \mathbf{k}_n + \gamma_1'(\mathbf{k}_2 - \mathbf{k}_1) + \gamma_2'(\mathbf{k}_3 - \mathbf{k}_2) + \ldots + \gamma'_{n-1}(\mathbf{k}_n - \mathbf{k}_{n-1});\\ \gamma_1', \gamma_2', \ldots, \gamma'_{n-1} = 0, \pm 1;\end{gathered} \tag{16}$$

where we should have $t = t_1$ and $\mathbf{k} = \mathbf{k}_1$ at $n = 1$. It must be borne in mind that if we choose in (16) a set of values for $\gamma_1', \gamma_2', \ldots, \gamma_{n-1}'$ in the relation for t (for k), then we must substitute only this set of values in the relation for k (for t).

It follows from all the foregoing that the system (1) produces as a result of n-pulse excitation superradiant coherent signals of the echo and free-induction type, characterized by appearance times and by wave vectors satisfying the relations (16). However, by varying the quantities $\theta_1', \theta_2, \ldots, \theta_n$ we can amplify these signals to a definite limit or to attenuate them until they vanish completely.

A sequence of n pulses can be characterized by effective wave vectors, which can be taken to the arbitrary combinations of the wave vectors in the right-hand side of the relation for k in (16). The relations (16) are satisfied only when $\gamma_1'\tau_1 + \gamma_2'\tau_2 + \ldots + \gamma_{n-1}'\tau_{n-1} \geq 0$, since these relations were obtained for $t \geq t_n$. Consequently, they determine the responses that appear after the n-th pulse. The very last of these responses appears at the instant of time $t = t_n + \tau_1 + \tau_2 + \ldots + \tau_{n-1} < 2t_n$ with a wave vector $\mathbf{k} = 2\mathbf{k}_n - \mathbf{k}_1$.

A sequence of n pulses can be characterized by the number of responses appearing after the n-th pulse. The maximum value M_n of such responses is given by

$$M_n = (3^{n-1} + 1)/2. \tag{17}$$

If we replace n by $\nu = 1, 2, \ldots, n-1$ in all the preceding relations and arguments and consider the instants of time t satisfying the condition $t_\nu \le t \le t_{0\nu+1}$, then we can analyze the responses that appear between the ν-th and the $\nu+1$-st pulses. The maximum number M_ν of such responses is determined by the equation (at $\tau_\nu \ge \tau_1 + \tau_2 \ldots + \tau_{\nu-1}$)

$$M_\nu = (3^{\nu-1} + 1)/2. \tag{18}$$

The maximum total number $M^{(n)}$ of the responses is:

$$M^{(n)} = \sum_{m=1}^{n} M_m = \frac{3^n + 2n - 1}{4}. \tag{19}$$

If the n-pulse excitation is such that the system (1) has a maximum number $M^{(n)}$ of responses that differ in their times of appearance and in their wave numbers, then some of these responses will always have equal intensities. Choosing sequences of n pulses, we can obtain echo signals at arbitrary prescribed instants of time and obtain the radiation of physical fields with arbitrary wave vectors.

Obviously, the theory considered in the present section can describe real systems if the following conditions are satisfied:

$$t_n < T_2, T_1; \qquad \sum_{m=1}^{n} \Delta t_m < T_{2\mathrm{rev}}; \tag{20}$$

where $T_{2\mathrm{rev}}$ is the time of the reversible transverse relaxation due to the scatter of the resonant frequencies as indicated in (1).

3. PULSE SEQUENCE OF THE CP TYPE

For a more detailed analysis of the possibilities of the multipulse-excitation technique, we consider the action of a CP sequence consisting of $n \ge 2$ pulses on the system (1). Such a sequence is characterized by $\theta_1 = \pi/2$; $\theta_2, \theta_3, \ldots, \theta_n = \pi$, and we then obtain from (15) (at $t \ge t_n$)

$$I_1(\mathbf{k}) = I_0(\mathbf{k}) N\{R(R+1) - {}^1/_4(2R+1)\,\mathrm{cth}\,[{}^1/_2\zeta(2R+1)]\,\mathrm{cth}\,({}^1/_2\zeta) + {}^1/_4\,\mathrm{cth}^2\,({}^1/_2\zeta)\};$$

$$I_2(\mathbf{k}, t) = I_0(\mathbf{k})\,[\mathrm{Sp}\,\rho_j R_{j(0)}]^2 \sum_j \sum_{l \ne j} \exp\Big\{i\Big[t - t_n + \sum_{m=1}^{n-1} (-1)^{n+m}\tau_m\Big] \times (\Delta\omega_j - \Delta\omega_l)\Big\} \exp\Big\{i\Big[\mathbf{k} + (-1)^n \mathbf{k}_1 - 2\sum_{m=2}^{n} (-1)^{n+m}\mathbf{k}_m\Big](\mathbf{r}_j - \mathbf{r}_l)\Big\}. \tag{21}$$

For a CP sequence, the summation over the particles in the formula for $I_1(\mathbf{k})$ is carried out exactly. The final expression given above for $I_1(\mathbf{k})$ illustrates the temperature dependence of the incoherent part of the power. The temperature dependence of the coherent part of the radiation power, as seen from (21), is determined by a factor equal to the square of the mean value of the longitudinal component of the effective spin of one of the particles at the instant of time t = 0. This mean value can be calculated from the formula

$$\mathrm{Sp}\,\rho_j R_{j(0)} = {}^1/_2\{(2R+1)\,\mathrm{cth}\,[{}^1/_2(2R+1)\zeta] - \mathrm{cth}\,({}^1/_2\zeta)\}. \tag{22}$$

It follows from (21) that the coherent part of the radiation power has a maximum when the relations

$$t = t_n - \sum_{m=1}^{n-1} (-1)^{n+m}\tau_m, \quad \mathbf{k} = (-1)^n\Big[-\mathbf{k}_1 + 2\sum_{m=2}^{n} (-1)^m \mathbf{k}_m\Big], \tag{23}$$

which are a particular case of (16), are satisfied. It follows from (23) that the first and most important distinguishing feature of a sequence of the CP type is that it is characterized by only one effective vector

$$\mathbf{k}_{\mathrm{eff}} = (-1)^n\Big[-\mathbf{k}_1 + 2\sum_{m=2}^{n} (-1)^m \mathbf{k}_m\Big],$$

which depends on the vectors of all the pulses multiplied by the largest possible (see (16)) numerical coefficients. Thus, a sequence of the CP type makes it possible to excite radiation (a physical field) with a wave vector $\mathbf{k} = \mathbf{k}_{\mathrm{eff}}$.

A second feature of a sequence of the CP type is that after the n-th pulse the system should produce not more than one response. A third feature of this sequence is that the intensities of the responses do not depend on the number of pulses and exceed the intensities of the responses excited by all other arbitrary sequences of pulses.

If the number n of all the pulses acting on the system is odd and (23) is satisfied, then the larger $\tau_2, \tau_4, \tau_6, \ldots, \tau_{n-1}$ and the smaller $\tau_1, \tau_3, \tau_5, \ldots, \tau_{n-2}$, the larger the time interval between the last n-th pulse and the response appearing after this pulse. If n is an even number and (23) is satisfied, then the larger $\tau_1, \tau_3, \tau_5, \ldots, \tau_{n-1}$ and the smaller $\tau_2, \tau_4, \tau_6, \ldots, \tau_{n-2}$, the larger the interval between the n-th pulse and the response that follows it. It follows from the foregoing that at small time intervals between the pulses, the time interval between the n-th pulse and the response that appears after it can be large as a result of the large n.

4. PULSE SEQUENCE OF THE WW TYPE

We consider the behavior of the system (1) when it is acted upon by a sequence of the WW type, consisting of $n \ge 2$ pulses. The parameters characterizing the pulses of the type WW sequence should have the following values:

$$\begin{gathered}\tau_1 = \tau, \quad \tau_2 = \tau_3 = \ldots = \tau_{n-1} = 2\tau;\\ \theta_1 = \theta_2 = \ldots = \theta_n = \pi/2; \quad \alpha_{j(1)} = \beta_1 \exp(i\mathbf{k}_1\mathbf{r}_j),\\ \alpha_{j(2)} = \alpha_{j(3)} = \ldots = \alpha_{j(n)} = \beta_2 \exp(i\mathbf{k}_2\mathbf{r}_j).\end{gathered} \tag{24}$$

Substituting (24) in (15), we obtain for the coherent part of the spontaneous-radiation power on the system (1) the expression (at $t \ge t_n$):

$$\begin{aligned} I_2(\mathbf{k}, t) = I_0(\mathbf{k})\,[\mathrm{Sp}\,\rho_j R_{j(0)}]^2 \sum_j \sum_{l \ne j} \sum_{\xi, \xi'} \{P_{(n)\xi} P_{(n)\xi'} \\ \times \exp[i(\mathbf{k} - 2\mathbf{k}_2 + \mathbf{k}_1)(\mathbf{r}_j - \mathbf{r}_l)] + Q_{(n)\xi} Q_{(n)\xi'} \exp[i(\mathbf{k} - \mathbf{k}_1)(\mathbf{r}_j - \mathbf{r}_l)] \\ + P_{(n)\xi} Q_{(n)\xi'} \beta_1^2 \beta_2^{*2} \exp[i(\mathbf{k} - 2\mathbf{k}_2 + \mathbf{k}_1)\mathbf{r}_j - i(\mathbf{k} - \mathbf{k}_1)\mathbf{r}_l] + \\ + Q_{(n)\xi} P_{(n)\xi'} \beta_1^{*2} \beta_2^2 \exp[i(\mathbf{k} - \mathbf{k}_1)\mathbf{r}_j - i(\mathbf{k} - 2\mathbf{k}_2 + \mathbf{k}_1)\mathbf{r}_l]\} \\ \times \exp\{i[t - t_n - (2\xi - 1)\tau]\Delta\omega_j - i[t - t_n - (2\xi' - 1)\tau]\Delta\omega_l\}; \\ \xi, \xi' = -n + 2, \quad -n + 3, \quad -n + 4, \ldots, n - 1. \end{aligned} \tag{25}$$

$P_{(n)\xi}$ and $Q_{(n)\xi}$ are dimensionless real numerical coefficients and can be calculated from the following formulas:

$$P_{(n)\xi}=(-1)^{\xi}\left[\sum_{\eta=|\xi-1|}^{n-2}(-1)^{\eta}2^{-\eta}f(\eta,\xi-1)+(-1)^{n-1}2^{-n+1}f(n-2,\xi-1)\right];$$

$$Q_{(n)\xi}=(-1)^{\xi-1}\left[\sum_{\eta=|\xi-1|}^{n-2}(-1)^{\eta}2^{-\eta}f(\eta,\xi-1)+(-1)^{n}2^{-n+1}f(n-2,\xi-2)\right];$$

$$f(\lambda,\mu)=\sum_{r}(-1)^{r}2^{2r}C_{\lambda-r}^{r}C_{2\lambda-4r}^{\lambda-\mu-2r};\qquad r=0,1,2,\ldots,\left[\frac{\lambda-\mu}{2}\right];$$

$$\eta=|\xi-1|,\quad |\xi-1|+1,\quad |\xi-1|+2,\ldots; \tag{26}$$

where C_y^x is the number of combinations of y taken x at a time, [x] is the integer part of x, i.e., the largest integer not exceeding x, $P_{(n)\xi}=0$ at $|\xi-1|>n-2$, and $Q_{(n)\xi}=0$ at $|\xi|>n-2$.

It is seen from (25) that the coherent part of the spontaneous-emission power $I_2(\mathbf{k}, t)$ will have a maximum when t and $\mathbf{k}$ assume simultaneously the following values:

$$t=t_n+(2\xi-1)\tau;\quad \mathbf{k}=2\mathbf{k}_2-\mathbf{k}_1,\mathbf{k}_1; \quad \xi=1,2,\ldots,n-1, \tag{27}$$

for when $t=t_n+(2\xi-1)\tau$ and $\mathbf{k}=2\mathbf{k}_2-\mathbf{k}_1$ in the expression contained in (25) after the two signs of summation over the particles, the term proportional to $P_{(n)\xi}^2$ will have a maximum value and will not depend on the indices of the summation with respect to the particles j and l; at $t=t_n+(2\xi-1)\tau$ and $\mathbf{k}=\mathbf{k}_1$, the same can be stated with respect to the term proportional to $Q_{(n)\xi}^2$. Of course, if $P_{(n)\xi}=0$ or $Q_{(n)\xi}=0$, then $I_2(\mathbf{k}, t)$ will not have corresponding maxima. The presence of maxima of the coherent part of the spontaneous-emission power means that the system produces superradiant coherent echo signals, the appearance times and the propagation directions of which are determined by relations (27).

The number of echo signals that appear after the n-th pulse and propagate in the direction $k=2k_2-k_1$ cannot be larger than $n-1$. The number of echo signals that appear after the n-th pulse and propagate in the direction $k=k_1$ cannot be larger than $n-2$, since $Q_{(n)n-1}=0$ always.

If we replace n by $\nu=2,3,\ldots,n-1$ in all the preceding relations and arguments of the present section, and consider instants of time t satisfying the condition $t_\nu\le t\le t_{0\nu+1}=t_\nu+2\tau$, then we can analyze the echo signals that appear between the ν-th and the $\nu+1$-st pulses. Obviously, two echo signals will appear between the ν-th and $\nu+1$-st pulses for $\nu\ge 3$ at the instant $t=t_\nu+\tau$; one of them will propagate in the direction $\mathbf{k}=2\mathbf{k}_2-\mathbf{k}_1$, and the other in the direction $\mathbf{k}=\mathbf{k}_1$. Only one echo signal in the direction $\mathbf{k}=2\mathbf{k}_2-\mathbf{k}_1$ will appear between the second and third pulses, since $Q_{(\nu)1}=0$ at $\nu=2$.

If the relaxation times characterizing a real excited system satisfy the inequalities $T_{2rev}<T_2<T_1$, then we can assume on the basis of[9] and the reasoning given in the introduction that when such a system is acted upon by a sequence of the WW type consisting of $n\gg 2$ optical and acoustic pulses, the effective phase-memory time of the system T_{2eff} will lengthen and will tend to T_1 when τ is decreased. The presence or the absence of this phenomenon can be verified experimentally by studying the character of the decrease of the intensities of the echo signals that appear between the pulses of a sequence of the WW type, in comparison with the values that can be obtained for these intensities with the aid of (25), by integrating $I_2(\mathbf{k}, t)$ over all the angles that determine the direction of the wave vector $\mathbf{k}$[2].

5. THREE-PULSE SCHEME FOR ACOUSTIC EXCITATION OF ELECTROMAGNETIC SPIN ECHO

It was shown in[6] that a two-pulse scheme of acoustic excitation of ordinary spin echo is possible if the splitting of the unperturbed spin-system spectrum is abruptly altered in the time interval between the acoustic pulses; this is not easy to do. On the other hand, the use of three acoustic pulses immediately eliminates all the difficulties that lie in the path of obtaining an effective wave vector equal to the wave vector $\mathbf{k}$ of the radiated electromagnetic field. In our case the frequencies of the sound and of the electromagnetic field should coincide.

In the NMR region, and in practice also in the EPR region, $|\mathbf{k}|\sim 0$. Therefore, according to (23), we should direct the vectors $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_3$ of the traveling sound waves in such a way as to have

$$2(\mathbf{k}_2-\mathbf{k}_3)-\mathbf{k}_1=0. \tag{28}$$

If $|\mathbf{k}_1|=|\mathbf{k}_2|=|\mathbf{k}_3|$, then the condition (28) can be satisfied in the following manner. We locate all three vectors in one plane: if $\mathbf{k}_1$ makes an angle 0° with the coordinate axis, then $\mathbf{k}_2$ and $\mathbf{k}_3$ should naturally make angles 75° 30′ and 104° 30′ with this axis. The condition (28) is obviously satisfied also in the case of not too different moduli of the wave vectors. On the other hand, if one of the quantities $|\mathbf{k}_1|$, $|\mathbf{k}_2|$, and $|\mathbf{k}_3|$ greatly exceeds each of the remaining ones, then it is necessary to use a scheme consisting of more exciting pulses, the minimum number n again being determined with the aid of formula (23).

The theory developed here is applicable to cases in which the spin-phonon interaction operator is linear in the effective spin (Co^{2+} ions in MgO, rare-earth ions in CaF_2). The conditions for setting up such experiments were discussed in detail in[6]. To realize this scheme, it is necessary to have a single crystal in the form of a trihedral prism with lateral surfaces polished and making the angles indicated by us.

CONCLUSION

The formulas obtained by us illustrate the promising nature of the technique of multipulse excitation of quantum systems. Insofar as we know, the presently existing pulse technique of optical and hypersonic excitation of a medium is perfectly suitable for the performance of the experiments considered in the present paper. It seems to us that the most promising realization of our theoretical relations will be the development of an optical computer. From the point of view of physical research, the important conclusion is that it is possible to lengthen the effective time of the optical transverse relaxation. Such an effect can be used to produce a "time magnifier," which makes it possible to investigate rapid physical processes of picosecond and shorter durations.

Finally, the multiples technique of optical and hypersonic (and in perspective also terasonic) excitation can be used to generate coherent beams of x rays, γ quanta, and elementary particles.

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