KINETIC PROCESSES IN A LASER PLASMA

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Kinetic processes in a high-temperature ($T_e \geq 1$ keV) laser plasma are investigated theoretically. The criterion for a strong field in which the plasma state deviates from the equilibrium state is derived. The kinetic equation for the plasma electron distribution function is investigated under strong-field conditions. The distribution function for electrons in a multiply-charged plasma ($z \gg 1$) is computed analytically and expressions are obtained for its parameters (temperature, ionization multiplicity, the self-radiation spectra) as functions of the intensity and duration of the laser pulse.

1. One of the most effective methods for the experimental investigation of a laser plasma consists in the determination of its macroscopic parameters from the emitted radiation and the characteristics of the state of ionization (ionization multiplicity, degree of ionization, etc.)[1]. For this reason, the determination of the kinetic processes in the plasma which govern the energy distribution of the electrons under strong laser-field conditions is important. This is necessary, since the interpretation of the experimental data obtained by the indicated method depends on the form of the electron distribution function.

In this paper we investigate the kinetic processes in a multiply ionized plasma under the conditions of sufficiently high radiation flux densities, when nonequilibrium ionization obtains[2]. The conditions under which ionization equilibrium is destroyed and the electron distribution function substantially differs from Maxwellian are obtained. The macroscopic parameters of the plasma are computed: the electron temperature T_e, the effective ionization multiplicity z as a function of the radiation flux density q_0, the time t, and the properties of the material. Expressions are given for the bremsstrahlung and recombination spectra of the plasma radiation in the case of a non-Maxwellian distribution function. The question of the radiation spectra of a fully ionized plasma (e.g., of the hydrogen plasma) is also discussed.

2. It is convenient to characterize the ionization state produced under the action of the laser radiation of the plasma by the parameter $\eta_0 = T_e/I(z)$, i.e., by the ratio of electron temperature T_e to the ionization potential I(z) for an ion of ionization multiplicity z. (It is assumed here and below that the plasma contains at each given monent of time ions with a narrow distribution over z[3], and under the quantity z will be meant its mean value.) As in the case of the optical breakdown of a gas, this ratio depends on the parameter

$$\beta_0 = I(z)\nu_i(z) / \varepsilon_0\nu_{eff}(z),$$

where $\nu_i(z)$ and $\nu_{eff}(z)$ are the inelastic and elastic collision rates of an electron with the ions,

$$\varepsilon_0 = e^2E_0^2 / 2m\omega^2 = 4\pi e^2 q_0 / cm\omega^2$$

is the oscillation energy of an electron in the field of a light wave of amplitude E_0 and angular frequency ω. In the region of relatively small radiation flux densities $q_0 \lesssim 10^{12}$ watt/cm² the quantity $\epsilon_0 \lesssim 0.1$ eV, $\beta_0 > 1$, the ratio' $\eta_0 < 1$, and the plasma is in a quasi-equilibrium ionization state. Physically, this is connected with the fact that the electron distribution function is cut off in this case at energies ϵ close to the threshold of the inelastic processes ($\epsilon \approx I(z)$, as in the case of thermodynamic equilibrium), so that the ionization takes place owing to the "tail" of the distribution function. The distribution of the plasma ions over z is then described at each given moment of time either by the Saha equation if triple recombination is the principal inverse process ($\eta_0 \approx 1/7 - 1/10$), or by the coronal distribution[4] in the case when photorecombination predominates ($\eta_0 \approx 1/4 - 1/5$). However, at large q_0 ($\beta_0 < 1$), because of the rapid "diffusion" of electrons into the region of energies exceeding I(z), we can expect a substantial disruption of the ionization equilibrium in the plasma. As a result, the electron temperature increases and the parameter η_0 may become larger than unity. The increase of η_0 leads to a sharp growth of the ionization rate ($\nu_i \sim \exp\{-1/\eta_0\}$), which for $\eta_0 > 1$ considerably exceeds the recombination rate, i.e., the process becomes unidirectional. Under these conditions the final state of the plasma (after the action of the laser pulse) is quite removed from the equilibrium state. In this case, for radiation pulse durations $\tau_p \approx 10^{-10}-10^{-9}$ sec, and for $q_0 \approx 10^{13}-10^{14}$ watt/cm², the ionization multiplicity, although less than the equilibrium value at the given temperature, attains values $z \approx 20-30$, whereas the temperature $T_e \approx 5-10$ keV. Physically, the situation described here indicates that as compared to the energy expended on ionization, the fraction of the radiation energy which goes into the thermal energy of the electrons increases with the radiation flux density.

The gap between the electron and ion temperatures, which is determined principally by the inelastic losses also becomes important at high q_0. Thus, for example, at $q_0 \approx 10^{12}$ watt/cm² $\Delta T \approx 4\pi Me^2q/3mw^2c \approx 1$ keV. In the general case, the electron distribution function in a strongly ionized plasma, $F(\epsilon, t)$, satisfies the kinetic equation

$$\frac{\partial F}{\partial t} = \left(\frac{\partial F}{\partial t}\right)_q + \left(\frac{\partial F}{\partial t}\right)_{in} + \left(\frac{\partial F}{\partial t}\right)_{ee} \tag{1}$$

The terms on the right hand side of Eq. (1) respectively describe the contributions of the laser radiation field, the inelastic collisions of the electrons with the ions (including ionization), and the electron-electron collisions to the change in $F(\epsilon, t)$. The quantity z, which in the model under consideration is for mathematical convenience assumed to be a continuous function of the time, is related to the distribution function by the relation

$$z(t)=1+\frac{1}{N_0}\int_0^\infty d\varepsilon\, F(\varepsilon, t), \qquad (2)$$

where N_0 is the density of the ions (z = 2 corresponds to a single ionization).

The ionization potential for an ion of ionization multiplicity z may be represented in the form $I(z) = I_0 z^2$ [5], where I_0 is a slowly varying function of z, which we shall assume to be approximately a constant.

The frequency of elastic collisions of electrons with ions of ionization multiplicity z is equal to

$$\nu_{eff}(\varepsilon, z) = (2\pi)^{1/2} N_0 e^4 (z-1)^2 \Lambda / m^{1/2} \varepsilon^{3/2}, \qquad (3)$$

where Λ is the Coulomb logarithm. For the inelastic collision frequency $\nu_i(\epsilon, z)$, we can use the Born approximation when $\beta_0 < 1$ [5]:

$$\nu_i(\varepsilon, z) = \nu_{im}(1) z^{-3} [I(z)/\varepsilon]^{1/2}, \qquad (4)$$

where $\nu_{im}(1)$ is the maximum frequency of inelastic collisions of electrons with neutral atoms.

Under these conditions the expressions for $(\partial F/\partial T)_q$ and $(\partial F/\partial t)_{in}$ are determined by [2] ($\hbar\omega/\epsilon \ll 1$, $I(z)/\epsilon < 1$)

$$\left(\frac{\partial F}{\partial t}\right)_q = -\frac{\varepsilon_0}{3}\frac{\partial}{\partial\varepsilon}\left(\nu_{eff}F - 2\nu_{eff}\varepsilon\frac{\partial F}{\partial\varepsilon}\right), \qquad (5)$$

$$\left(\frac{\partial F}{\partial t}\right)_{in} = -\frac{I(z)\nu_i(\varepsilon,z)}{2\varepsilon}\left(F - 2\varepsilon\frac{\partial F}{\partial\varepsilon}\right). \qquad (6)$$

The collision term $(\partial F/\partial t)_{ee}$, describing the contribution of the electron-electron collisions, can be represented in the Fokker-Planck approximation [6] in the form

$$\left(\frac{\partial F}{\partial t}\right)_{ee} = (4\pi e^2)^2\left(\frac{2\varepsilon}{m}\right)^{1/2}$$
$$\times\left[\frac{F^2}{8\pi\varepsilon} - 2\varepsilon\frac{\partial\psi}{\partial\varepsilon}\frac{\partial}{\partial\varepsilon}\left(\frac{\partial F}{\partial\varepsilon} - \frac{F}{2\varepsilon}\right) - \frac{\psi}{\varepsilon}\left(\frac{\partial F}{\partial\varepsilon} - \frac{F}{2\varepsilon}\right)\right], \qquad (7)$$

$$\psi(\varepsilon) = -\frac{1}{24\pi}\left(3\int_0^\varepsilon F\, d\varepsilon - \frac{1}{\varepsilon}\int_0^\varepsilon \varepsilon F\, d\varepsilon + 2\varepsilon^{1/2}\int_\varepsilon^\infty \varepsilon^{-1/2} F\, d\varepsilon\right).$$

Since in the case being considered the parameter β_0 characterizing the magnitude of the light field depends on ϵ and z, it is necessary to analyze the condition $\beta_0 < 1$ in greater detail, and to establish the condition, not dependent on ϵ and z, for a strong field and nonequilibrium ionization.

With that end in view, let us introduce a new variable $\eta = \epsilon/I(z) = \epsilon/I_0 z^2$. Then the relations (3) and (4) will have the form

$$\nu_{eff}(\eta, z) = \frac{\overset{0}{\nu}_{eff}}{z\eta^{3/2}}, \quad \overset{0}{\nu}_{eff} = \frac{\sqrt{2\pi}\, N_0 e^4 \Lambda}{\sqrt{m}\, I_0^{3/2}}, \quad \nu_i(\eta, z) = \frac{\nu_{im}(1)}{z^3\eta^{1/2}} \qquad (8)$$

and the inequality $\beta_0 < 1$, the form

$$\eta\beta_0' < 1, \quad \beta_0 = \eta\beta_0', \quad \beta_0' = \frac{\nu_{im}(1) I_0}{\varepsilon_0 \nu_{eff}^0}. \qquad (9)$$

It follows from the relation (9) that if $\beta_0' \ll 1$, then, owing to inelastic collisions, the electron distribution function terminates at $\eta \approx 1/\beta_0' \gg 1$, i.e., when $\epsilon \gg I(z)$. Consequently, the condition $\beta_0' \ll 1$, which does not depend on ϵ and z, is a sufficient condition for the validity of the Born approximation for the frequency ν_i, and the majority of the electrons will, for $\beta_0' \ll 1$, be concentrated in the region $\epsilon > I(z)$. It is also evident that the inequality $\beta_0' \ll 1$ leads to the condition $\eta_0 > 1$, i.e., it determines the magnitude of the field in which a nonequilibrium ionization obtains. It is physically clear that in the case being considered the form of the distribution function $F(\epsilon, t)$ is mainly determined by the competition between the electron diffusion process in energy space under the action of the light field and the electron-electron collision mechanism.

Owing to electron-electron collisions, in the region of not too high fields (or not too small β_0') the distribution function $F(\epsilon, t)$ will be nearly Maxwellian with a temperature that depends on the time and the magnitude of the field. When the magnitude of the light field is increased ($\beta_0' \ll 1$) the function $F(\epsilon, t)$ is determined mainly by the electron-field interaction process and is, generally speaking, non-Maxwellian in the entire energy range. First, we give an approximate condition which separates these cases. In order of magnitude

$$\left(\frac{\partial F}{\partial t}\right)_{ee} \approx (N_e\sigma_{ee}v_e)F = (N_0 z\sigma_{ee}v_e)F, \qquad \left(\frac{\partial F}{\partial t}\right)_q \approx \varepsilon_0 N_0 z^2 \sigma_{ee} v \frac{\partial F}{\partial\varepsilon}$$

where σ_{ee} is the Coulomb cross section of the electron-electron collisions. Consequently, the distribution function will be nearly Maxwellian when $(\partial F/\partial t)_{ee} \gg (\partial F/\partial t)_q$, i.e., $F \gg z\epsilon_0 \partial F/\partial\epsilon$, or finally

$$T_e > z\varepsilon_0. \qquad (10)$$

The condition (10) can be more rigorously derived for the "tail" of the distribution function, i.e., in the energy region $\epsilon > T_e$, and this is most important for the interpretation of the spectra of the plasma radiation.

Changing in the expression (7) to the variable $\eta = \epsilon/I_0 z^2$ and expanding it in powers of $\langle\eta\rangle/\eta$ (where $\langle\eta\rangle$ is the mean value), we obtain in the zeroth approximation

$$\left(\frac{\partial F}{\partial t}\right)_{ee} = -\frac{\nu_{ee}^0\langle\eta\rangle}{z^2}\left[\frac{\partial^2}{\partial\eta^2}\left(\frac{F}{\eta^{1/2}}\right) + \frac{\partial}{\partial\eta}\left(\frac{F}{\eta^{1/2}}\right)\right], \qquad (11)$$

where $\nu_{ee}^0 = 2^{3/2}\pi e^4 N_0/m^{1/2} I_0^{3/2}$. When the change $\eta = \epsilon/I_0 z^2$ is made in the relation (5) it takes the form

$$\left(\frac{\partial F}{\partial t}\right)_q = \frac{2}{3}\frac{\overset{0}{\nu}_{eff}\varepsilon_0}{z^3 I_0}\frac{\partial^2}{\partial\eta^2}\left(\frac{F}{\eta^{1/2}}\right). \qquad (12)$$

Comparison of (11) and (12) shows that the distribution function will have a Maxwellian "tail" provided

$$\varepsilon_0 \ll 3\sqrt{\pi}\Lambda^{-1} z I_0 \langle\eta\rangle. \qquad (13)$$

Since $T_e = \langle\eta\rangle I_0 z^2$, (13) coincides with (10) except for the factor $3\sqrt{\pi}\Lambda^{-1} \sim 1$.

It is physically clear that the condition (13) is applicable to a completely ionized hydrogen plasma (z = 2), and is always valid (within the limits of validity of the kinetic equations under consideration). Consequently, the radiation of a hydrogen plasma, the energy of whose quanta exceeds the temperature, is certainly determined by the Maxwellian "tail" of the electron distribution function.

Let us make numerical estimates, using the formulas (9) and (13):

$$\varepsilon_0[\mathrm{eV}] = \frac{1.7\cdot 10^{13} q_0[\mathrm{watt/cm^2}]}{(\nu/3\cdot 10^{14})^2},$$
$$\beta_0' = \frac{10^{10} I_0^{5/2}[\mathrm{eV}]\,(\nu/3\cdot 10^{14})^2}{q_0[\mathrm{watt/cm^2}]}. \qquad (14)$$

It follows from (14) that for the radiation of a neodymium laser ($\nu = 3\times 10^{14}$ sec^{-1}) and for $I_0 \approx 5-10$ eV, the condition $\beta_0' \ll 1$ is fulfilled when $q_0 \gtrsim 10^{12}-10^{13}$ watt/cm^2, whereas the inequality (13) is valid only when $q_0 < 10^{14}-10^{15}$ watt/cm^2 ($z \approx 20$, $\langle\eta\rangle \approx 1-2$). Thus, in the range of radiation flux densities $10^{12} < q_0 < 10^{14}$ watt/cm^2 the ionization equilibrium in the plasma is destroyed, but the electron distribution function remains nearly Maxwellian with $T_e > I(z)$. In the region $q_0 > 10^{14}$ watt/cm^2 ($\nu = 3\times 10^{14}$ sec^{-1}), or in the case of radiation in the infrared region (e.g., for a CO_2 laser with $\nu = 3\times 10^{13}$ sec^{-1}, $q_0 > 10^{12}$ watt/cm^2) the condition (13) may turn out to be unfulfilled, and the electron distribution function deviates substantially from the Maxwellian. In this situation it is necessary to seek the exact solution of Eq. (1) with $(\partial F/\partial t)_{ee} = 0$.

3. In the general case the electron distribution function can be represented in the form

$$F(\varepsilon, t) = N_0 f(\varepsilon, z)\exp\left\{\int_0^t \gamma(z)\,dt\right\}, \quad \int_0^\infty f(\varepsilon, z)\,d\varepsilon = 1. \qquad (15)$$

It is then easy to show that

$$\frac{dz}{dt} = z\gamma(z), \quad \gamma(z) = \int_0^\infty f(\varepsilon, z)\nu_i(\varepsilon, z)\,d\varepsilon. \qquad (16)$$

A. Let us first consider the case of relatively weak light fields $10^{12} < q_0 < 10^{14}$ watt/cm^2, corresponding to the fulfillment of the conditions (10) and (13). Then the distribution function $f(\epsilon, z)$ is close to the Maxwellian distribution with a z-dependent temperature:

$$f(\varepsilon, z) = \frac{2\varepsilon^{1/2}}{\pi^{1/2} T_e^{3/2}(z)}\exp\left\{-\frac{\varepsilon}{T_e(z)}\right\}. \qquad (17)$$

The quantity $\gamma(z)$, which has the meaning of an avalanche-development "constant," is equal to

$$\gamma(z) = \int_0^\infty f(\varepsilon, z)\nu_i(\varepsilon, z)\,d\varepsilon = \frac{2\nu_{im}(1) I_0^{1/2}}{\pi^{1/2} z^2 T_e^{1/2}}. \qquad (18)$$

Further, multiplying Eq. (1) by ϵ and integrating it with the function (17), we obtain an equation expressing the energy conservation law for an electron gas:

$$\frac{3}{2}\frac{d}{dt}N_0 z T_e = \frac{4}{3^{1/2}}\frac{N_0\varepsilon_0 \overset{0}{\nu}_{eff}}{\eta_0^{3/2}(t)} - \frac{2}{\pi^{1/2}}\frac{N_0 I_0 \nu_{im}(1)}{\eta_0^{1/2}(t)}, \qquad (19)$$

where $\eta_0(t) = T_e(z)/I_0 z^2(t)$. Adding the equation $dz/dt = \gamma(z)z$ to Eq. (19), we obtain a system for the determination of T_e and z as functions of the magnitude of the light field and time. The system of equations under consideration admits of an exact solution of the form

$$z = 4^{1/3}[\nu_{im}(1)t]^{1/3}(\beta_0')^{1/12} \sim q_0^{-1/12}t^{1/3},$$
$$T_e = 1.75[\nu_{im}(1)t]^{2/3} I_0(\beta_0')^{-1/3} \sim q_0^{1/3}t^{2/3}. \qquad (20)$$

The magnitude of the ratio $\eta_0 = T_e/I_0z^2$ then turns out to be time independent and equal to

$$\eta_0 = 0.7/\sqrt{\beta_0'}. \qquad (21)$$

The solutions (20) permit us to express the conditions (10) and (13) in terms of the parameters of the laser pulse and the material. Substituting (20) into (10) and (13), we have

$$\varepsilon_0/I_0 \ll [\nu_{im}(1)\tau_p]^{4/5}[\nu^0_{eff}/\nu_{im}(1)]^{7/5}, \qquad (22)$$

where τ_p is the duration of the laser pulse.

Let us now make numerical estimates, setting $q_0 \approx 10^{13}$ watt/cm^2 ($\nu = 3\times 10^{14}$ sec^{-1}), $\tau_p = 10^{-9}$ sec, $I_0 = 5$ eV, $N_0 = 10^{20}$ cm^{-3}, and $\nu_{im}(1) = 10^{12}$ sec^{-1}. We obtain with the aid of (14), (20), and (21)

$$\beta_0' = 6\cdot 10^{-2}, \quad z = 15, \quad T_e \approx 3\,\mathrm{keV}, \eta_0 \approx 2.7.$$

The inequality (22) is in this case very well satisfied.

B. As follows from (22), the electron distribution function may substantially deviate from the Maxwellian when we go over to higher-power and shorter pulses ($q_0 > 10^{14}$ watt/cm^2, $\tau_p \approx 10^{-10}-10^{-11}$ sec). Under these conditions the principal role is played by the interaction of the electrons with the radiation field, and the electron-electron collisions may be neglected. The kinetic equation (1) then has an analytic solution which allows us to find the parameters of the plasma and determine the form of the spectra of the radiation.

Representing the distribution function $F(\epsilon, t)$ in the form (15) together with the conditions (16), we arrive in Eq. (1) to the new variables $\eta = \epsilon/I_0z^2$ and z. Then we shall have

$$\nu_{eff}(\eta, z) = \frac{\overset{0}{\nu}_{eff}}{z\eta^{3/2}}, \quad \nu_i(\eta, z) = \frac{\nu_{im}(1)}{z^3\eta^{1/2}},$$
$$\left(\frac{\partial F}{\partial t}\right)_q = \frac{2}{3}\frac{\overset{0}{\nu}_{eff}\varepsilon_0}{z^3 I_0}\frac{\partial^2}{\partial\eta^2}\frac{F}{\eta^{1/2}}, \qquad (23)$$
$$\left(\frac{\partial F}{\partial t}\right)_{in} = \frac{\nu_{im}(1)}{z^3}\frac{\partial}{\partial\eta}\frac{F}{\eta^{1/2}}.$$

In terms of the variables η and z the function $f(\epsilon, z)$ has the form

$$f(\varepsilon, z) = A(z)\varphi(\eta). \qquad (24)$$

In fact, A(z) is determined from the normalization condition

$$A(z) = 1\Big/ I_0 z^2\int_0^\infty \varphi(\eta)\,d\eta. \qquad (25)$$

Further, we find with the aid of the relation (16) the expression for $\gamma(z)$:

$$\gamma(z) = \int_0^\infty f\nu_i\,d\varepsilon = \nu_{im}(1)J_0/z^2, \qquad (26)$$

$$J_0 = \int_0^\infty \eta^{-1/2}\varphi\,d\eta \Big/ \int_0^\infty \varphi\,d\eta. \qquad (27)$$

Substituting now the function $F(\epsilon, t)$ into Eq. (1) (with $(\partial F/\partial t)_{ee} = 0$) and taking into account the expressions (15) and (23)–(26), we obtain the equation for the function $\varphi(\eta)$:

$$\frac{d^2}{d\eta^2}\left(\frac{\varphi}{\eta^{1/2}}\right) + \frac{3}{2}\beta_0'\frac{d}{d\eta}\left(\frac{\varphi}{\eta^{1/2}}\right) + \frac{3}{2}J_0\beta_0'\left(\varphi + 2\eta\frac{\partial\varphi}{d\eta}\right) = 0. \qquad (28)$$

Thus, the formulated problem reduces to the solution of the system (27) and (28).

The substitution $\Phi = \varphi/\eta^{1/2}$ reduces Eq. (28) to the form

$$\frac{d^2\Phi}{d\eta^2} + \left(\frac{3}{2}\beta_0' + 3J_0\beta_0'\eta^{3/2}\right)\frac{d\Phi}{d\eta} + 3J_0\beta_0'\eta^{1/2}\Phi = 0. \qquad (28')$$

The term $\frac{3}{2}\beta_0' d\Phi/d\eta$ in Eq. (28') is connected with the allowance for the influence on the distribution function of the effect of deceleration of electrons during inelastic collisions with ions. However, in the case under consideration ($\beta_0' \ll 1$) the effect of deceleration affects the form of the distribution function only in the energy region $\eta_0 \approx 1/\beta_0' \gg 1$, a region which is of no interest to us. For this reason, this term may be neglected in Eq. (28′). We recall that in the present problem inelastic collisions leading to ionization are taken into account in the determination of the quantity $\gamma(z)$. Setting in (28′)

$$\Phi = \eta^{1/2} e^{-y} W(y), \quad y = {}^{2}/_{5} J_0 \beta_0' \eta^{5/2},$$

we obtain a universal equation, not containing physical parameters:

$$y^2 \frac{d^2W}{dy^2} + y\frac{dW}{dy} - W\left(y^2 - \frac{1}{5}y + \frac{1}{25}\right) = 0. \tag{29}$$

We now show that in the present case we can, without solving Eq. (29), derive the expressions for the "temperature" T_e and ionization multiplicity z up to constant coefficients of the order of unity. Indeed, substituting the expression $\varphi = \eta e^y W(y)$ into (27), we obtain

$$J_0 = J_1(\beta_0')^{1/4}, \quad J_1 = \left(\frac{3}{5}\right)^{1/4} Y_{-2/5}/Y_{-1/5},$$
$$Y_n = \int_0^\infty y^{-n} e^{-y} W(y)\, dy. \tag{30}$$

Further, using the relations

$$\langle\eta\rangle = \int_0^\infty \eta\varphi\, d\eta \Big/ \int_0^\infty \varphi\, d\eta,$$

(26), and (16), we find the expressions for z, T_e and $\langle\eta\rangle$:

$$\langle\eta\rangle = J_2(\beta_0')^{-1/2}, \quad J_2 = ({}^{5}/_{3}J_1)^{2/5} Y_{1/5} / Y_{-1/5},$$
$$z = (3J_1)^{1/3}(\beta_0')^{1/12}[\nu_{im}(1)t]^{1/3} \sim q_0^{-1/12} t^{1/3}, \tag{31}$$
$$T_e = \langle\eta\rangle I_0 z^2 \sim q_0^{1/3} t^{2/3}.$$

Comparison of the formulas (20) and (31) shows that in spite of the formal difference in form of the distribution functions, the final expressions for z and T_e in the last case and in the case of the Maxwellian electron distribution differ only by numerical factors of the order of unity. This circumstance is not accidental, but connected, as will be shown below, with the fact that the function

$$f(\varepsilon, z) = \frac{1}{I_0 z^2}\varphi(\eta) \Big/ \int_0^\infty \varphi\, d\eta$$

depends only on ϵ and $T_e(\eta_0, z)$, i.e., does not depend on q_0 and z explicitly. For this reason, mean values computed with the aid of $f(\epsilon, z)$ (through integration from 0 to ∞), depend up to numerical factors on T_e and z in the same way as in the case of the Maxwellian distribution function. However, if the computation of some characteristics of the plasma requires the integration of $f(\epsilon, T_e)$ between the limits (I, ∞), where I is some characteristic energy, then a function depending on the ratio I/T_e appears as a coefficient in the corresponding expression. The form of the indicated function then depends very much on the form of $f(\epsilon, T_e)$. For this reason, the spectra of the bremsstrahlung and the recombination radiation, for example, may be substantially different in the two cases considered.

For the computation of the bremsstrahlung spectrum in the case of a non-Maxwellian distribution function, we use the expression for the effective radiation dq_ν[3]

$$dq_\nu = \frac{32\pi^2 z^2 e^2}{3^{3/2} m^2 c^3 v_e} d\nu \tag{32}$$

and, with the aid of the relation[3]

$$j_\nu^T d\nu = \int_{v_e^{min}}^\infty N_0 F(\varepsilon, t)\, v_e dv_e dq_\nu$$

$(m(v_e^{min})^2/2 = h\nu)$, we obtain an expression for the spectral density of the bremsstrahlung j_ν^T:

$$j_\nu^T = \frac{32\pi^2 N_0 e^6 z^2}{3\cdot 6^{1/2} m^{3/2} c^3 I_0} \int_{h\nu/I_0 z^2}^\infty \varphi\eta^{-1/2} d\eta \Big/ \int_0^\infty \varphi\, d\eta. \tag{33}$$

Similarly, we obtain for the recombination spectrum, in accordance with[3],

$$j_\nu^p = 2\pi\hbar\sigma_0\left(\frac{2}{m}\right)^{1/2} N_0^2 I_0 \frac{z^3}{n^3}\varphi\left(\frac{h\nu}{I_0 z^2} - \frac{1}{n^2}\right)\left[\left(h\nu - \frac{I_0 z^2}{n^2}\right)^{1/2} \int_0^\infty \varphi\, d\eta\right]^{-1}. \tag{34}$$

In order to obtain explicit expressions for the radiation spectra (33) and (34), let us consider Eq. (29). Analysis shows that Eq. (29) is to a high degree of accuracy satisfied by the function

$$W(y) = K_0(y), \tag{35}$$

where $K_0(y)$ is the Macdonald function. The solution (35) permits us to determine the unknown constant J_1 in the relation (31):

$$J_1 = 2^{1/5}\left[\frac{\Gamma(^3/_5)}{\Gamma(^4/_5)}\right]^2 \frac{\Gamma(1.1)}{\Gamma(1.3)} = 2.1,$$
$$J_2 = \left(\frac{5}{3J_1}\right)^{2/5} \frac{1}{2^{2/5}}\left[\frac{\Gamma(^6/_5)}{\Gamma(^4/_5)}\right]^2 \frac{\Gamma(1.3)}{\Gamma(1.7)} = 0.5,$$

where $\Gamma(x)$ is the gamma function. Substituting J_1 into (31), we obtain the final expressions for z, T_e, and $\langle\eta\rangle$:

$$z = (6.3)^{1/3}(\beta_0')^{1/12}[\nu_{im}(1)t]^{1/3} \sim q_0^{-1/12} t^{1/3},$$
$$T_e = 1.7 I_0(\beta_0')^{-1/3}[\nu_{im}(1)t]^{2/3} \sim q_0^{1/3} t^{2/3}, \tag{31'}$$
$$\langle\eta\rangle = 0.5(\beta_0')^{-1/2}.$$

When (35) is taken into account the function $\varphi(\eta)$ takes the form

$$\varphi(\eta) = \eta K_0\left(8\left(\frac{\eta}{\langle\eta\rangle}\right)^{5/2}\right) \exp\left\{-8\left(\frac{\eta}{\langle\eta\rangle}\right)^{5/2}\right\},$$
$$\int_0^\infty \varphi\, d\eta = \frac{2\Gamma^2(^4/_5)\langle\eta\rangle^2\sqrt{\pi}}{5\cdot 8^{4/5}\Gamma(1.3)}. \tag{36}$$

The distribution functions $f(\epsilon, z)$ and $F(\epsilon, T_e)$ are respectively equal to

$$f(\varepsilon, z) = \frac{8^{4/5}\Gamma(1,3)}{{}^2/_5\pi^{1/2}\Gamma^2(^4/_5)} \frac{\varepsilon}{T_c^2} K_0\left(8\left(\frac{\varepsilon}{T_e}\right)^{5/2}\right) \exp\left\{-8\left(\frac{\varepsilon}{T_e}\right)^{5/2}\right\},$$
$$F(\varepsilon, t) = N_0 z f(\varepsilon, z). \tag{37}$$

Computation of the quantity j_ν^T for the bremsstrahlung spectrum (33) with the aid of (36) leads to the expression

$$j_\nu^T = \frac{32\pi^2}{3\sqrt{6}} \frac{\Gamma(1.3)}{\Gamma^2(^4/_5)} \frac{N_0^2 e^6 z^3}{m^{3/2} c^3 T_e^{1/2}} \Gamma\left(0.1, 16\left(\frac{h\nu}{T_e}\right)^{5/2}\right), \tag{38}$$

where $\Gamma(x, y)$ is the incomplete gamma function.

In contrast to the known formula for the bremsstrahlung spectrum in the case when the electron distribution is Maxwellian, the expression (38) contains the function $\Gamma(0, 1, 16\,(h\nu/T_e)^{5/2})$ instead of the exponential factor $e^{-h\nu/T_e}$. When $16\,(h\nu/T_e)^{5/2} \gg 1$,

$$\Gamma(0.1,\ 16(h\nu/T_e)^{5/2}) \sim \exp\left\{-16\left(\frac{h\nu}{T_e}\right)^{5/2}\right\} \Big/ 16^{9/10}\left(\frac{h\nu}{T_e}\right)^{9/4},$$

i.e., the intensity of the radiation decreases with increase of the frequency much more rapidly than in the equilibrium case. The integrated power of the bremsstrahlung and the recombination radiation differs from the usual expressions only by numerical constants of the order of unity.

We note in conclusion that the expressions obtained in the present paper for the electron distribution function and the plasma parameters are based on the consideration of pair interactions between the plasma particles. However, as is well known, under certain conditions the important role in the plasma is played by collective processes allowance for which can, in principle, result in some changes in the electron distribution function. Although this problem has not as yet been fully investigated, the principal result of the theory developed here, namely, the deviation in strong fields of the state of the plasma from the thermodynamic equilibrium state, should remain valid. In the particular case when the effect of induced Compton scattering of light in the laser plasma is taken into account, the electron distribution function differs substantially from the Maxwellian[7].

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