KINETIC THEORY OF PARAMETRIC EXCITATION OF SURFACE WAVES IN A SEMI-FINITE PLASMA

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A theory of parametric excitation of quasistatic surface waves by a weak high-frequency electric field in a semi-finite plasma is developed. The case is considered when the external high-frequency field intensity vector is parallel to the plasma boundary, the field frequency is close to the natural frequency of the high-frequency quasistatic surface oscillations and reflection of particles from the boundary of the plasma is specular. A dispersion equation is derived which describes plasma instability with respect to surface wave buildup. Threshold values of the external field strength above which such instabilities arise are found and the oscillation increments in the near-threshold region are calculated.

1. At the present time it can be regarded as established that the anomalous character of the interaction of an external HF field with a plasma is due to parametric excitation of plasma oscillations[1]. In the kinetic theory of parametric resonance, developed in[2], the plasma was assumed to be unbounded and homogeneous. Under real conditions, however, the plasma always has boundaries, and in many cases, e.g., in the one considered below, the presence of the boundaries exerts a decisive influence on the excitation conditions and on the wave propagation in a parametrically unstable medium. The hydrodynamic theory of excitation of surface oscillations in a strong HF electric field, in which thermal motion of the particles and the non-potential character of the surface waves was completely neglected, was developed by Ferlenghi and one of the authors[3]. It is known, however, that parametric instabilities and the associated effects of anomalous interaction of HF fields with a plasma can develop already at relatively small values of the external-field intensity, when allowance for thermal motion of the particles is important. This is precisely why it is necessary to construct for parametric resonance in a bounded plasma a kinetic theory, that makes it possible, in particular, to find the threshold values of the intensity of the external HF field needed for the buildup of surface waves.

In the present paper we develop a kinetic theory of the dispersion properties of a semi-bounded plasma for the case when the field frequency is close to the natural frequence of the quasi-particle HF surface oscillations. It is well known[4] that the damping of a HF surface wave exceeds the frequency of the natural low-frequency surface oscillations. Therefore the parametric instabilities of the plasma with respect to buildup of surface waves have always a decay character, unlike the situation that can be realized when volume oscillations build up. Another distinguishing feature of a semi-bounded plasma is that the low-frequency surface oscillations are weakly damped only at relatively short wavelengths. Such singularities lead to rather large values of the threshold intensity of the external field needed for the buildup of the low-frequency mode of the natural surface oscillations. Because of this, the parametric interaction of the HF field with the semi-bounded plasma (in the frequency region under consideration) is determined completely by the instability against the buildup of high-frequency surface and non-natural low-frequency oscillations.

2. We consider a fully ionized plasma bounded by an infinite plane surface. The field intensity vector $\mathbf{E}_0$ of the HF electric field

$$\mathbf{E}_0(t) = \mathbf{E}_0 \sin \omega_0 t \tag{2.1}$$

is assumed to be parallel to the plane of the boundary. We consider henceforth HF fields ω_0 close to the frequency of the natural surface oscillations $\omega_p/\sqrt{2}$ (here $\omega_p^2 = \omega_{Le}^2 + \omega_{Li}^2$, $\omega_{L\alpha} = (4\pi e_\alpha^2 n_\alpha/m_\alpha)^{1/2}$ is the Langmuir frequency of particles of type α). At such values of ω_0, the external HF electromagnetic field penetrates into the plasma to a depth $l \sim c/\omega_p$. We shall therefore investigate below the excitation of short-wave oscillations with characteristic length λ of variation of the field normal to the boundary; this length is smaller than the penetration depth l, so that we can use for the external HF field the coordinate-independent expression (2.1). It is known[4] that the characteristic scale of variation of the field of the surface HF wave normal to the boundary coincides with the length of this wave along the boundary, which equals $2\pi/k_\parallel$ (here $k_\parallel$ is the wave number of the surface waves traveling along the plasma boundary). Thus, the condition of homogeneity of the HF field leads to the inequality $k_\parallel c > \omega_p$. The field of the HF surface wave is in this case almost potential.

To describe parametric excitation of almost longitudinal surface oscillations, we use a kinetic equation with a collision integral in the form proposed by Landau[5] for small perturbations of the distribution function δf_α of plasma particles of type α. We consider the case of specular reflection of the particles from the surface that bounds the plasma, when the solution of the system of kinetic equations and Maxwell's equations for the perturbation of the electric field in the plasma (which occupies the half-space $z > 0$) can be sought with the aid of the following continuation of the distribution function $\delta f_\alpha(\mathbf{r}, \mathbf{v}, t)$ of the electric field $\mathbf{E}(\mathbf{r}, t)$, and of the magnetic field $B(\mathbf{r}, t)$ into the region $z < 0$:

$$\delta f_\alpha(-z, -v_z) = \delta f_\alpha(z, v_z),$$
$$E_z(-z) = -E_z(z), \quad E_{x,y}(-z) = E_{x,y}(z), \tag{2.2}$$
$$B_z(-z) = B_z(z), \quad B_{x,y}(-z) = -B_{x,y}(z).$$

This makes it possible to use the ordinary Fourier transformation of the distribution functions and of the fields. Representing the electric and magnetic fields in the form of an expansion in the harmonics of the frequency of the external field (2.1) and eliminating from Maxwell's equation the Fourier component of the magnetic field, we obtain the following system of equations:

$$\left(k^2\delta_{rs} - k_r k_s - \frac{\omega_n^2}{c^2}\delta_{rs}\right) E_s^{(n)}(\mathbf{k}) - i\frac{4\pi\omega_n}{c^2} j_r^{(n)}(\mathbf{k}) \tag{2.3}$$
$$= 2ie_{rzs}B_s^{(n)}(z=0)\frac{\omega_n}{c}, \quad \omega_n = \omega + i\gamma + n\omega_0.$$

Here e_{rsp} is an absolutely antisymmetrical unit tensor of third rank, $\mathbf{j}^{(n)}(\mathbf{k})$ is the n-th harmonic of the perturbation of the electric field

$$\mathbf{j}(\mathbf{k}, t) \equiv \sum_\alpha e_\alpha \int d\mathbf{v}_\alpha \mathbf{v}_\alpha \delta f_\alpha(\mathbf{k}, \mathbf{v}_\alpha, t) = \sum_{n=-\infty}^{\infty} \exp(-i\omega_n t)\,\mathbf{j}^{(n)}(\mathbf{k}),$$

for which we can obtain from the system of kinetic equations the following expression

$$j_r^{(n)}(k) = -\frac{i}{4\pi}\sum_\alpha \sum_{m,p=-\infty}^{\infty} J_{n-m}(\mathbf{k}\mathbf{r}_{E,\alpha}) J_{p-m}(\mathbf{k}\mathbf{r}_{E,\alpha})$$
$$\times \left\{\omega_m \mathscr{E}_{s,\alpha}^{(p)} \left[\left(\delta_{rs} - \frac{k_r k_s}{k^2}\right)\delta\varepsilon_\alpha^{tr}(\omega_m, \mathbf{k}) + \frac{k_r k_s}{k^2}\delta\varepsilon_\alpha^{l}(\omega_m, \mathbf{k})\right]\right.$$
$$\left. + \frac{v_{E,\alpha}^r}{(\mathbf{k}\mathbf{v}_{E,\alpha})}\mathbf{k}\vec{\mathscr{E}}_\alpha^{(p)}\delta\varepsilon_\alpha^{l}(\omega_m, \mathbf{k})(\omega_n - \omega_m)\right\}. \tag{2.4}$$

Here $\mathbf{v}_{E,\alpha} = e_\alpha \mathbf{E}_0/m_\alpha\omega_0$ is the amplitude of the velocity of the oscillations of particles of type α in the external HF field; $\delta\epsilon_\alpha^{tr}(\omega, \mathbf{k})$ and $\delta\epsilon_\alpha^{l}(\omega, \mathbf{k})$ is the partial contribution of the particles of type α to the transverse and longitudinal dielectric constant, respectively:

$$\varepsilon_\alpha^{tr}(\omega, \mathbf{k}) = 1 + \sum_\alpha \delta\varepsilon_\alpha^{tr}(\omega, \mathbf{k}), \quad \varepsilon^{l}(\omega, \mathbf{k}) = 1 + \sum_\alpha \delta\varepsilon_\alpha^{l}(\omega, \mathbf{k}),$$
$$\vec{\mathscr{E}}_\alpha^{(n)}(\mathbf{k}) = \mathbf{E}^{(n)}(\mathbf{k}) - \frac{1}{2c}[\mathbf{v}_{E,\alpha}, \mathbf{B}^{(n+1)}(\mathbf{k}) + \mathbf{B}^{(n-1)}(\mathbf{k})], \tag{2.5}$$

$J_n(\mathbf{k}, \mathbf{r}_{E,\alpha})$ is a Bessel function, and $\mathbf{r}_{E,\alpha} = \mathbf{v}_{E,\alpha}/\omega_0$. Solving the system of equations (2.3) and (2.4), we obtain the electric field in the plasma $\mathbf{E}(\mathbf{r}, t)$ with the aid of the inverse Fourier transform:

$$\mathbf{E}^{(n)}(\mathbf{k}_\parallel, z) = \int_{-\infty}^{\infty} \frac{dk_z}{2\pi}\exp(-ik_z z)\mathbf{E}^{(n)}(\mathbf{k}),$$

($\mathbf{k} = \{\mathbf{k}_\parallel, k_z\}$), expressed in terms of the values of the fields on the boundary ($z = +0$). Assuming that the plasma is bounded by a medium ($z < 0$) with a dielectric constant $\epsilon_0 > 0$[1], we obtain from Maxwell's equations the following connection between the values of the field on the left of the boundary ($z = -0$):

$$e_{ijs}q_j^{(n)}E_s^{(n)}(z=-0) = \frac{\omega_n}{c}B_i^{(n)}(z=-0),$$
$$\mathbf{q}^{(n)}\mathbf{E}^{(n)}(z=-0) = 0, \quad \mathbf{q}^{(n)} = \mathbf{k}_\parallel + i\mathbf{e}_z\varkappa^{(n)}, \quad \varkappa^{(n)} = (k_\parallel^2 - \varepsilon_0\omega_n^2/c^2)^{1/2}. \tag{2.6}$$

Here $\mathbf{e}_z$ is a unit vector along the z axis. Using these conditions, it is easy to show that the right-hand side of (2.3) takes the form

$$2i\frac{\omega_n}{c}e_{izs}B_s^{(n)}(z=-0)$$
$$= \frac{2}{\varkappa^{(n)}}[q_i^{(n)}(\mathbf{k}_\parallel\mathbf{E}^{(n)}(z=-0)) - (\varkappa^{(n)})^2E_i^{(n)}(z=-0)]. \tag{2.7}$$

We note that the tangential components of the harmonics of the electric field are continuous. In the considered case of quasistatic oscillations ($k_\parallel c > \omega_p$), the nonpotential corrections to the electric field of the excited waves are small. In addition, the intensity of the HF electric field will be assumed below to be relatively small ($|\mathbf{k}\cdot\mathbf{r}_{E,e}| \ll 1$). When calculating the small nonpotential corrections we can therefore assume the external field to be equal to zero. Since the minimum of the threshold intensity of the HF field and the maximum of the increment are reached for wave vectors $\mathbf{k}_\parallel \parallel \mathbf{E}_0$, it becomes possible to neglect the second term in (2.5). Thus, accurate to terms that are quadratic in the amplitude of the external HF field, we obtain from (2.3) and (2.4) the following system of equations:

$$(\mathbf{k}_\parallel\mathbf{E}^{(n)}(\mathbf{k}))\{\varepsilon^l(\omega_n, \mathbf{k}) + {}^1/_4(\mathbf{k}_\parallel\mathbf{r}_{E,e})^2[-2\delta\varepsilon_e^l(\omega_n, \mathbf{k}) + \delta\varepsilon_e^l(\omega_{n+1}, \mathbf{k})$$
$$+ \delta\varepsilon_e^l(\omega_{n-1}, \mathbf{k})]\} - {}^1/_2\mathbf{k}_\parallel\mathbf{r}_{E,e}(\mathbf{k}_\parallel\mathbf{E}^{(n+1)}(\mathbf{k}))[\delta\varepsilon_e^l(\omega_n, \mathbf{k}) - \delta\varepsilon_e^l(\omega_{n+1}, \mathbf{k})]$$
$$- {}^1/_2\mathbf{k}_\parallel\mathbf{r}_{E,e}(\mathbf{k}_\parallel\mathbf{E}^{(n-1)}(\mathbf{k}))[\delta\varepsilon_e^l(\omega_{n-1}, \mathbf{k}) - \delta\varepsilon_e^l(\omega_n, \mathbf{k})] = (\mathbf{k}_\parallel\mathbf{E}^{(n)}(z=+0))\frac{\varepsilon_0}{\varkappa^{(n)}}$$
$$\times\left[\frac{k_\parallel^2}{k^2} - \frac{\varepsilon^l(\omega_n, \mathbf{k})\omega_n^2k_z^2}{c^2k^4}\right]. \tag{2.8}$$

We shall consider below a case when the frequency of the external HF field ω_0 is close to the natural frequency of the HF surface waves, and the frequency ω and the increment γ of the excited oscillations are small in comparison with ω_0. In this case the only harmonics that are not small are $(0, \pm 1)$. Retaining only such terms in the system (2.8), we can obtain, with allowance for the continuity of the tangential components of the electric-field harmonics, the following dispersion equation:

$$D_0 = \frac{1}{4}(\mathbf{k}_\parallel\mathbf{r}_{E,e})^2\left[\frac{1}{D_1} + \frac{1}{D_{-1}}\right]\frac{k_\parallel}{\varepsilon_0\pi}\int_{-\infty}\frac{dk_z}{k^2}$$
$$\times\frac{(1+A)\delta\varepsilon_i^l(\omega+i\gamma, \mathbf{k})[1+\varepsilon_0+\delta\varepsilon_e^l(\omega+i\gamma, \mathbf{k})]}{\varepsilon^l(\omega+i\gamma, \mathbf{k})}$$
$$D_n \equiv 1 + \frac{\varepsilon_0}{\pi\varkappa^{(n)}}\int_{-\infty}^{\infty}dk_z\left[\frac{k_\parallel^2}{k^2\varepsilon^l(\omega_n, \mathbf{k})} - \frac{\omega_n^2k_z^2}{c^2k^4}\right]$$
$$A \equiv \frac{k_\parallel}{\pi}\int_{-\infty}\frac{dk_z}{k^2\varepsilon^l(\omega+i\gamma, \mathbf{k})} -$$
$$-\frac{k_\parallel}{\pi\delta\varepsilon_i^l(\omega+i\gamma, \mathbf{k})}\int_{-\infty}\frac{dk_z\delta\varepsilon_i^l(\omega+i\gamma, \mathbf{k})}{k^2\varepsilon^l(\omega+i\gamma, \mathbf{k})} \tag{2.9}$$

To simplify the subsequent calculations, we put $\epsilon_0 = 1$. We note that the need for taking into account small nonpotential corrections in the resonant denominators $D_{\pm 1}$ is connected with the fact that they describe the spatial dispersion of the surface high-frequency oscillations. This dependence turns out to be appreciable

[1] It assumed that the space-time dispersion of the dielectric constant ϵ_0 is negligible.

in the considered case of resonance ($\omega_0 \approx \omega_p/\sqrt{2}$). The dispersion equation (2.9) has two types of unstable solutions ($\gamma > 0$). One of them is characterized by the frequency $\omega = 0$ and will henceforth be called aperiodic. For the other, periodic type of oscillations the frequency is $\omega \gtrsim \gamma$.

3. We consider first the case of the aperiodic instability ($\omega = 0$). Since the high-frequency surface installations are weakly damped only for $k_{\parallel} r_{De} < 1$, the quantities $D_{\pm 1}$ and the dispersion equation (2.9) can be greatly simplified in this limit:

$$D_{\pm 1} = 1 + \left[1 - \frac{\omega_p^2}{(\omega + i\gamma \pm \omega_0)^2}\right]^{-1} + 4C_1 k_{\parallel} r_{De} - 4iC_2 k_{\parallel} r_{De} - \frac{\omega_p^2}{8k_{\parallel}^2 c^2}. \quad (3.1)$$

The spectrum of the surface waves, in which the constants C_1 and C_2 enter, was investigated by Romanov[4]. These coefficients, calculated at $\epsilon_0 = 1$ with high accuracy, are equal to[2)]

$$C_1 = 1{,}22; \quad C_2 = 0{,}176. \quad (3.2)$$

Being interested in the case of neo-threshold instability, when $\gamma \ll \nu_{ei}$, $k_{\parallel}^2 v_{Te}^2/\nu_{ei}$, $k_{\parallel} v_{Ti}$, we use the following expressions for the partial contributions to the dielectric constant of the particles of type α:

$$\delta\varepsilon_i^l(\omega, k) \approx \frac{1}{k^2 r_{Di}^2}\left(1 + i\left(\frac{\pi}{2}\right)^{1/2} \frac{\omega}{k v_{Ti}}\right),$$
$$\delta\varepsilon_e^l(\omega, k) \approx \frac{1}{k^2 r_{De}^2}\left(1 + i\varphi\left(\left|\frac{e}{e_i}\right|\right) \frac{\omega \nu_{ei}}{k^2 v_{Te}^2}\right). \quad (3.3)$$

Here φ is the smooth function of the charge ratio, with $\varphi \approx 1.44$ at $|e_i| = |e|$ and $\varphi \approx 32/3\pi$ at $|e_i| \gg |e|$. Substituting (3.3) in the dispersion equation (2.9) and calculating the integrals that enter in it, we obtain the increment and the limiting value of the intensity of the HF electric field:

$$\gamma = \frac{\omega_0}{32\pi k_{\parallel}^2} \frac{(\mathbf{k}_{\parallel}\mathbf{E}_0)^2 - (\mathbf{k}_{\parallel}\mathbf{E}_0)_{\lim}^2}{n_e T_e + n_i T_i}\left(1 + \frac{4}{3(2\pi)^{1/2}} \frac{\tilde{\gamma}}{k_{\parallel} v_{Ti}} \frac{r_{Di}^2}{r_{De}^2 + r_{Di}^2}\right)^{-1},$$
$$\frac{(\mathbf{k}_{\parallel}\mathbf{E}_0)_{\lim}^2}{4\pi k_{\parallel}^2 (n_e T_e + n_i T_i)} = -4\frac{\Delta^2 + \tilde{\gamma}^2}{\omega_0 \Delta}; \quad (3.4)$$

$$\Delta = \omega_0 - 2^{-1/2}\omega_p(1 + C_1 k_{\parallel} r_{De} - \omega_p^2 / 8k_{\parallel}^2 c^2), \quad (3.5)$$

$$\tilde{\gamma} = C_2 \frac{\omega_p}{\sqrt{2}} k_{\parallel} r_{De} + \frac{\nu_{ei}}{2}, \quad \nu_{ei} = \frac{4}{3} \frac{(2\pi)^{1/2} e^2 e_i^2 n_i L}{m_e^{1/2} T_e^{3/2}}. \quad (3.5')$$

Here ν_{ei} is the frequency of the electron-ion collisions, and $\mathbf{E}_{0,\lim}(\mathbf{k}_{\parallel})$ is the value of the HF field intensity at which $\gamma(\mathbf{E}_0, \mathbf{k}_{\parallel}) = 0$. As seen from (3.4) and (3.5), the aperiodic instability exists in the region of frequency deviations $\delta(\delta = 1 - \omega_p/2^{1/2}\omega_0)$ and wave vectors $\mathbf{k}_{\parallel}$ satisfying the condition $\Delta < 0$. It is easy to see from (3.4) that the maximum of the increment is reached for wave vectors $\mathbf{k}_{\parallel}$ which are collinear with $\mathbf{E}_0$. At a specified value of the detuning δ there are excited on the threshold oscillations whose wave vector satisfies, accurate to quantities of the order of $(C_2/C_1)^2$, the equation

$$(k_{\parallel} r_{De})^3 - \tilde{\delta}(k_{\parallel} r_{De})^2 - \frac{1}{8(C_1 - C_2)} \frac{v_{Te}^2}{c^2} = 0,$$
$$\tilde{\delta} = \frac{1}{C_1 - C_2}\left(\delta + \frac{\nu_{ei}}{2\omega_0}\right). \quad (3.6)$$

Substituting the solution of (3.6) in (3.4) and (3.5), we can obtain, respectively, the maximum value of the near-threshold increment and the minimum limiting (threshold) value of the HF field intensity. For frequency deviations satisfying the inequalities

$$1 > |\tilde{\delta}| > {}^1/_2 (v_{Te}/c)^{2/3}, \quad (3.7)$$

the threshold instability of the HF field is determined by the expression

$$\frac{E_{0,\text{thr}}^2}{4\pi(n_e T_e + n_i T_i)} = 8\left[C_2 \tilde{\delta} + \frac{\nu_{ei}}{2\omega_0}\right], \quad \tilde{\delta} > 0. \quad (3.8)$$

Near the threshold there are excited oscillations with wavelength

$$(k_{\parallel} r_{De})_{\text{thr}} = \tilde{\delta}. \quad (3.9)$$

Where negative values of the frequency deviations, $\tilde{\delta} < 0$, satisfying (3.7), we obtain

$$\frac{E_{0,\text{thr}}^2}{4\pi(n_e T_e + n_i T_i)} = 4\left\{\frac{C_2}{(C_1 - C_2)^{1/2}} \frac{v_{Te}}{c(-\tilde{\delta})^{1/2}} + \frac{\nu_{ei}}{\omega_0}\right\},$$
$$(k_{\parallel} r_{De})_{\text{thr}} \approx (8(C_1 - C_2))^{-1/2} v_{Te} / c(-\tilde{\delta})^{1/2}. \quad (3.10)$$

Finally, for small deviations

$$|\tilde{\delta}| < {}^1/_2 (v_{Te}/c)^{2/3}. \quad (3.11)$$

The values of $E_{0,\text{thr}}$ and $k_{\parallel,\text{thr}}$ are determined from the equations

$$\frac{E_{0,\text{thr}}^2}{4\pi(n_e T_e + n_i T_i)} = 4\left\{\frac{C_2}{(C_1 - C_2)^{1/3}}\left(\frac{v_{Te}}{c}\right)^{2/3} + \frac{\nu_{ei}}{\omega_0}\right\},$$
$$(k_{\parallel} r_{De})_{\text{thr}}^3 = \frac{v_{Te}^2}{8(C_1 - C_2)c^2}. \quad (3.12)$$

The maximum of the increment (3.4) is reached at $k_{\parallel} \approx k_{\parallel,\text{thr}}$ and $E_{0,\lim} \approx E_{0,\text{thr}}$, where $k_{\parallel,\text{thr}}$ and $E_{0,\text{thr}}$ are determined from (3.8)–(3.12).

As seen from the derived expressions, the dependence of the threshold HF field intensity on the frequency deviation differs considerably from that obtaining for the aperiodic instability against the buildup of volume oscillations[2]. This difference is due both to the specific character of the damping of the surface waves and to allowance for the nonpotential behavior, which significantly influences the dispersion of the excited high-frequency oscillations. With decreasing ω_0 in the resonance region, the value of $E_{0,\text{thr}}$ decreases. At small negative $\tilde{\delta}$ we have for $E_{0,\text{thr}}$ (3.10), in order of magnitude,

$$\frac{E_{0,\text{thr}}^2}{16\pi(n_e T_e + n_i T_i)} \sim C_2 \frac{v_{Te}}{c} + \frac{\nu_{ei}}{\omega_0}. \quad (3.13)$$

We see thus that, for example for the plasma parameters used in experiments on the interaction of HF fields with plasma[6], the threshold HF field intensity in the entire range of frequency deviations is determined completely by the collisionless damping of the surface oscillations. We note that for a correct investigation of parametric resonance in the region of external-field frequencies ω_0 much smaller than $\omega_p/\sqrt{2}$, where the surface oscillations are almost transverse and long-wave with a wavelength on the order of the dimensions of the skin layer l ($l \approx c\omega_p$), it is necessary to take into account the inhomogeneity of the external HF field.

4. Proceeding to analyze the periodic ($\omega \gtrsim \gamma$) in-

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2) The authors are grateful to N. E. Andreev for help in the numerical calculations of the coefficients C_1 and C_2.

stability, let us investigate first the possibility of the buildup of natural low-frequency surface oscillations. We note that the parametric instability corresponding to the decay of an external field into high-frequency and low-frequency surface waves is impossible since the damping decrement $\tilde{\gamma}$ (3.5) greatly exceeds the frequency of the natural ion-acoustic surface oscillations. We consider therefore the non-decay periodic instability, using the dispersion equation (2.9). Substituting in this equation the following expressions for the dielectric constants (kv_{Ti}, $(k^2v_{Ti}^2\nu_{ii})^{1/3} \ll |\omega + i\gamma| \ll kv_{Te}$):

$$\delta\varepsilon_e'(\omega,\mathbf{k}) = 1 + \frac{1}{k^2 r_{De}^2}\left(1 + i\left(\frac{\pi}{2}\right)^{1/2}\frac{\omega}{kv_{Te}}\right), \quad (4.1)$$

$$\delta\varepsilon_i'(\omega,\mathbf{k}) = -\frac{\omega_{Li}^2}{\omega^2}\left(1 - i\frac{8}{5}\frac{\nu_{ii}k^2v_{Ti}^2}{\omega^3}\right),$$

$$\nu_{ii} = \frac{4}{3}\frac{\pi^{1/2}e_i^4 n_i L}{m_i^{1/2}T_i^{3/2}},$$

separating the real and imaginary parts, and assuming ω to be close to the frequency of the ion-acoustic surface oscillations, we obtain

$$\omega = \omega_s + \frac{(\mathbf{k}_\parallel \mathbf{r}_{E,e})^2}{4}\frac{\omega_0\Delta}{k_\parallel^4 r_{De}^4}\frac{[k_\parallel^5 r_{De}^5\omega_{Li}(\Delta^2+\tilde{\gamma}^2) - 2\omega_s\tilde{\gamma}\gamma_s]}{(\Delta^2+\tilde{\gamma}^2)^2}, \quad (4.2)$$

$$\gamma = -\gamma_s + \frac{(\mathbf{k}_\parallel \mathbf{r}_{E,e})^2}{4}\frac{\omega_0\Delta}{k_\parallel^4 r_{De}^4}\frac{[\gamma_s(\Delta^2+\tilde{\gamma}^2) + 2\omega_{Li}^2 k_\parallel^6 r_{De}^6\tilde{\gamma}]}{(\Delta^2+\tilde{\gamma}^2)^2}. \quad (4.3)$$

In the long-wave limit $k_\parallel r_{De} \ll 1$ of interest to us, the frequency of the natural ion-acoustic surface oscillations of a non-isothermal ($T_e \gg T_i$) plasma is $\omega_s = \omega_{Li}k_\parallel r_{De}$. In the region

$$(k_\parallel r_{De})^4 \gg \omega_{Li}/\omega_{Le} \quad (4.4)$$

such oscillations are weakly damped and have a decrement

$$\gamma_s = \left(\frac{\pi}{8}\right)^{1/2}\frac{\omega_{Li}}{\omega_{Le}}\omega_s + \frac{4}{5}\frac{r_{Di}^2}{r_{De}^2}\nu_{ii}. \quad (4.5)$$

It is seen from (4.3) that instability is possible only when $\Delta > 0$. Taking the condition (4.4) into account, we obtain a limitation on the frequency-deviation region in which the instability in question is possible:

$$1 > \delta > (\omega_{Li}/\omega_{Le})^{1/4} \quad (4.6)$$

Putting $\gamma = 0$ in (4.3) and minimizing with respect to the wave number $k_\parallel$ the resultant expression for $E_{0,\lim}(k_\parallel)$, we arrive at the following expression for $E_{0,\mathrm{thr}}$ and $k_{\parallel,\mathrm{thr}}$:

$$\frac{E_{0,\mathrm{thr}}^2}{4\pi n_e T_e} = \frac{4}{3}\left(\frac{\pi}{3}\right)^{1/2}\frac{C_2^2 C_1}{\delta}, \qquad (k_\parallel r_{De})_{\mathrm{thr}} \approx \frac{\delta}{C_1}. \quad (4.7)$$

The obtained value of $E_{0,\mathrm{thr}}$ exceeds the corresponding quantity in (3.8) for the buildup of the aperiodic instability.

Let us consider the region of frequency deviations for which an inequality inverse to (4.6) is satisfied. Recognizing that in this case the frequency ω greatly exceeds ω_s, and using for the dielectric constants the expression (4.1), we obtain from (2.9) the following dispersion equation:

$$\frac{(\omega+i\gamma)^2}{\omega_{Li}^2} + i\frac{\omega_s(\omega+i\gamma)}{\omega_{Li}^2} - \frac{(\mathbf{k}_\parallel\mathbf{r}_{E,e})^2}{8}\frac{\omega_0\Delta}{\Delta^2 - (\omega+i\gamma+i\tilde{\gamma})^2} = 0. \quad (4.8)$$

From this we can easily find an expression for the spectrum of the unstable oscillations:

$$\omega^2 = \frac{(\mathbf{k}_\parallel\mathbf{r}_{E,e})^2}{8}\frac{\omega_0\Delta\omega_{Li}^2}{\Delta^2+\tilde{\gamma}^2}, \quad (4.9)$$

$$\gamma = -\frac{\omega_s}{2} + \frac{(\mathbf{k}_\parallel\mathbf{r}_{E,e})^2}{8}\frac{\omega_0\Delta\tilde{\gamma}\omega_{Li}^2}{(\Delta^2+\tilde{\gamma}^2)^2}. \quad (4.10)$$

Assuming $\gamma = 0$, we obtain from (4.9) and (4.10) for $E_{0,\lim}$ and $\omega_{\lim}$:

$$\omega_{\lim}^2 = \frac{\omega_s}{2\tilde{\gamma}}(\Delta^2+\tilde{\gamma}^2),$$

$$\frac{(\mathbf{k}_\parallel\mathbf{E}_0)_{\lim}^2}{4\pi\mathbf{k}_\parallel^2 n_e T_e} = \frac{(\Delta^2+\tilde{\gamma}^2)^2}{\omega_s\tilde{\gamma}\omega_0\Delta}. \quad (4.11)$$

Minimizing $E_{0,\lim}$ with respect to $k_\parallel$, we obtain the following relation for the threshold intensity of the HF field:

$$\frac{E_{0,\mathrm{thr}}^2}{4\pi n_e T_e} = \frac{16}{3\sqrt{3}}\frac{\tilde{\gamma}^2(k_{\parallel,\mathrm{thr}})}{\omega_0\omega_s(k_{\parallel,\mathrm{thr}})}, \quad (4.12)$$

$$\omega^2(k_{\parallel,\mathrm{thr}}) = {}^2/_3\,\omega_s(k_{\parallel,\mathrm{thr}})\tilde{\gamma}(k_{\parallel,\mathrm{thr}}),$$

while $(k_\parallel r_{De})_{\mathrm{thr}}$ as a function of $\tilde{\delta}$, accurate to quantities of $(C_2/C_1)^2$, is obtained from the equation

$$\Delta = \tilde{\gamma}/3^{1/2}. \quad (4.13)$$

Thus, for example, under conditions when the inequality

$$\nu_{ei}/\omega_0 > C_2 v_{Te}/c \quad (4.14)$$

is satisfied, the expression for $E_{0,\mathrm{thr}}$ determined by formula (4.12) reaches the minimum value

$$\min\frac{E_{0,\mathrm{thr}}^2}{4\pi n_e T_e} = \frac{32C_2}{3\sqrt{3}}\frac{\nu_{ei}}{\omega_{Li}} \quad (4.15)$$

at a frequency deviation

$$\delta \approx \nu_{ei}(3^{1/2}C_1 + 2C_2)/2C_2\omega_0 \quad (4.16)$$

and a corresponding value of the wave number

$$(k_\parallel r_{De})_{\mathrm{thr}} = \nu_{ei}/2C_2\omega_0. \quad (4.17)$$

At the frequency deviation (4.16) we obtain from (4.10) the following expression for the maximum increment in the near-threshold field region (4.15)

$$\gamma_{max} = \frac{3\sqrt{3}}{128C_2^2}\frac{\omega_{Li}^2}{\omega_0}\frac{E_0^2 - E_{0,\mathrm{thr}}^2}{4\pi n_e T_e}, \quad (4.18)$$

Thus, it follows from the foregoing analysis that only the non-natural low-frequency branches are excited when the surface oscillations build up, unlike the case of excitation of volume oscillations in a non-isothermal plasma, when the decay form of parametric instability can be realized. We note, finally, that in the limit of sufficiently large HF field intensities, when the dissipative effects are negligible, we obtain from (4.8) a dispersion equation that differs from the corresponding results of the preceding paper[3] in that both thermal motion and the small non-potentiality of the high-frequency oscillations are taken into account.

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