# Emission by Charged Particles in the Field of a Plane Electromagnetic Wave in a Medium

**V. M. Arutyunyan and G. K. Avetisyan**

*Institute of Physical Research, Academy of Sciences, Armenian SSR*

Submitted July 28, 1971

Zh. Eksp. Teor. Fiz. 62, 1639–1647 (May, 1972)

The emission by charged particles in the field of a plane electromagnetic wave in a medium with a refractive index $n_0 > 1$ is considered. The features of "spontaneous" emission by a particle in a wave capture regime are investigated. Noncoherent emission at the frequency of the fundamental wave is strongly suppressed in comparison with the ordinary Cerenkov radiation. An initially homogeneous particle beam is modulated in the field of a linearly polarized wave. The self-consistent problem is used to show that the energy lost by decelerating particles (for the case $v_0 > c/n_0$) is forcedly transferred to the wave, which is amplified as the result. Various conditions of light amplification in a medium with constant $n_0$ are determined. Numerical computation shows that over the length of $\sim 1$ cm the radiation transferred to the wave exceeds (by an order) the power of the fundamental wave.

1. According to earlier papers[1-4] a reflection and capture of a particle by the wave takes place in a medium with a refraction index $n_0 > 1$ when a charged particle interacts with a plane electromagnetic wave whose field exceeds a certain critical value. Thus an "external" (in relation to the wave) particle cannot penetrate the wave, while an "internal" particle cannot escape from the wave. After reflection the "external" particle accelerates its initial velocity is less than the phase velocity of light ($v_0 < c/n_0$), or decelerates (if $v_0 > c/n_0$), releasing energy to the wave in the form of stimulated Cerenkov radiation. The "internal" particle oscillates about the equilibrium phase and moves along with the wave on the average; $\langle v_x \rangle = c/n_0, \langle v_y \rangle = 0$, i.e., it periodically loses and acquires energy within the wave. If the wave is linearly polarized the frequency of these oscillations depends on the initial phase of the particle, so that the initially homogeneous beam becomes modulated. In the case of circular polarization this frequency is constant so that all particles are simultaneously either accelerated or decelerated. In this manner one can amplify light (or accelerate a particle) in such a case even in a medium with constant $n_0$ (the theory of such an amplification is fully analogous to that of a quantum amplifier).

Since all particles coherently emit into the wave upon deceleration, the gain depends on the beam density and if the latter is high the incremental energy of the wave may exceed the energy of the fundamental emission. Relatively high gain is possible if the refractive index is varied along the direction of wave propagation. Light is thus continuously amplified in the direction of increasing $n(x)$ (and the particles are accelerated in the direction of decreasing $n(x)$). As for the "spontaneous" emission by a particle in the capture mode, coherence is disrupted since the particle velocity oscillates within the wave leading to a strong damping of ordinary Cerenkov radiation at the fundamental wave frequency[5,6].

2. We consider emission in a medium by a particle captured by a plane electromagnetic wave. Let the wave propagate along the $x$ axis. Then the particle equations of motion are

$$\frac{dp_x}{dt} = \frac{e}{c} n_0 (v_y E_y + v_z E_z),$$

$$\frac{dp_y}{dt} = e\left(1 - n_0 \frac{v_x}{c}\right) E_y, \qquad \frac{dp_z}{dt} = e\left(1 - n_0 \frac{v_x}{c}\right) E_z, \tag{2.1}$$

where $n_0$ is the refractive index of the medium at the fundamental wave frequency $\omega_0$. We assume initially that the wave is linearly polarized along the $y$ axis:

$$E = E_y = E_0 \cos(\omega_0 n_0 x / c - \omega_0 t). \tag{2.2}$$

We also assume that the initial velocity of the particle is directed along $x$ and differs little from the Cerenkov velocity:

$$v_0 = v_{0x} = c(1 + \mu) / n_0, \quad \mu \ll 1. \tag{2.3}$$

It follows directly from (2.1) that for $\mu = 0$ we have

$$v_x = v_{0x} = c / n_0, \; v_y = 0, \; x = x_0 + ct / n_0$$

($x_0$, $y_0$, and $z_0$ define the position of the particle for $t = 0$ and $y_0 = z_0 = 0$). For $\mu \ll 1$ we represent the solution of (2.1) in the form $v_x = c(1 + \mu u_x)/n_0$, $v_y = c\mu u_y$. Linearizing (2.1) with respect to $\mu$ we find

$$u_x' = \frac{e(n_0^2 - 1)^{3/2}}{mcn_0^2} E_0 \cos \Phi_0 u_y,$$

$$u_y' = -\frac{e(n_0^2 - 1)^{1/2}}{mc} E_0 \cos \Phi_0 u_x,$$

$$\Phi_0 = \omega_0 n_0 x_0 / c,$$

which we solve for the initial conditions $u_{0x} = 1$ and $u_{0y} = 0$. Thus we find the velocity of the particle in the capture mode:

$$v_x = \frac{c}{n_0}(1 + \mu \cos \Omega t),$$

$$v_y = -\frac{c}{(n_0^2 - 1)^{1/2}} \mu \sin \Omega t, \tag{2.4}$$

$$\Omega = e(n_0^2 - 1) E_0 |\cos \Phi_0| / mcn_0. \tag{2.5}$$

The computation is based on the approximation

$$\mu\omega_0 / \Omega \ll 1, \tag{2.6}$$

which does not hold for $\cos \Phi_0 = 0$. The stability in the capture mode (2.4) is readily understood if we change to a system of coordinates that travels with the particle. In such a system $E' = 0$ and $B' = B(n_0^2 - 1)^{1/2}/n_0$, so that for small deviations from the equilibrium position the particle revolves about field $B'$. Since stability is provided by the magnetic field $B'$ the point $\cos \Phi_0$

$= 0$ $(B' = 0)$ is obviously unstable.

According to (2.4) the velocity of the particle oscillates within the wave at a frequency $\Omega$ that depends on the initial phase $\Phi_0$. If we have a beam of particles, then different particles fall into the well with different phases and have different frequencies. This means that the beam becomes spatially modulated, resulting in modulation of current density. According to (2.5) the modulation frequency is

$$\Omega \approx \omega_0(n_0^2 - 1)\xi|\cos\Phi_0|,$$

where $\xi = eE_0/mc\omega_0$ is the relativistically invariant parameter of intensity. Since it is realistic to expect that $\xi \ll 1$ even for high-power lasers, it follows that $\Omega \ll \omega_0$.

The intensity of radiation from a charge in a medium at a frequency $\omega$, in a frequency interval $d\omega$ and solid angle do, is given by the expression[7]*

$$dI(\omega) = \frac{e^2 n(\omega)}{4\pi^2 c^3}\omega^2 d\omega\, do \left| \int_{-\infty}^{+\infty} [\nu v] e^{i\mathbf{kr}(t) - i\omega t} dt \right|^2,$$

where $\mathbf{k} = n(\omega)\omega\boldsymbol{\nu}/c$, $\boldsymbol{\nu}$ is the unit vector along the wave vector of the radiation, and $n(\omega)$ is the refractive index at the frequency $\omega$. Assuming a linear approximation in $\mu$ (and also considering that $\mu\omega/\Omega \ll 1$) and using (2.4) we obtain

$$dI(\omega) = dI_0(\omega) + dI_+(\omega) + dI_-(\omega), \quad (2.7)$$

$$dI_0(\omega) = \frac{e^2}{cn_0}\left(1 - \frac{n_0^2}{n^2(\omega)}\right)\omega\, d\omega, \quad (2.8)$$

$$dI_\pm(\omega) = \mu^2 \frac{e^2}{4c}\frac{1}{n_0(n_0^2 - 1)} \cdot \left\{ n_0^2 + \frac{n_0^2 + n^2(\omega) - 2}{2}\left[\frac{\omega^2}{\Omega^2} - \frac{n_0^2}{n^2(\omega)}\left(1 \mp \frac{\Omega}{\omega}\right)^2\right]\right\}\omega d\omega, \quad (2.9)$$

and in (2.9)

$$\omega = \pm \frac{\Omega}{1 - n(\omega)\cos\theta/n_0}, \quad (2.10)$$

where $\theta$ is the angle between the emission direction and the x axis. The function $dI_0(\omega)$ corresponds to Cerenkov radiation by a particle moving with the velocity $v = c/n_0$ inside the wave. The function $dI_\pm(\omega)$ determines the radiation due to oscillatory motion.

The approximation $\mu\omega/\Omega \ll 1$ used in the computation, which is equivalent to (2.6), is necessary for the analytic solution of the problem (in the general case the particle velocity is expressed by elliptic integrals, making it difficult to obtain an analytic solution).

Since $\Omega$ depends on the initial phase of the particle, in the case of a particle beam in a linearly polarized wave a whole frequency spectrum is radiated at every angle $\theta$, rather than a single frequency as in the ordinary Cerenkov radiation. We compare the radiation at the fundamental wave frequency $\omega_0$ with the ordinary Cerenkov radiation at the same frequency (without the field). The function $dI_0(\omega)$ vanishes at $\omega = \omega_0$ and in $dI_-(\omega)$ the conservation law breaks down at the frequency $\omega_0$. We obtain from (2.9) for $n(\omega) = n_0$

$$dI_+(\omega_0) = \frac{e^2}{2cn_0}\mu^2\frac{\omega_0}{\Omega}\omega_0 d\omega. \quad (2.11)$$

If $v = c(1 + \mu)/n_0$ is substituted into the Tamm-Frank formula and the approximation linear in $\mu$ is used, we obtain

$$dI(\omega_0) = \frac{2e^2}{cn_0}\mu\omega_0 d\omega. \quad (2.12)$$

A comparison of (2.11) with (2.12) shows that the emission by a particle at the frequency $\omega_0$ in the capture mode is much weaker than the ordinary Cerenkov radiation (since $\mu\omega_0/\Omega \ll 1$). This reduction of emission is due to disruption of coherence, owing to oscillation of the particle velocity inside the wave. The fundamental frequency $\omega_0$ in the capture mode is emitted at the angle $\theta \approx (2\Omega/\omega_0)^{1/2}$. The ordinary Cerenkov angle $\theta_C \approx (\mu/2)^{1/2}$ and since $\mu \ll \Omega/\omega_0$ we get $\theta \gg \theta_C$, i.e., the angle of emission of the fundamental wave frequency increases in comparison to the ordinary Cerenkov angle (without field). For the remaining frequencies $(n \neq n_0)$ the emission is mainly determined by the term $dI_0(\omega)$ which practically coincides with the Tamm-Frank formula.

We now consider the case when the fundamental wave is circularly polarized:

$$E_y = E_0\cos\left(\frac{\omega_0 n_0}{c}x - \omega_0 t\right), E_z = E_0\sin\left(\frac{\omega_0 n_0}{c}x - \omega_0 t\right). \quad (2.13)$$

Linearizing the equations of motion in the field (2.13) with respect to $\mu$, taking (2.3) into account, we find the particle velocity in the capture mode:

$$v_x = \frac{c}{n_0}(1 + \mu\cos\Omega t), \quad (2.14)$$

$$v_y = -\mu\frac{c}{\sqrt{n_0^2 - 1}}\cos\Phi_0\sin\Omega t, \quad v_z = -\mu\frac{c}{\sqrt{n_0^2 - 1}}\sin\Phi_0\sin\Omega t,$$

where $\Omega = e(n_0^2 - 1)E_0/mcn_0$ no longer depends on the initial phase $\Phi_0$. The computation of the spectral emission distribution from (2.14) yields exactly formulas (2.7)–(2.10) for the linear polarization case, except for the fact that in our case $\Omega$ is constant for all particles within the wave, and in the case of a beam of particles a single particular frequency is emitted at a given angle. In other respects the emission resembles the case of linear polarization.

3. We consider stimulated emission of a particle beam captured by a wave. Equations (2.4) and (2.14) show that the particle energy in the wave field

$$\mathscr{E} = \mathscr{E}_0 + \mu\frac{\mathscr{E}_0}{n_0^2 - 1}\cos\Omega t, \quad \mathscr{E}_0 = \frac{mc^2 n_0}{\sqrt{n_0^2 - 1}} \quad (3.1)$$

oscillates between the values

$$\mathscr{E}_{min} = \mathscr{E}_0\left(1 - \frac{\mu}{n_0^2 - 1}\right) \text{ and } \mathscr{E}_{max} = \mathscr{E}_0\left(1 + \frac{\mu}{n_0^2 - 1}\right)$$

so that

$$\Delta\mathscr{E} = 2\mu\frac{mc^2 n_0}{(n_0^2 - 1)^{3/2}}. \quad (3.2)$$

According to (3.1) the particle periodically loses and acquires energy inside the wave. We can prove rigorously that energy lost by decelerating particle is forcedly transferred to the wave (this, in particular, is the cause of the strong suppression of "spontaneous" emission at the fundamental frequency $\omega_0$).

We now proceed to a quantitative analysis of the self-consistent problem. In this case the field amplitude varies slowly with x and t. Let the wave be at first circularly polarized:

$$E_y(x, t) = E(x, t)\cos(\omega_0 n_0 x/c - \omega_0 t),$$
$$E_z(x, t) = E(x, t)\sin(\omega_0 n_0 x/c - \omega_0 t). \quad (3.3)$$

*$[\nu v] \equiv \nu \times v$.

We select the boundary conditions in the form

$$E_y(0,\ t) = E_0\cos\omega_0 t,\ E_z(0,\ t) = -E_0\ \sin\omega_0 t. \quad (3.4)$$

We assume that particles cross the boundary of the medium $x = 0$ at a time $t = t_0$ with an initial velocity $v_0 = v_{0x} = c(1+\mu)/n_0$.

Linearizing the equations of motion and taking (3.3) into account we find the velocity of a single particle in the field:

$$v_y = -\frac{c}{\sqrt{n_0^2-1}}\mu\cos(\omega_0 t_0)\sin\left[\frac{e(n_0^2-1)}{mcn_0}\int_{t_0}^{t} E(t',x)\,dt'\right], \quad (3.5)$$

$$v_z = \frac{c}{\sqrt{n_0^2-1}}\mu\sin(\omega_0 t_0)\sin\left[\frac{e(n_0^2-1)}{mcn_0}\int_{t_0}^{t} E(t',x)\,dt'\right].$$

To determine the current created by the flow of particles we assume that all space is continuously filled with charged particles. Then at the time $t_0$ in the point $x$ there are only particles for which $t_0 = t - n_0x/c$ (with an accuracy to $\mu\omega_0/\Omega \ll 1$). Thus we have

$$j_y(x,t) = -\mu\frac{ec\rho_0}{\sqrt{n_0^2-1}}\cos\left(\frac{\omega_0 n_0}{c}x-\omega_0 t\right)\sin\left[\frac{e(n_0^2-1)}{mcn_0}\times\int_{t-n_0x/c}^{t} E\left(t',\frac{c}{n_0}(t'-t)+x\right)dt'\right],$$

$$j_z(x,t) = -\mu\frac{ec\rho_0}{\sqrt{n_0^2-1}}\sin\left(\frac{\omega_0 n_0}{c}x-\omega_0 t\right)\sin\left[\frac{e(n_0^2-1)}{mcn_0}\times\int_{t-n_0x/c}^{t} E\left(t',\frac{c}{n_0}(t'-t)+x\right)dt'\right], \quad (3.6)$$

where $\rho_0$ is the average particle density in the initial beam that we consider constant (for $\mu \ll 1$ we can neglect the change in $\rho_0$).

We are interested only in the stimulated part of the emission and therefore we do not consider the scalar potential field and the axial field along the $x$ axis. Substituting (3.6) into Maxwell's equation and taking into account the slow variation of field amplitude: $|\partial E/\partial t| \ll \omega_0|E|$, $|\partial E/\partial x| \ll \omega_0 n_0|E|/c$, we obtain a simplified equation for the self-consistent field:

$$\frac{\partial E}{\partial x}+\frac{n_0}{c}\frac{\partial E}{\partial t} = \frac{2\pi e\rho_0}{n_0\sqrt{n_0^2-1}}\mu\sin\left[\frac{e(n_0^2-1)}{mcn_0}\int_{t-n_0x/c}^{t} E\left(t',\frac{c}{n_0}(t'-t)+x\right)dt'\right]. \quad (3.7)$$

Equation (3.7) assumes a simpler form in the wave coordinates $\tau = t - n_0x/c$, $\eta = x$, $E(t, x) = f(\tau, \eta)$. Then

$$\frac{\partial}{\partial\eta}f(\tau,\eta) = \frac{2\pi e\rho_0}{n_0\sqrt{n_0^2-1}}\mu\sin\left[\frac{e(n_0^2-1)}{mcn_0}\int_0^{\eta} f(\tau,\eta')\,d\eta'\right]. \quad (3.8)$$

A simple analytic solution is obtained for the case of a monochromatic incident wave: $f(\tau, 0) = E_0$. In the case, according to (3.8), $f(\tau,\eta) = f(\eta)$ is independent of $\tau$ and the quantity

$$\varphi = \frac{e(n_0^2-1)}{mc^2}\int_0^{\eta} f(\eta')\,d\eta' \quad (3.9)$$

is expressed by the nonlinear equation of anharmonic oscillations

$$\varphi'' = \frac{2\pi e^2\rho_0\sqrt{n_0^2-1}}{mc^2n_0}\mu\sin\varphi, \quad (3.10)$$

whose general solution is the incomplete elliptic integral of the first kind:

$$\frac{1}{2}(n_0^2-1)\frac{eE_0x}{mc^2} = \int_0^{\varphi/2}\frac{dz}{\sqrt{1+\beta^2\sin^2 z}},\quad \beta^2 = \frac{8\pi\mu}{n_0(n_0^2-1)^{3/2}}\frac{mc^2\rho_0}{E_0^2}. \quad (3.11)$$

We investigate particular cases of (3.10). In the linear case, when $\varphi \ll 1$, we have

$$E(x) = E_0\begin{cases}\operatorname{ch}(x/l), & \mu > 0\\ \cos(x/l), & \mu < 0.\end{cases} \quad (3.12)$$

Thus for $\mu > 0$ the light is amplified exponentially. For $\mu < 0$ there is no amplification on the average. The quantity

$$l = \left(\frac{mc^2n_0}{2\pi e^2\sqrt{n_0^2-1}\,\rho_0\mu}\right)^{1/2} \quad (3.13)$$

contained in (3.11) is the coherent amplification length. Analysis of the above formulas shows that the linear regime occurs for $E \lesssim e\lambda_0\rho_0(mc^2/\epsilon_0)^3$ and that $l \gtrsim mc^2(\epsilon_0/mc^2)^2/e^2\lambda_0\rho_0$ in that case. Here $\lambda_0 = 2\pi c/\omega_0$ is the wavelength of the fundamental emission. In the saturation region, (3.8) gives

$$E(x) = E_0 + \mu\frac{2\pi mc^2\rho_0}{n_0(n_0^2-1)^{3/2}}\frac{1}{E_0}\left\{1-\cos\left[(n_0^2-1)\frac{eE_0x}{mc^2}\right]\right\}. \quad (3.14)$$

The resulting wave-energy increment corresponds to the energy lost by particles (per unit volume) according to (3.2):

$$\Delta W = \rho_0\Delta\varepsilon = 2\mu\varepsilon_0\rho_0/(n_0^2-1). \quad (3.15)$$

Equation (3.14) is valid for $E \gtrsim e\lambda_0\rho_0\epsilon_0/mc^2$.

Equation (3.8) is fully analogous to the quantum-amplifier equation. The parameter $\mu$ plays here the role of the population excess.

We now examine the case in which the fundamental wave is linearly polarized along the $y$ axis:

$$E_y = E(x,\ t)\cos(\omega_0 n_0 x/c - \omega_0 t). \quad (3.16)$$

We use the same method to find the velocity of a single particle in such a field:

$$v_x = \frac{c}{n_0}\left\{1+\mu\cos\left[\int_{t_0}^{t}\Omega(t',x)\,dt'\right]\right\}, \quad (3.17)$$

$$v_y = -\mu\frac{c}{(n_0^2-1)^{1/2}}\sin\left[\int_{t_0}^{t}\Omega(t',x)\,dt'\right],$$

where the modulation frequency

$$\Omega(t,x) = \frac{e(n_0^2-1)}{mcn_0}E(x,t)\cos\omega_0 t_0$$

depends here on the initial phase $\Phi_0 = \omega_0 t_0$. Therefore in this case all harmonics are present in the emission from a particle beam. Similarly, if we determine the current produced all particles and expand it in a Bessel function series, we find that the stimulated portion of the emission (due to current $j_y$) contains only odd harmonics, and the noncoherent portion (axial field along the $x$ axis) contains only even harmonics. Just as in the case of circular polarization, we consider the coherent emission. Substituting the current

$$j_y(x,t) = -\frac{ec\rho_0}{(n_0^2-1)^{1/2}}\mu\sum_{s=-\infty}^{+\infty} i^{s-1}J_s(Z)\exp\left\{is\omega_0\left(\frac{n_0}{c}x-t\right)\right\},$$

$$s = 2k-1;\ k = 0, \pm 1, \pm 2\ldots$$

$$Z(x,t)=\frac{e(n_0^2-1)}{mcn_0}\int\limits_{t-n_0x/c}^{t}E\left(t',\frac{c}{n_0}(t'-t)+x\right)dt' \quad (3.18)$$

into Maxwell's equation and simplifying, we obtain for the amplitudes $E_s(x, t)$

$$2is\omega_0\left(\frac{n_0}{c}\frac{\partial E_s}{\partial x}+\frac{n_s^2}{c^2}\frac{\partial E_s}{\partial t}\right)+\frac{s^2\omega_0^2}{c^2}(n_s^2-n_0^2)E_s = i^s\frac{4\pi e\rho_0 s\omega_0}{c(n_0^2-1)^{1/2}}\mu J_s(Z), \quad (3.19)$$

where $n_s$ is the index of refraction of the medium at the s-th harmonic.

We consider (3.19) for the cases of present and absent velocity matching. If $n_s \neq n_0$ (no match), we obtain because of the slow amplitude variation

$$E_s=i^s\mu\frac{4\pi ec\rho_0}{(n_0^2-1)^{1/2}}\frac{1}{s\omega_0}\frac{1}{n_s^2-n_0^2}J_s(Z). \quad (3.20)$$

This formula shows that the radiation field weakly depends on the harmonic number. On the other hand, in the case of a match ($n_s = n_0$) we have

$$\frac{\partial E_s}{\partial x}+\frac{n_0}{c}\frac{\partial E_s}{\partial t}=i^{s-1}\mu\frac{2\pi e\rho_0}{n_0(n_0^2-1)^{1/2}}J_s(Z). \quad (3.21)$$

For the first harmonic (coherent portion) all results follow exactly the case of circular polarization in (3.11)–(3.13) except for the fact that the coherent gain length increases by a factor of $\sqrt{2}$ in this case: $l_{lin} = l_{cr}\sqrt{2}$. To determine emission for the remaining harmonics in the matching case we consider the problem for a given field. Then for large x ($e(n_0^2 - 1)E_0x/mc^2 \gg 1$) we obtain

$$E_s=i^{s-1}\mu\frac{2\pi mc^2\rho_0}{n_0(n_0^2-1)^{3/2}}\frac{1}{E_0}. \quad (3.22)$$

Hence the harmonic radiation power is (in order of magnitude)

$$P_s=\frac{c}{8\pi}|E_s|^2\approx e^2c\lambda_0^{-4}(\lambda_0^3\rho_0)^2(\varepsilon_0/mc^2)^2. \quad (3.23)$$

Let us obtain some estimates. The estimating formulas as well as (3.15) and (3.23) have been obtained for the condition $\mu \sim \xi(mc^2/\epsilon_0)^2$ which follows from the capture condition. Since the coherent length in the linear regime increases as the square of energy and the particle losses depend logarithmically on the energy, energy increase is of no particular benefit to the amplification of weak signal. The optimum energy is $\epsilon_0 \sim mc^2$. Then $l \sim 10^{12}(\lambda_0\rho_0)^{-1}$. For example, $l \sim 1$ cm for $\lambda_0 \sim 1\ \mu$ and $\rho_0 \sim 10^{16}$ cm$^{-3}$, and $l \sim 1$ mm for $\lambda_0 \sim 10\ \mu$. To reduce electron energy loss it is desirable to use gases that resonate at the incident radiation frequency (large index of refraction with relatively low densities). For the above parameters, the linear regime occurs up to fields of $10^5$ V/cm. In the saturation regime for $\rho_0 \sim 10^{16}$ cm$^{-3}$ additional emission input to the wave amounts to $10^8$ W/cm$^2$, which is larger by one order than the power of incident emission. For the same parameter values harmonic emission (in the match case) is of the order of $10^7$ W/cm$^2$. These computations show that the amplification effect (in the case of constant $n_0$) can be utilized in the optical range if we have an electron beam with an energy of several MeV and a density ($10^{15}$–$10^{16}$) cm$^{-3}$.

4. The situation is considerably improved if we use a medium with a variable index of refraction n(x). In this case the particles continuously decelerate and the light is continuously amplified in the direction of increasing n(x). On the other hand, the particles are accelerated in the direction of decreasing n(x). In this case the solution of the self-consistent problem is exceedingly difficult and therefore our results are presented in the given-field approximation. The most favorable situation occurs for a particle with $P_{0y} \neq 0$. The change of energy of an equilibrium particle is then given by the relation

$$\varepsilon-\varepsilon_{s0}=eE_0(y-y_0)\cos\Phi_s, \quad (4.1)$$

Hence it follows that $\cos\Phi_s > 0$ corresponds to the particle acceleration regime, and $\cos\Phi_s < 0$ to the light amplification regime. Equation (4.1) determines the actual transverse dimension of the system. The character of variation of n(x) is given by

$$\frac{1}{2}\left\{\frac{n(0)}{n^2(0)-1}-\frac{n(x)}{n^2(x)-1}\right\}+\frac{1}{4}\ln\frac{[n(x)+1][n(0)-1]}{[n(0)+1][n(x)-1]} = -\frac{eE_0cP_{0y}(x-x_0)\cos\Phi_s}{m^2c^4+c^2P_{0y}^2}. \quad (4.2)$$

The longitudinal dimension of the system is determined by the relation

$$\varepsilon_s^2=(m^2c^4+c^2P_{0y}^2)\frac{n^2(x)}{n^2(x)-1}. \quad (4.3)$$

Equations (4.1)–(4.3) are valid for an equilibrium particle whose phase satisfies the relation $\ddot{\Phi}_s = \dot{\Phi}_s = 0$ ($v_x = c/n(x)$). If $E_0 \sim 10^7$ V/cm and the transverse dimension of the laser $y - y_0 \sim 1$ cm, then $\Delta\epsilon \sim 10$ MeV for each electron. Selecting $cP_{0y}$ we can make the axial dimension ($x - x_0$) of the order of the transverse dimension (at least a few times larger). This shows that a beam with a total number of electrons of $10^{12}$ (and energy of ten MeV) emits an additional energy of the order of 1 j into the wave.

[1]V. M. Arutyunyan and G. K. Avetisyan, Preprint IFI 71-01.
[2]V. M. Arutyunyan and G. K. Avetisyan, DAN Arm. SSR **52**, 5 (1971).
[3]V. M. Arutyunyan and G. K. Avetisyan, Kvantovaya élektronika (Quantum Electronics) (in press).
[4]V. M. Arutyunyan and G. K. Avetisyan, Preprint IFI 71-03.
[5]V. L. Ginzburg and V. Ya. Éidman, Zh. Eksp. Teor. Fiz. **36**, 1823 (1959) [Sov. Phys.-JETP **9**, 1300 (1959)].
[6]I. M. Frank, Yad. Fiz. **7**, 1100 (1968) [Sov. J. Nucl. Phys. **7**, 660 (1960)].
[7]M. L. Ter-Mikaelyan, Vliyanie sredy na élektromagnitnye protsessy pri vysokhikh énergiyakh (The Effect of Medium on Electromagnetic Processes at High Energies) Izd. AN Arm. SSR, 1969.

Translated by S. Kassel
187.