Gradient Excitation of Periodic Structures and of Transverse Waves

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It is shown that in a conducting medium periodic structures and transverse waves may be excited in the absence of a magnetic field by a damped electromagnetic or sound wave beam. The structure may be either one- or two-dimensional. The properties of the structures are investigated as well as the distribution of the magnetic and electric fields in them. Besides these structures, other structures are possible, in which the concentration and field are distributed periodically. These structures may be one-, two-, or three-dimensional. Finally the radiation emitted by a crystal in which transverse waves are excited is determined.

1. We have shown in a number of papers[1-3] that a stationary flux (a flux of electromagnetic waves, henceforth called for brevity optical, a sound beam, heat flow, or an electric current) passing through a conducting medium placed in an external magnetic field parallel to the flux, excites in this medium under certain conditions, static transverse structures characterized by a periodic distribution of the magnetic electric fields, of the charge and of the lattice deformations, and also transverse waves that can lead to microwave radiation (or radiation of longer wavelength) from the crystal. (As shown in[3], the structures can be produced also in the absence of a magnetic field, if the opto-electric coefficients depend sufficiently strongly on the temperature.)

It turns out that a damped optical or acoustic flux can always excite structures or waves even in the absence of an external magnetic field, if the flux density and the attenuation are strong enough. In addition to the indicated structures, which we shall call magnetic, the damped flux can excite structures of a different type, which we shall call concentration structures, and which are characterized by a periodic distribution of the concentration of the electric-field carriers and of the lattice displacements. These structures may be charged in conductors with carriers of like sign, and quasineutral in conductors with carriers of both signs.

In order for either a magnetic or a concentration structure to be excited by a damped flux, the circuit must be closed in the flux direction. Depending on the flux intensity, they can be either one- or two-dimensional. In either case they are produced in the plane perpendicular to the flux. There exists, however, another possibility of exciting concentration structures, wherein the damping of the flux is immaterial, and the circuit in the flux direction must be open. This possibility is realized when the flux heats the carriers, so that their mobility, and hence their opto-electrical coefficients, depend on the opto-electric field, and via this field on the flux intensity. In this case the structures can be one-, two-, or three-dimensional.

A particular case are the vibrational structures that occur in the presence of an electric field, and a magnetic field parallel to it, in crystals having a slow rate of surface recombinations. In these structures, the amplitude oscillation frequency vanishes at definite values of the electric and magnetic fields.

In conclusion, we consider the radiation from a crystal in which transverse waves are present.

2. We consider a conductor (with dimensions L_x, L_y, L_z) in which there exists a stationary flux of density I in the z direction. If the flux is inhomogeneous in this direction, then when $|\mathrm{div}\,\mathbf{I}|$ exceeds a certain critical value the medium is unstable against the formation of periodic structures or waves. This can be understood in the following manner. Assume that a fluctuating magnetic field H'_x has been produced in a medium with $I_z(z)$. This field (as shown in[3]) leads to the appearance of a solenoidal electric field $E'_y \sim [\mathbf{I}\times\mathbf{H}']_y$, which depends on z, and which again produces according to Maxwell's equation a field H'_x. If the circuit is closed in the z-direction (so that the fluctuation current does not vanish on the boundaries, and therefore the fluctuations may be independent of z), and if L_x, $L_y = L_\perp > L_z$ then, just as in[3], periodic structures are produced in the xy plane; in the opposite case, transverse waves are excited.

In the presence of a flux and a magnetic field, the electric field consists of ohmic and opto-electric (acousto-electric) components:*

$$\mathbf{E}=\rho\mathbf{j}+\rho_1[\mathbf{jH}]+\rho_2\mathbf{H}(\mathbf{jH})+\gamma\mathbf{I}+\gamma_1[\mathbf{IH}]+\gamma_2\mathbf{H}(\mathbf{IH});$$

here ρ, ρ_1, and ρ_2 are the ordinary, Hall, and focusing resistances, γ, γ_1, and γ_2 are the opto-electric (acousto-electric) coefficients, and j is the current density. Linearizing this equation in the absence of an external magnetic field H_0 and using Maxwell's equations (neglecting the displacement current), we get

$$\frac{\partial\mathbf{H}'}{\partial t}=\frac{c^2}{4\pi}\rho\Delta\mathbf{H}'-c|\gamma_1|(\mathbf{I}\nabla)\mathbf{H}'-c|\gamma_1|\,\mathrm{div}\,\mathbf{I}\cdot\mathbf{H}'.$$

(It is shown in[4] that $\gamma_1 > 0$.)

We consider first the region $z \ll \delta$, where δ is the characteristic beam attenuation length. For a light beam we have $I = I_0e^{-2z/\delta}$, where δ is the thickness of the skin layer. Putting $H' \sim \exp(\mathbf{k}\cdot\mathbf{r}-\omega t)$, we get

$$\omega=c|\gamma_1|(\mathbf{kI}_0)-ic^2k^2\rho/4\pi+2ic|\gamma_1|I_0/\delta.$$

Just as in[3], at $L_x > L_y > L_z$ the imaginary part of ω first vanishes, i.e., instability sets in, if

$$I_0\geqslant I_{0c1}=c\rho\delta k_x^2/8\pi|\gamma_1|.$$

This is accompanied by appearance of a periodic structure, i.e., by a time-constant magnetic field that is periodically distributed along the large side of the crystal. In addition, there appear periodic distributions of the current, of the lattice displacements, of the

*$[\mathbf{jH}]\equiv\mathbf{j}\times\mathbf{H}$.

charge, and of the electric field. We call such structures magnetic. When the flux I increases further to a value

$$I_0 \geqslant I_{0c2} = c\rho\delta(k_x^2 + k_y^2)/8\pi|\gamma_1|$$

a doubly-periodic structure appears, in which all the indicated quantities depend periodically on the two directions transverse to the beam. Finally, at

$$I_0 \geqslant I_{0c3} = c\rho\delta(k_x^2 + k_y^2 + k_z^2)/8\pi|\gamma_1|,$$

oscillations appear, characterized by a frequency

$$\omega = c|\gamma_1|k_z I_0.$$

It was shown in[3] that a sufficiently strong dependence of the opto-electric coefficients on the temperature can lead to excitation of structures in the absence of a magnetic field. The ratio of I_{0c} in the case of gradient excitation to the value of I_c obtained in[3] is in this case equal to $\pi\delta p/L_\perp$, i.e., it is in practice much smaller than unity.

In a cylindrical conductor whose length L is much smaller than the radius R, a quasiperiodic field distribution

$$H' \sim J_0\left(2{,}4\frac{I}{I_{0c}}\frac{r}{R}\right)$$

in the radial direction is produced, just as in[3], when

$$I_0 \gg I_{0c4} = \frac{c\rho\delta}{8\pi|\gamma_1|}\left(\frac{2.4}{R}\right)^2$$

(J_m is a Bessel function of order m and 2.4 is the first root of J_0). With further increase of the flux, a doubly-periodic distribution is produced,

$$H' \sim J_m(\alpha_{mn}r/R)\cos m\varphi$$

(φ is the azimuthal angle and α_{mn} is the n-th root of the Bessel function J_m), in which all the quantities are periodic in the angle φ and quasi-periodic in the radius. The amplitude of the produced magnetic field is determined from a quasilinear theory analogous to that developed in[3]. The excitation turns out to be soft, and at a small supercriticality, $I - I_c \ll I_c$, the amplitude of the magnetic field is

$$H' = \frac{c}{2\pi p\mu}\frac{L_\perp}{\delta}\left[\frac{3(I-I_c)}{I_c}\right]^{1/2}, \quad p = 1, 2, \ldots$$

(μ is the mobility). Just as in[3], the mode p is attenuated when the mode $p-1$ is excited.

The presence of an external magnetic field $\mathbf{H}_0$ directed along the large dimension L_x transverse to the flux leads to excitation of waves with helicon frequency $ck_x^2 H_0\rho_1/4\pi$. Unlike the earlier studies,[1-3] where the waves appeared only at $k_z \neq 0$, we do not have this limitation here. At $\mathbf{H}_0$ directed along the smaller transverse dimension, oscillations are produced with frequency

$$2ck_y\frac{\gamma I}{H_0}\frac{\partial \ln\gamma}{\partial \ln H^2}.$$

When $\mathbf{H}_0 \parallel \mathbf{I}$ there are two mechanisms for the excitation of structures or waves. The first is described in[3] and is not connected with the inhomogeneity of the flux; the second has been described above. In the case of magnetic excitation, the ratio to I_{0c_1} of the critical density is equal to

$$\frac{\pi\delta p}{L_\perp}\min\left|\frac{\gamma_2 H_0}{\gamma_1}\right| \approx \frac{\pi\delta p}{L_\perp}\min\left|\frac{\mu H_0}{c}\ \frac{c}{\mu H_0}\right|$$

(In the last equation we used the results of[4].)

We consider now the structures without restricting ourselves to the condition $z \ll \delta$. For H_z' we obtain the equation

$$\frac{d^2H_z'}{dz^2} + k^2\left[\frac{I_0}{I_c}e^{-2z/\delta} - 1\right]H_z' = 0.$$

I_c should be taken to mean I_{0c_1} or I_{0c_2} for gradient excitation, or the I_c obtained in[3] for magnetic excitations. If $I_0 < I_c$, no structures are produced even if $z \ll \delta$.

The employed linear theory is valid only if $I_0 \approx I_{0c}$. We introduce the notation

$$\Delta z = -\frac{\delta}{2}\ln\frac{I_0}{I_c}, \quad z^* = z + \Delta z.$$

We then obtain

$$H_z'(z^*) = C_1 J_{k\delta}(k\delta\exp(-z^*/\delta)) + C_2 N_{k\delta}(k\delta\exp(-z^*/\delta)),$$

where N is the Neumann function. Recognizing that $H_z' \to 0$ as $z^* \to \infty$ and, in addition, $k\delta \ll 1$, we obtain for $z \gg \delta$:

$$H' \sim \exp[-kz(I_0/I_c)^{k\delta/2}].$$

But under our conditions $k\delta \approx \pi p\delta/L_\perp < 1$. Therefore $H' \sim e^{-kz}$, i.e., the magnetic field attenuates more weakly than the flux I. This can be understood by considering a limiting case. Assume that at $z < 0$ the beam is $I = I_0 = I_c$ and there exist structures $H_y' \sim \cos k_x x$. When $z > 0$ the flux is $I = 0$ and we have in this region $\Delta H' = 0$. Taking into account the continuity of H_y' at $z = 0$, we find that at $z > 0$ the field is $H_y' \sim [\exp(-k_x z)]\cos k_x x$.

We consider now the shapes of the current, field, and lattice-displacement lines in the doubly-periodic structures (under magnetic or gradient excitation). In a rectangular sample we get from the expressions derived in[3]

$$j_x' = -\frac{ck_y}{4\pi}H_{0z}'\cos k_x x\sin k_y y,$$

$$j_y' = \frac{ck_x}{4\pi}H_{0z}'\sin k_x x\cos k_y y$$

(H_{0z}' is the component of the amplitude of the magnetic field of the structures), we obtain an equation for the current lines:

$$\cos k_x x\cos k_y y = C_1$$

(C_1 = const). Hence

$$j_y' = C_1\,\mathrm{tg}\,k_x x, \quad j_x' = -(\cos^2 k_x x - C_1^2)^{1/2}$$

or

$$j_x' = -C_1\,\mathrm{tg}\,k_y y, \quad j_y' = -(\cos^2 k_y y - C_1^2)^{1/2}.$$

C_1 varies in the interval from -1 to $+1$. If $-\pi/2 \leq k_x x$ and $k_y y < \pi/2$, then $C_1 > 0$, and if $\pi/2 \leq k_x x < 3\pi/2$ and $-\pi/2 \leq k_y y < \pi/2$, then $C_1 < 0$, etc., i.e., the entire rectangular cross section of the sample perpendicular to the flux breaks up into sections with $C_1 > 0$ and $C_1 < 0$. The current lines are shown in Fig. 1a for the case $k_x = k_y$. The lines of H_x', H_y', and of the electric potential coincide with the lines j_x' and j_y'. The equation for the electric field lines is

$$\sin k_y y = C_2\sin k_x x.$$

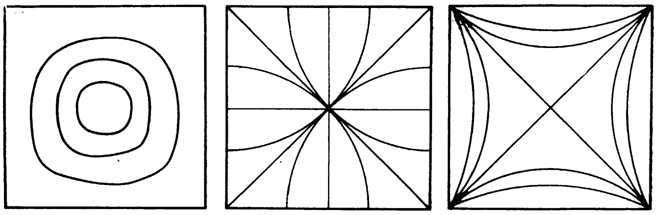

FIG. 1. Lines of the current (a), electric field (b), and lattice displacements (c) for a rectangular sample.

The sign of the constant C_2 varies in the same way as that of C_1 with variation of $k_x x$ and $k_y y$, but the value of C_2 ranges from $-\infty$ to $+\infty$. Unlike the current lines, all the electric-field lines pass through the center of each section. Their form is shown in Fig. 1b. The diagonals are the lines at $C_2 = \pm 1$; at $C_2 = \pm\infty$ the lines are vertical, and at $C_2 = 0$ they are horizontal. The equation for the lattice displacements is

$$\cos k_y y = C_3 \cos k_x y.$$

C_3 ranges from $-\infty$ to $+\infty$ in each section. The lines are shown in Fig. 1c. At $C_3 = \pm 1$ the lines are the diagonals of the section; at $C_3 = 0$ the lines are horizontal, and at $C_3 = \pm\infty$ they are vertical.

In the case of a cylindrical sample, the shape of the sections changes, and they are bounded by concentric circles and radii. The angle between the radii and the number of circles are equal to the index of the Bessel function describing the quasiperiodic radial distribution. The current lines are shown in Fig. 2.

3. In addition to the magnetic structures considered in[3] and in Sec. 2, a flux can excite structures of an entirely different type, connected with a periodic redistribution of the carrier density (we call them concentration structures). There are two different occurrences of such structures: (a) gradient excitation of charged and quasineutral structures, (b) excitation of concentration structures by heating the carriers.

We consider first gradient excitation. If a light flux $I - I_0 e^{-2z/\delta}$ is incident on a sample with carriers of only one kind, and the sample circuit is closed in the flux direction, then an opto-electric field is produced in it:

$$E_0 = -\frac{4\pi}{c}\frac{\sigma_1}{\sigma} I \frac{R_L}{R_0 + R_L}$$

(σ and σ_1 are the ordinary and Hall conductivities, R_L is the load resistance, and R_0 is the sample resistance). If the flux does not heat the electrons, then the equations (ν is the recombination frequency)

$$\operatorname{div} \mathbf{E}' = -4\pi e n'/\varepsilon,$$

$$\frac{\partial n'}{\partial t} - \nu n' + \operatorname{div}\left\{-D\nabla n' + \mu \mathbf{E}_0 n' + \frac{4\pi}{c}\mu_1 \mathbf{I} n' + \mu n \mathbf{E}'\right\} = 0$$

yield in the region $z \ll \delta$

$$\omega = -iDk^2 - i\nu - i\frac{4\pi\sigma}{\varepsilon} + \frac{R_0}{R_0 + R_L}(\mathbf{kI})\frac{4\pi\mu_1}{c}\left(2i\frac{\delta}{|\mathbf{k}|} - 1\right).$$

In analogy with Sec. 2, we find that if $L_x > L_y > L_z$,

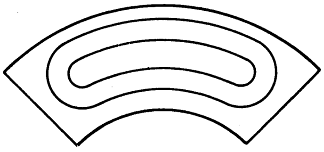

FIG. 2. Current lines in a cylindrical sample.

then at

$$I_0 > I_{0c5} = \frac{R_L + R_0}{R_0}\frac{\delta c^2}{4\pi\mu^2}\left[Dk^2 + \frac{4\pi\sigma}{\varepsilon} + \nu\right]$$

a one-dimensional periodic structure is produced. With further increase of the flux, a doubly-periodic structure is produced, as in Sec. 2, followed by the appearance (unlike in Sec. 2 and in[1-3]) of longitudinal concentration waves and fields. In a cylindrical sample, a quasi-periodic radial distribution, characterized by a Bessel function of zero order, is first produced. With increasing flux, a doubly-periodic radial (quasiperiodic) and azimuthal distribution is produced.

In a piezoelectric crystal, a structure of the lattice displacements u is produced simultaneously with the concentration and field structure. The ratio

$$\left|\frac{kun}{n'}\right| = \frac{8\pi\beta}{\rho s^2}\frac{enL_\perp}{\varepsilon}$$

(β is the piezoelectric coefficient, ρ is the density, and s is the speed of sound) can approach unity. The nonlinear theory shows that the excitation is soft in this case and when $I - I_c \ll I_c$ we have

$$\frac{n'}{n} = \frac{L_\perp}{2\pi\rho\delta}\left(3\frac{I - I_c}{I_c}\right)^{1/2}.$$

In the case of carriers of both signs, quasineutral structures can be excited. We confine ourselves to crystals in which the recombination, which is described by the expression $\nu^*(n_+ n_- - n_i^2)/N$ (n_i, N, and ν^* are constants, $n_\pm$ are the carrier densities) is more effective than the capture. From the linearized continuity equations we obtain in the quasineutral approximation $\operatorname{div}(j'_+ - j'_-) = 0$ the equation

$$\frac{1}{D_0}\frac{\partial n'}{\partial E} = \Delta n' - \frac{\tilde{\nu}}{D}n' - |\tilde{\gamma}| \operatorname{div} \mathbf{I} \cdot n' + |\tilde{\gamma}|(\mathbf{I}\nabla)n' = 0,$$

$$|\tilde{\gamma}| = \frac{4\pi}{c}\frac{\mu_1\mu_-}{\mu_+ + \mu_-}\frac{\mu_{1+}n_+ - \mu_{1-}n_-}{(\mu_+ n_+ + \mu_- n_-)^2}(n_+ - n_-) + \frac{\mu_-\mu_{1+}n_- + \mu_+\mu_{1-}n_+}{c(\mu_+ n_+ + \mu_- n_-)}$$

(D_0 is the coefficient of ambipolar diffusion). In the region $z \ll \delta$ we find that at a current density outside the crystal (this density is $c/\delta\omega$ times larger than the current density inside the crystal)

$$I_{0c6} = \frac{c^3}{4\pi\mu^2}\left(\frac{\tilde{\nu}}{\tilde{\omega}} + \frac{D_0\pi^2 p^2}{L_\perp^2\tilde{\omega}}\right)$$

($\tilde{\omega}$ is the frequency of the incident light; $\tilde{\nu} = \nu^*(n_+ + n_-) = \nu^*(n_+ + n_-)/N$), concentration structures are excited. Their excitation regime is soft. At $z \gg \delta$, the structures attenuate in the direction of the z axis in both cases in proportion to $\exp(-k_\perp z)$, i.e., much more slowly than the beam, just as in the case of magnetic structures.

The equation for the field lines and their shape is analogous to that in Sec. 2 and Fig. 1b. If the light flux heats the electrons and the circuit along the z axis is open, then, taking into account the dependence of the mobility on the electric field and substituting $\mu_1 = a\mu^2/c$, where a is a number that depends on the scattering mechanism, we obtain from the Poisson and continuity equations

$$\frac{\partial n'}{\partial t} = D\Delta n' - \frac{4\pi\sigma}{\varepsilon}\left[1 + 2\frac{\partial \ln\mu}{\partial \ln E^2} - 4a\frac{\partial \ln\mu}{\partial \ln E^2}\right]n' - \tilde{\nu}n'.$$

When $a \gtrsim 1$ and $\partial \ln \mu/\partial \ln E^2 > 1/2$, the structures can be one-, two-, or three-dimensional. This is possible when the electrons are momentum-scattered by charged impurities and energy-scattered by acoustic phonons; in this case $\partial \ln \mu/\partial \ln E^2 = 3/4$. The critical value of I is determined from the condition

$$(4a-2)\frac{\partial \ln \mu}{\partial E^2}(\gamma I)^2 = 1 + \frac{\tilde{\nu}}{Dk_D^2} + \frac{\pi^2}{k_D^2}\left[\frac{p_x^2}{L_x^2} + \frac{p_y^2}{L_y^2} \mp \frac{p_z^2}{L_z^2}\right]$$

(k_D is the Debye radius). Since $k_D L \gg 1$, three-dimensional structures are produced practically immediately. The nonlinear theory leads in this case to a quadrature that can be calculated only numerically. An analysis at a small supercriticality shows that the excitation is soft. The equations for the field lines are

$$\sin k_x x = C_4 \sin k_y y, \quad \sin k_x x = C_5 \sin k_z z,$$

i.e., they are the same in the xy, xz, and yz planes as in the case of Sec. 2.

4. A perfectly unique phenomenon is possible when an electric current parallel to the external magnetic field flows through the conductor. This can give rise to structures whose amplitudes oscillate in time or, when the conditions are right, to standing waves of the concentration and of the field. We consider a cylinder (of length much larger than the radius R) with almost equal carriers densities n_+ and n_-, in which the recombination occurs only on the surface at a very slow rate S, such that $SR \ll D$, in an external weak magnetic field $\mu_\pm H \ll c$. The continuity equations in the quasineutral approximation show that a stationary distribution of the concentration and of the field, which is proportional to

$$e^{i(kz\mp\varphi)} J_1\left(\left(k^2 + \frac{ieEk}{T}\frac{n_- - n_+}{n_- + n_+}\right)^{1/2} r\right)$$

(T is the temperature in ergs), is possible if the electric and magnetic fields have values E_0 and H_0 determined by the equations

$$\sqrt{\frac{3}{32}}\frac{eE_0R}{T}\frac{\pi Rp}{L}\left|\frac{n_- - n_+}{n_- + n_+}\right| = 1,$$

$$\sqrt{24}\left|\frac{n_- - n_+}{n_- + n_+}\right|\frac{c}{(\mu_- + \mu_+)H_0} = 1.$$

If $E = E_0 + \Delta E$ with $\Delta E \ll E_0$, then the structure oscillates at a frequency

$$\frac{\mu_-\mu_+}{\mu_- + \mu_+}\frac{n_- - n_+}{n_- + n_+}k\Delta E;$$

if $H = H_0 + \Delta H$ with $\Delta H \ll H_0$, then the oscillation frequency is

$$\mu_-\mu_+\Delta H k E_0 / c.$$

If $\Delta E < 0$ or $\Delta H < 0$, then the structure attenuates slowly, and if $\Delta E > 0$ and $\Delta H > 0$, then the structure increases; the amplitude approaches the value

$$n(\Delta E / E_0)^{1/2}.$$

5. In[1,2] we investigated the microwave radiation produced by the passage of current, and in Sec. 2 and in[3] we have shown that a light flux can also excite transverse waves. In[3], however, it was assumed that the wave vector is given by $k = \pi p/L_z$, where p = 1, 2, and 3, an assumption which is not justified. We present here a rigorous analysis.

When the flux passing through the conducting medium reaches a definite value, growing oscillations are produced in the medium and are described by the dispersion equation (at $\mathbf{I} \parallel \mathbf{H} \parallel \mathbf{k} \parallel Oz$)

$$\omega + c\gamma_1 Ik \pm \frac{c^2k^2\rho_1 H_0}{4\pi} - i\frac{c^2k^2\rho}{4\pi} \pm i\gamma_2 ckIH_0 = 0;$$

The ± signs correspond to the two possible wave polarizations. Inside a crystal we have

$$H_x' = \sum_{j=1}^{4} C_j \exp(ik_j z),$$

where k_j are the different values of the wave vector, determined by the last equation (1, 2 for the plus sign, and 3, 4 for the minus sign). Outside the crystal, in the regions $z < 0$ and $z > L$, the wave vector is $k = \omega/c$. Continuity of the tangential components of the electric and magnetic fields at z = 0 and z = L leads to a system of eight homogeneous equations, and the frequency is determined from the condition that its determinant vanish. The general equation for the frequency is quite complicated, and we consider therefore particular cases.

If the frequency is such that $\omega \gg ck_j$ or $\omega \ll ck_j$ for all j, then the determinant does not vanish. On the other hand $\omega \ll ck_{1,3}$ and $\omega \approx ck_{2,4}$, which requires the condition $\omega < \sigma(\gamma_1 I)^2$, then the determinant is proportional to

$$(e^{i(k_1-k_4)L} - 1)(e^{i(k_3-k_2)L} - 1).$$

Equating this expression to zero, we find, by substituting k_j, that $\mathrm{Im}\,\omega \geq 0$ at $\gamma_2 H_0 I > 2\pi cp/L_z$ and $\mathrm{Re}\,\omega = \pi c(\gamma_1 I)p/2L \pm c^2\rho_1 H\pi p^2/L_z^2$, p = 1, 2, 3. This result agrees with[3] if $k = 2\pi p/L$; the earlier conclusion yielded $k = \pi p/L$.

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