Nonlinear Landau Damping of Sound Waves in Conductors

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A nonlinear theory is developed for the propagation in conductors of sound waves whose length is much less than the mean free path of the conduction electrons. The only restriction of the theory on the sound intensity is that the interaction energy between the electron and the sound wave be small in comparison with the characteristic electron energy. It is shown that in this situation, the higher harmonics of the effective field of the wave can be neglected; the basic mechanism of the nonlinearity is the distortion of the distribution function for electrons moving in phase with the wave, due to interaction with the sound; the nonlinear effects can then be very important. The Boltzmann kinetic equation for the electron system interacting with the sound is solved by the method of characteristics. The coefficient of absorption (amplification) of the sound and the acousto-electric current are calculated with the help of the resultant distribution function and the heating of the electrons by the sound wave is estimated. The limits of application of such considerations are analyzed. The results of the theory are in agreement with experiment.[6]

1. INTRODUCTION

A nonlinear theory is developed here for the propagation in a conductor of an ultrasonic waves whose length $2\pi/q$ is much less than the free path length of the carriers l:

$$ql \gg 1. \tag{1}$$

While the propagation of small-amplitude ultrasound has been studied in sufficient detail,[1-5] we know of only a single experimental[6] and two theoretical works[7,8] devoted to nonlinear effects under conditions in which the inequality (1) is satisfied. An important assumption used in these researches is the assumption of the smallness of amplitude of the higher harmonics of the electron distribution function in the field of the sound wave, and their effect on the absorption of sound at the fundamental frequency. The requirement of smallness of the higher harmonics is the essential limitation of the region of applicability of the results of[7,8]. Our aim is to construct a theory of sound absorption and the acousto-electric effect, the only small parameter of which (a parameter depending on the sound intensity) is the ratio of the interaction energy of the electrons with the sound wave to the characteristic energy of the electron.

Let us consider qualitatively the phenomena which occur in a conductor when a sound wave propagates in it. As a consequence of the electron-phonon interaction, the sound wave is accompanied by a longitudinal wave of the effective field acting on the electrons, a wave which is propagated with the sound speed w. The wave of the effective field materially affects only those electrons whose velocity in the direction of sound propagation is close to the sound velocity, inasmuch as the mean field acting on the rest of the electrons is small. Therefore the interaction of the electrons with the sound is determined by the properties of the distribution function f_p in the "resonance" region of velocities—the region of velocities close to the sound velocity, and also by the width of this region. In the presence of the sound wave, some of the electrons are "captured" by the sound wave—the motion of these electrons in the system of coordinates attached to the wave is finite in the wave propagation direction x. The relative number of such electrons is of the order of $e\varphi_0/\epsilon$, where φ_0 is the amplitude of the potential of the wave of the effective field, and e and ϵ are respectively the charge and characteristic energy of the electron. The captured electrons oscillate in the field of the wave. The frequency of these oscillations is of the order

$$\omega_0 \sim q\tilde{v} = q\sqrt{e\varphi_0/m}, \tag{2}$$

where $\tilde{v} = \sqrt{e\varphi_0/m}$ is the characteristic velocity of the oscillatory motion of the captured electrons. However, each scattering act changes the direction of the velocity of the electron and withdraws it from the captured group. Therefore, in order that the group of captured electrons carry out the oscillatory motion, the condition

$$\omega_0\tau_p \gg 1 \tag{3}$$

must be satisfied.

Condition (3) means that the electrons manage to execute many oscillations in the field of the wave in the time τ_p between collisions. Thus, (3) is the condition for formation of the group of captured electrons. In its turn, the formation of a group of captured electrons leads to the distortion of the distribution function in the resonance region of velocities, the width of which is of the order $|v_x - w| \sim \tilde{v}$, and, consequently, to nonlinear effects in the sound absorption.

If, on the other hand, $\omega_0\tau_p \ll 1$, then no groups of captured electrons are formed, the width of the resonance region of velocities is determined by the collisions and is of order $1/q\tau_p$. Here, if the condition

$$e\varphi_0/\varepsilon \ll 1 \tag{4}$$

is simultaneously satisfied, the linear theory of Landau damping of the sound wave is valid.[1] Thus the quantity $\omega_0\tau_p$ is a parameter that determines the role of the nonlinear effects at large values of ql. Iterations in terms of this parameter at $\omega_0\tau_p \ll 1$ were constructed in[7].[1)]

[1)]We note that the parameter $\omega_0\tau_p$ is equal to the ratio of the characteristic velocity of the captured electrons $\tilde{v}$ to the collision width of the interaction region.

Inasmuch as $\omega_0\tau_p \sim ql\sqrt{e\varphi_0/\bar{\epsilon}} \gg \sqrt{e\varphi_0/\bar{\epsilon}}$, a region of sound intensity exists,

$$(ql)^{-2} \ll e\varphi_0/\bar{\epsilon} \ll 1, \tag{5}$$

in which the conditions (3) and (4) are satisfied simultaneously. A group of captured electrons is then formed but the number of electrons in it is small. In spite of this, we shall show that the distribution function is distorted in the resonance region and the nonlinear effects are very important. We shall construct a theory of sound absorption and of the acousto-electric effect, assuming satisfaction of condition (4). It will be shown that the amplitudes of the higher harmonics of the potential are small for such sound intensities. However, the higher harmonics of the distribution function are important in the resonance range of velocities. The reason here is the following. The effective potential of the wave is created by all the electrons, by virtue of Poisson's equation; the fraction of captured electrons among them is small. At the same time, the interaction of the electrons with the wave takes place in a narrow range of velocities, where the distortion of the distribution function is significant. The contribution of the higher harmonics of the distribution function to the sound absorption is a quantity of the same order as the corrections to the fundamental harmonic due to distortion of the slow part of the distribution function, which was taken into account in[7]. Therefore, a theory of the quasilinear type constructed in Sec. 3 of[7] has actually no region of applicability, although it gives the correct order of the absorption coefficient and its dependence on the intensity of the wave.

We note the essential difference between the considered case and the hydrodynamic case ($ql \ll 1$). In the latter, the higher harmonics of the potential of the wave and the distribution function are of the same order and make contributions to the sound absorption that are also of the same order.

2. THE ELECTRON DISTRIBUTION FUNCTION

In analogy with[7], we assume that the sound wave is propagated along an axis of symmetry of not less than third order and that the interaction of the electrons with the sound has a piezoelectric character.[2] The set of equations describing the propagation of the sound wave consists of the equation of elasticity theory

$$\rho\frac{\partial^2 u}{\partial t^2} = c\frac{\partial^2 u}{\partial x^2} - \beta\frac{\partial^2\varphi}{\partial x^2} \tag{6}$$

and Poisson's equation

$$-\varepsilon_0\frac{\partial^2\varphi}{\partial x^2} - 4\pi\beta\frac{\partial^2 u}{\partial x^2} = 4\pi en. \tag{7}$$

Here u is the displacement of the lattice, β the piezoelectric modulus, ϵ_0 the dielectric constant, ρ the density of the crystal, c the modulus of elasticity, and n the excess concentration of carriers. To make the set of equations closed it is necessary to determine the connection between the excess concentration of carriers and the perturbation $e\varphi$. As is well known, the excess concentration n is expressed in terms of the Wigner density f_W

$$n = \frac{2}{(2\pi\hbar)^3}\int d^3p(f_W - F_0), \tag{8}$$

which, under our assumptions, satisfies the kinetic equation

$$\frac{\partial f_W}{\partial t} + v_x\frac{\partial f_W}{\partial x} + \hat{I}f_W + \frac{1}{\hbar q}\frac{\partial(e\varphi)}{\partial x}\left(f_W\left(p_x - \frac{\hbar q}{2}\right) - f_W\left(p_x + \frac{\hbar q}{2}\right)\right) = 0. \tag{9}$$

Here F_0 is the equilibrium electron distribution function and the operator $\hat{I}$, for $\bar{\epsilon} \gg \hbar^2q^2/m$, takes the form of the usual classical collision operator.[8]

It is not difficult to see that the characteristic region of change of the Wigner density is of the order of $m/q\tau_p$ for $\omega_0\tau_p \ll 1$ and $\sqrt{me\varphi_0}$ for, $\omega_0\tau_p \gg 1$. Therefore, upon satisfaction of the conditions

$$\hbar^2q^2/m \ll \max(\hbar/\tau_p, e\varphi_0) \tag{10}$$

we can carry out in Eq. (9) an expansion in powers of $\hbar q$, and limit ourselves to the first order.[3] Then (9) reduces to the classical Boltzmann equation

$$\frac{\partial f_p}{\partial t} + v_x\frac{\partial f_p}{\partial x} - \frac{\partial(e\varphi)}{\partial x}\frac{\partial f_p}{\partial p_x} + \hat{I}f_p = 0, \tag{11}$$

where f_p is the classical distribution function of the electrons.

It follows from the discussions above that the limits of applicability of the classical description of the interaction of the electron system with the sound, which we shall use in what follows, are determined by the conditions (10). In the case

$$\hbar^2q^2/m \gg \max(\hbar/\tau_p, e\varphi_0) \tag{12}$$

appreciable corrections appear in the energy conservation law for the interaction of the electron with the sound quantum, these corrections are associated with the finiteness of the momentum of the sound quantum. Account of these corrections leads to the separation of a group of discrete values of the momentum of the electron that is free to interact with the sound. Condition (12) allows us to construct a quantum-mechanical perturbation theory in the parameter $e\varphi_0 m/\hbar^2q^2$. As was done in[8]. It is interesting to observe that the classical description again becomes suitable when the sound intensity increases, inasmuch as the region of significant change in the Wigner density f_W increases as a result of the interaction with the sound and becomes greater than the momentum of the sound quantum.

Thus, the complete set of equations for the sound wave and the electrons consists of Eqs. (6)–(8) and (11). Of these, only Eq. (11) is nonlinear. We shall assume that the amplitudes of the higher harmonics of the potential φ of the wave are small. We shall substantiate this assumption below. The nonlinear part of the problem then reduces to the solution of the kinetic equation and the determination of the nonlinear response of the concentration to the perturbation $e\varphi$. Inasmuch as the damping distance appreciably exceeds the wavelength and the free path length of the carriers, it is reasonable to assume that the amplitude of the wave φ_0 is independent of the coordinates in the solution of Eq. (11).

[2]The case of deformation interaction is considered similarly. The results for it will be given below.

[3]It can be shown that the corrections to the solution of Eq. (9) to take into account of the next powers of the expansion in $\hbar q$ are small in terms of the parameter

$$\min(\hbar q^2\tau_p/m, \quad \hbar^2q^2/me\varphi_0).$$

Thus, we determine the local distribution function and the local absorption coefficient Γ as a function of the sound intensity S. The latter should be obtained from the solution of the differential equation

$$dS/dx = -\Gamma(S)S(x), \quad S_{x=0} = S_0. \tag{13}$$

In accordance with what was pointed out above, we set $\varphi = \varphi(x - wt)$ and seek a solution in the form

$$f_p(x - wt) = F_0(\varepsilon + e\varphi) + g_p(x - wt). \tag{14}$$

The equation for g_p has the form

$$(v_x - w)\frac{\partial g_p}{\partial y} - e\frac{d\varphi}{dy}\frac{\partial g_p}{\partial p_x} + \hat{I}g_p = ew\frac{d\varphi}{dy}\frac{\partial F_0}{\partial \varepsilon}, \tag{15}$$

where $y = x - wt$; $\hat{I}$ represents the collision operator of the electrons. For an arbitrary quasi-elastic mechanism of scattering,

$$\hat{I}g_p = \sum_{p'}(W_{p'p}g_p - W_{pp'}g_{p'}) = \nu_p g_p - \sum_{p'} W_{pp'}g_{p'}, \quad \nu_p = \frac{1}{\tau_p}. \tag{16}$$

$W_{pp'}$ is the transition probability between the states p and p'.

The quantity ν_p depends on the energy of the electron. Here, since electrons with small longitudinal velocities interact with the sound, it can be assumed that ν_p depends only on the energy of transverse motion and does not depend on the longitudinal velocity. Accordingly, Eq. (16) is one-dimensional and can be reduced to an integral. Introducing the nondimensional variables

$$\xi = q(x - wt), \quad s = (v_x - w)/\tilde{v}, \quad \psi(\xi) = \varphi(\xi)/\varphi_0, \quad E = {}^1/_2 s^2(\xi) + \psi(\xi), \quad a = (\omega_0\tau_p)^{-1}, \tag{17}$$

we represent (16) in the form

$$\hat{L}g_p = U, \quad \hat{L} = s\frac{\partial}{\partial \xi} - \frac{d\psi}{d\xi}\frac{\partial}{\partial s} + a, \tag{18}$$

$$U = U_0 + \frac{1}{q\tilde{v}}\sum_{p'} W_{pp'}g_{p'}, \quad U_0 = e\varphi_0\frac{w}{\tilde{v}}\frac{d\psi}{d\xi}\frac{\partial F_0}{\partial \varepsilon}.$$

We construct the inverse operator by the method of characteristics, satisfying the periodicity condition $g_p = \hat{L}^{-1}U$. Here, two groups of electrons naturally separate: the trapped electrons, which execute finite motion in the set of coordinates attached to the sound wave, and the untrapped electrons. By virtue of the different character of the motion the inverse operator $\hat{L}^{-1}$ takes on different forms for these groups. For the untrapped electrons ($E > 1$), imposing the periodic boundary conditions

$$g(s, \xi) = g(s, \xi + 2\pi), \tag{19}$$

we have

$$g(s, \xi) = \frac{\operatorname{sign} s}{\exp[aA(E)\operatorname{sign} s] - 1} \times \int_{\xi}^{\xi+2\pi} d\xi' U(\xi')G(\xi')\exp\left[a(\operatorname{sign} s)\int_{\xi}^{\xi'} G(\xi'')d\xi''\right], \tag{20}$$

$$A(E) = \int_0^{2\pi} G(\xi)d\xi, \quad G(\xi) = [2(E - \psi(\xi))]^{-1/2}.$$

For the trapped electrons, the boundary conditions are established as follows. The trajectory of the electron for $E < 1$ has two turning points, which are the roots of the equation

$$\psi(\xi) = E. \tag{21}$$

We denote the turning points closest to the point ξ on the left and on the right by ξ_1 and ξ_2, respectively. Inasmuch as the electrons are reflected at these points from the walls of the potential well created by the sound wave, we require the satisfaction of the following conditions:

where
$$g^+(E, \xi_1) = g^-(E, \xi_1), \quad g^+(E, \xi_2) = g^-(E, \xi_2), \quad g^+(E, \xi) = g(E, \xi)|_{s>0}, \quad g^-(E, \xi) = g(E, \xi)|_{s<0}. \tag{22}$$

It is easy to establish the fact that the conditions (22) are equivalent to the requirement of continuity of the velocity distribution function. With the use of (22), we obtain

$$g^+(s, \xi) = 2\operatorname{sh}^{-1}(aB)\left[e^{aB}\int_{\xi_1}^{\xi} U(\xi')G(\xi')\exp\left(-a\int_{\xi'}^{\xi} G(\xi'')d\xi''\right)d\xi' + e^{-aB}\int_{\xi}^{\xi_2} U(\xi')G(\xi')\exp\left(a\int_{\xi}^{\xi'} G(\xi'')d\xi''\right)d\xi' + e^{aC}\int_{\xi_1}^{\xi_2} U(\xi')G(\xi')\exp\left(-a\int_{\xi}^{\xi'} G(\xi'')d\xi''\right)d\xi'\right]. \tag{23}$$

The expression for g^- is obtained from (23) by replacing a by $-a$ and multiplying by -1. In Eq. (23),

$$B(E) = \int_{\xi_1}^{\xi_2} G(\xi)d\xi, \quad C(E, \xi) = \int_{\xi}^{\xi_2} G(\xi')d\xi' - \int_{\xi_1}^{\xi} G(\xi')d\xi'. \tag{24}$$

In the case of a wave of sinusoidal shape,

$$A(E) = 2\sqrt{\frac{2}{E+1}}K\left(\sqrt{\frac{2}{E+1}}\right), \quad B(E) = 2K\left(\sqrt{\frac{E+1}{2}}\right), \quad C(E, \xi) = 2F\left[\arcsin\left(\sqrt{\frac{E+1}{2}}\cos\frac{\xi}{2}\right), \sqrt{\frac{E+1}{2}}\right], \tag{25}$$

where K and F are elliptic integrals.

We have obtained integral equations for the distribution function $g(s, \xi)$. As follows from the explicit form of the operator $\hat{L}^{-1}$, their solution for untrapped and trapped electrons have the properties of continuity and periodicity (the solution (23) possesses periodicity by virtue of the periodicity of the function $\psi(\xi)$). The integrals entering into (20) and (23) diverge at $E = 1$. However, it is easy to note that the distribution function is continuous for $E \to 1$, since the singularities of the numerator and denominator are eliminated. Here the solutions for the untrapped and trapped electrons transform into each other in continuous fashion for $E - 1 \to \pm 0$. If $a \gg 1$, the range of velocities is important in (20) and (23) in which $\sqrt{E} \sim a \gg 1$. Therefore, the contribution of the trapped electrons is unimportant, and one can expand the function for the untrapped electrons in powers of $E^{-1/2}$. Such a procedure leads to the linear theory and to the corrections to it obtained in[7] by iteration of the kinetic equation.

In the case of strong nonlinearity ($a \ll 1$) the operator $\hat{L}^{-1}$ can be expanded in a power series in a:

$$g_p = \left(\frac{1}{a}\hat{L}_{-1}^{-1} + \hat{L}_0^{-1} + a\hat{L}_1^{-1} + \ldots\right)\left(U_0 + \sum_{p'}\frac{W_{pp'}g_{p'}}{q\tilde{v}}\right). \tag{26}$$

We shall solve Eq. (26) by iteration with respect to the arrival term. It is not difficult to establish the fact that account of the arrival term is important only if its acted upon by the operator $a^{-1}\hat{L}_{-1}^{-1}$, and it is enough

to limit ourselves in this term to the part of the distribution function g that varies slowly with time. The discarded terms are small in the parameters ($\sqrt{e\varphi_0/\epsilon}$, $1/ql$), since they are connected with scattering acts that leave the electron in the small phase volume of the "resonance" region. To first order in a, we have

$$g_p = \frac{1}{\nu_p}\sum_{p'} W_{pp'}\{g_{p'}\}_\xi + (\hat{L}_0^{-1} + a\hat{L}_1^{-1})U_0, \tag{27}$$

where $\{g_p\}_\xi$ is the distribution function averaged over the period. The equation for $\{g_{p'}\}_\xi$ is obtained from (27) by averaging over the period of the sound wave

$$\hat{I}\{g_p\}_\xi = \{\nu_p(\hat{L}_0^{-1} + aL_1^{-1})U_0\}_\xi. \tag{28}$$

Using the explicit form of the operator $\hat{L}^{-1}$, we get

a) untrapped electrons

$$g(s,\xi) = \frac{1}{\nu_p}\sum_{p'} w_{pp'}\{g_{p'}\}_\xi + \frac{\operatorname{sign} s}{A(E)}\int_\xi^{\xi+2\pi} U_0 G\, d\xi' \int_\xi^{\xi'} G\, d\xi'' + \frac{a}{2}\Big[\frac{1}{A(E)}\int_\xi^{\xi+2\pi} U_0 G\, d\xi' \Big(\int_\xi^{\xi'} G\, d\xi''\Big)^2 - \int_\xi^{\xi+2\pi} U_0 G\, d\xi' \int_\xi^{\xi'} G\, d\xi''\Big]; \tag{29}$$

b) trapped electrons

$$g^{\pm}(s,\xi) = \frac{1}{\nu_p}\sum_{p'} w_{pp'}\{g_{p'}\}_\xi + \frac{a}{2B}\int_{\xi_1}^{\xi_2} U_0 G\, d\xi' \Big(\int_\xi^{\xi_1} G\, d\xi''\Big)^2 + \operatorname{sign} s \int_\xi^{\xi_2} U_0 G\, d\xi' - \frac{a}{2}\Big[\int_\xi^{\xi_2} U_0 G\, d\xi' \Big(\int_{\xi'}^{\xi'} G\, d\xi''\Big)\Big(1 + \frac{C(\xi)}{B}\Big) + \int_{\xi_1}^{\xi} U_0 G\, d\xi' \Big(\int_{\xi'}^{\xi} G\, d\xi''\Big)\Big(1 - \frac{C(\xi)}{B}\Big)\Big]. \tag{30}$$

The expansions (29) and (30) are not applicable in the narrow range of energies $|E - 1| \lesssim \exp(-1/a)$, which is not important for the calculation of the physical quantities of interest, inasmuch as the distribution function is continuous.

3. SOUND ABSORPTION. THE ROLE OF HIGHER HARMONICS

We found the distribution function of the electrons in a variable external field with the potential $\varphi(x - wt)$. We assume that the periodic displacement $u = u_0 \cos \omega t$ is given on the boundary of the crystal. It is then easy to see that the system (6)—(8), (11) is satisfied by the wave

$$\varphi = \varphi_0 \cos(qx - \omega t) \tag{31}$$

with amplitude φ_0 that depends slightly on the coordinates. Actually, substituting (31) in (29) and (30), and then (29), (30) in (8), we obtain the result that the higher harmonics of the concentration have the order[4] in

$$\max((e\varphi_0/\bar{\epsilon})^n, \quad aw/\bar{v}) \ll 1 \tag{32}$$

comparison with the amplitude of the fundamental. In view of Poisson's equation, this statement is valid also for the harmonics of the potential, which justifies the assumption made above.

The sound absorption coefficient is determined by the reactive part of the response of the concentration to the perturbation (31)

$$K_q''(\omega) = \frac{2[e\varphi_0]^{-1}}{(2\pi\hbar)^3}\int d^3p \int_0^{2\pi} \frac{(-\sin\xi)}{\pi} g_p(\xi)\, d\xi. \tag{33}$$

Equation (33) is obtained with account of the fact that

$$\int_0^{2\pi} (-\sin\xi) F_0(\epsilon + e\varphi_0 \cos\xi)\, d\xi = 0.$$

On the other hand, the active part of the concentration response, which determines the screening, is produced by the function $F_0(\epsilon + e\varphi_0 \cos\xi)$. With accuracy to within $e\varphi_0/\bar{\epsilon}$, we have

$$4\pi e^2 \epsilon_0^{-1} K_q'(\omega) = \varkappa^2, \tag{34}$$

where κ is a quantity that is the reciprocal of the Debye screening radius, equal to $\sqrt{4\pi n_0 e^2/\epsilon_0 T}$ in the case of Boltzmann statistics and $[(4me^2/\epsilon_0\hbar^2) \times (3n_0/\pi)^{1/3}]^{1/2}$ in the case of Fermi statistics. Therefore,

$$\frac{\Gamma}{\Gamma_0} = \frac{K_q''(\omega)}{\eta w/\bar{v}}, \quad \Gamma_0 = \chi q \eta \frac{4\pi e^2}{\epsilon_0 q^2}\frac{w}{\bar{v}}\Big(1 + \frac{\varkappa^2}{q^2}\Big)^{-2}. \tag{35}$$

Here Γ_0 is the linear absorption coefficient, $\bar{v} = v_F$, $\eta = m\hbar^{-2}(3n_0/\pi)^{1/3}$ in the case of Fermi statistics; $\bar{v} = \sqrt{2T/m}$, $\eta = \sqrt{\pi} n_0/T$ in the case of Boltzmann statistics. The results for piezoelectric and deformation interactions differ by the values of the dimensionless coupling constant χ. For the piezoelectric interaction $\chi = 4\pi\beta^2/\epsilon_0 c$, for the deformation interaction $\chi = \Lambda^2 \epsilon_0 q^2/4\pi e^2 c$.

Using (29) and (30), we have

$$\frac{\Gamma}{\Gamma_0} = \gamma_1 \langle a \rangle, \quad \langle a \rangle = \int d^2p_\perp \frac{\partial F_0}{\partial \varepsilon} a(p_\perp) \Big/ \int d^2p_\perp \frac{\partial F_0}{\partial \varepsilon}, \tag{36}$$

where γ_1 is a number of the order unity, obtained by integration of (29), (30) over ξ and E with the weight $(-\sin\xi)$. Numerical calculation gives the value 1.1 for γ.

4. ACOUSTO-ELECTRIC CURRENT AND THE HEATING OF THE ELECTRONS BY THE SOUND WAVE

The acousto-electric current is equal to

$$j^{ae} = e\sum_p v\{f_p\}_\xi, \tag{37}$$

where $\{f_p\}_\xi$ is the slowly varying part of the electron distribution function. Inasmuch as $F_0(\epsilon + e\varphi)$ does not contain a current part, one can put $\{g_p\}_\xi$ in place of $\{f_p\}_\xi$ in (37). The quantity $\{g_p\}_\xi$ satisfies Eq. (28). For the calculation of j^{ac}, we multiply (28) by the velocity vector v and sum over the surface of constant energy. Since

$$\sum_p v\hat{I}\{g_p\}_\xi \delta(\varepsilon - \varepsilon_p) = \frac{1}{\tau_{tr}(\varepsilon)}\sum_p v\{g_p\}_\xi \delta(\varepsilon - \varepsilon_p), \tag{38}$$

where τ_{tr} is the transport relaxation time, the acousto-electric current is expressed directly in terms of the right side of (28). Finally, we get a relation of the type of the Weinreich relation[9]

$$j^{ae} = k_s \Gamma S \sigma / e n_0 w. \tag{39}$$

for the acousto-electric current. Here σ is the electric conductivity, n_0 the electron concentration.

$$k_s = \Big\langle \frac{\nu_p}{\nu_{tr}} \Big\rangle \Big/ \langle \nu_p \rangle \Big\langle\Big\langle \frac{1}{\nu_{tr}} \Big\rangle\Big\rangle,$$

[4] Terms of the order $(e\varphi_0/\bar{\epsilon})^n$ arise in the expansion of the function $F_0(\epsilon + e\varphi)$ in powers of $(e\varphi_0/\bar{\epsilon})$.

$$\left\langle\!\left\langle \frac{1}{\nu_{tr}} \right\rangle\!\right\rangle = \frac{2}{(2\pi\hbar)^3}\int d^3p \frac{\partial F_0}{\partial \varepsilon}\frac{mv^2}{3}\left(\frac{1}{\nu_{tr}(\varepsilon)}\right); \quad \nu_{tr} = \frac{1}{\tau_{tr}}. \tag{40}$$

The coefficient k_S in the case of Fermi statistics is equal to unity and in the case of Boltzmann statistics depends on the scattering mechanism: it is equal to $\frac{1}{8}$ for scattering from ionized impurities and to $\frac{3}{2}$ for scattering from acoustic phonons.

It is convenient to estimate the heating of the electrons directly from the kinetic equation, averaged over the time:

$$\int_0^{2\pi} d\xi \left(\frac{d\psi}{d\xi}\frac{\partial g(\xi)}{\partial p_x} + \hat{I}g(\xi)\right) = 0. \tag{41}$$

Then, averaging (41) over the surface of constant energy, we have

$$\langle \hat{I}\{g\}_\xi \rangle_\varepsilon = \left(\int d^3p\,\delta(\varepsilon - \varepsilon_p)\right)^{-1} \int d^3p\,\delta(\varepsilon - \varepsilon_p) v \left\{ g \frac{d\psi}{d\xi}\right\}_\xi. \tag{42}$$

Substituting (29) and (30) in this expression, it is easy to estimate the heating parameter. In order of magnitude, it is equal to

$$a\left(\frac{e\varphi_0}{\varepsilon}\right)^2\left(\frac{w}{\bar{v}}\right)^2\frac{\tau_\varepsilon}{\tau_p}ql \sim \left(\frac{e\varphi_0}{\varepsilon}\right)^{3/2}\left(\frac{w}{\bar{v}}\right)^2\frac{\tau_\varepsilon}{\tau_p}, \tag{43}$$

where τ_ϵ is the energy relaxation time of the electrons.

Thus, for $e\varphi_0/\bar{\epsilon} \ll 1$, the heating of the electrons by the sound wave is unimportant (usually, $(w/\bar{v})^2\tau_\epsilon/\tau_p \lesssim 1$). We take this opportunity to remark that the electron heating by the sound wave is incorrectly treated in[7] even within the framework of the iteration method. The point is that the distribution function f_p in[7] is written in the form $F_0(\epsilon) + f_p^{(1)}(x, t)$ in contrast with (14). In such a splitting of the function f_p it is impossible to neglect the arrival term when $f_p^{(1)}$ is acted upon by the collision operator, inasmuch as $f_p^{(1)}$ has a smooth part in momentum space. We are grateful to P. E. Zil'berman who pointed out this circumstance to us. A similar omission is evidently contained in[10], in which, by a method similar to that used by us, the problem of the absorption of a plasma wave in weakly ionized plasma is solved. This, it seems to us, leads to an inaccurate expression for the distribution function.

5. EFFECT OF THE EXTERNAL ELECTRIC FIELD

We have calculated the sound absorption coefficient and the acousto-electric current. If the crystal is placed in an external constant electric field ϵ, the sound absorption increases if the drift velocity of the electrons is sufficiently large. It can be shown that the nonlinear amplification factor is obtained from the nonlinear sound absorption coefficient by the replacement of the sound velocity w by $w - k_S v_{dr}$, where $v_{dr} = \Sigma v\{f_p\}_\xi$. This result is correct in a moderately strong electric field, when the change of the electron energy in the external field over one wavelength is much less than the potential energy of the electron in the field of the wave, i.e., $\epsilon \ll q\varphi_0$. In the opposite case, the external field decreases considerably the effective depth of the potential wells and consequently weakens the nonlinearity. A detailed analysis of this phenomenon which is correct in order of magnitude, is given in[11].

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