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Dispersion of Spin Waves in Thin Metallic Films

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This investigation concerns the nature of certain singularities in the spin-wave spectrum of thin films of ferromagnetic metals. The singularities are due to dimensional quantization of the conduction electrons.

THERE are now quite a large number of papers devoted to the theory of spin waves in thin films[1-6]. The first work in this direction was the article of Klein and Smith, in which a theory of the spontaneous magnetization of films was constructed[1]. The magnon spectrum in films was also investigated in connection with investigations on the absorption of a high-frequency magnetic field by films of ferromagnetic materials (spin-wave resonance)[2]. It was shown that allowance for the finiteness of the specimen dimensions and imposition of boundary conditions on its surface transform the spectrum of possible values of spin-wave energy into a discrete one. The further improvement of the theory of spin waves in films reduced basically to a taking into account of the actual boundary conditions that occur on the surface of the film[3,4]; it was noted that the boundary conditions exert a great influence on the spin-wave spectrum. The investigation of spin-wave resonance in films was the subject of papers by Kaganov (see, for example,[5,6]), in which the influence of the finite conductivity of the ferromagnet on the magnon spectrum was also taken into account.

In ferromagnetic metals, the conduction electrons exert a great influence on the magnon spectrum. Between the spin waves and the conduction electrons there is an interaction that consists in absorption and emission of spin waves by conduction electrons, and that is accompanied by inversion of the spin of the conduction electron. It leads, in particular, to a finite relaxation time both of the electrons and of the magnons, and causes threshold effects and attenuation of the spin waves. The present article considers the influence of this interaction on the spin-wave spectrum in a thin film of a ferromagnetic metal in zero magnetic field, at $T = 0$, with allowance for dimensional quantization of the conduction electrons. In the calculation of the magnon spectrum, we shall suppose that there are in the metal under consideration conduction electrons (s-electrons) and d(f)-electrons localized on atoms of the lattice. The s-d interaction leads to a partial polarization of the conduction electrons. The Fermi energies corresponding to the two possible orientations of the spins of the s-electrons (each reckoned from its own "bottom") differ in our case by a quantity of the order of $(\theta\epsilon_F)^{1/2}$, where θ is the Curie energy and ϵ_F is the Fermi energy of the conduction electrons. This quantity determines the order of magnitude of the energy of interaction between the s-electrons and the spin waves[7].

The spin-wave spectrum is determined from the equation (see[8])

$$\omega - \omega_n(\mathbf{k}) - \Pi(\omega_n(\mathbf{k})) = 0, \tag{1}$$

where Π is the polarization operator and $\omega_n(\mathbf{k})$ is the energy of a spin wave (without allowance for its interaction with the s-electrons). The latter can be written, for the isotropic case, in the form

$$\omega_n(k) = \theta(k^2 + k_z^2) / p_0^2, \tag{2}$$

where $k^2 = k_x^2 + k_y^2$ and $k_z = \pi\hbar n/L$ $(n = 0, 1, 2, \ldots)$, L is the film thickness, and p_0 is the Fermi momentum of a conduction electron. The polarization operator $\Pi(\omega_n(\mathbf{k}))$ can be calculated by use of a method described in[8]:

$$\Pi(\omega_n(\mathbf{k})) = -\frac{g^2 a_+ a_-}{(2\pi)^2 L} \sum_{\nu=-N}^{N}{}' \int d^2\mathbf{p} \left[\frac{n(\varepsilon_{\nu+n}^{+}(\mathbf{p}+\mathbf{k})) - n(\varepsilon_\nu^{-}(\mathbf{p}))}{\omega_n(\mathbf{k}) - \varepsilon^{+}_{\nu+n}(\mathbf{p}+\mathbf{k}) + \varepsilon^{-}(\mathbf{p}) + i\delta} - \frac{n(\varepsilon_\nu^{+}(\mathbf{p})) - n(\varepsilon_\nu^{-}(\mathbf{p}))}{\omega_n(\mathbf{k}) - \varepsilon_\nu^{+}(\mathbf{p}) + \varepsilon_\nu^{-}(\mathbf{p}) + i\delta} \right]. \tag{3}$$

Here the prime on the summation sign means that the terms of the sum with $\nu = \pm 1, \pm 2, \ldots$; g^2 is an electron-magnon interaction constant; a_+ and a_- are renormalization constants of the order of unity; $n(\epsilon)$ is the Fermi distribution function at zero temperature; and $\epsilon^+(p)$ and $\epsilon^-(p)$ are the energies of an s-electron with positive and negative spin orientations, respectively, in the thin film. This differs significantly from the energy of an electron in an infinite crystal; in the effective-mass approximation, it has the form

$$\varepsilon_\nu^{\pm}(p) = (p_x^2 + p_y^2) / 2m^* + \varepsilon_1 \nu^2 - p_\pm^2 / 2m^*, \tag{4}$$

where $p_\pm$ are of the cutoff momenta of s-electrons in the positive and negative spin orientations, $\epsilon_1 = \pi^2\hbar^2/2m^*L^2$, and N is determined from the relation $N^2\epsilon_1 = \epsilon_F$. The calculation (4) assumes also the equality of the effective masses m^* of the s-electrons. Equation (3) differs from the corresponding equation for magnons in an unbounded crystal[7] by replacement of integration by a summation with respect to the discrete electron momentum values $p_z = \pi\hbar\nu/L$.

We shall now show that allowance for dimensional quantization of the conduction electrons in a thin film significantly affects the spectrum of spin waves in the film. For this purpose, we go over to the calculation of the polarization operator $\Pi(\omega_n(\mathbf{k}))$. On carrying out the integration in (3), we get to within terms of order k/p_0

$$\operatorname{Re}\Pi(\omega_n(\mathbf{k})) = -\frac{g^2\hbar}{2\pi L} \sum_{\nu=-N}^{N}{}' \left\{ \frac{1}{k^2}\left[\sqrt{X_+} - \sqrt{X_-}\right] - \frac{\varepsilon_0 + \omega_n(\mathbf{k})}{\sqrt{X_+}} - \frac{(p_{+\nu}^2 - p_{-\nu}^2)/2m^*}{\varepsilon_0 - \omega_n(\mathbf{k})} + 1 \right\}, \tag{5}$$

$$X_\pm = (\varepsilon_0 - 2\varepsilon_1 \nu n + \omega_n(k))^2 - p_{\pm\nu}^2 k^2 / m^{*2},$$

$$p_{\pm\nu}^2 = p_\pm^2 - 2m^*\varepsilon_1\nu^2.$$

Here $\epsilon_0 \sim \sqrt{\theta\epsilon_F}$ is the separation of the Fermi surface. Equation (5) is valid for $X_{\pm} > 0$; in the contrary case, the imaginary part of ω, which determines the attenuation of the spin waves, becomes large:

$$\operatorname{Im}\omega = \frac{g^2\hbar}{2\pi L}\sum_{\nu=-N}^{N}\left\{\frac{1}{k^2}\left[\sqrt{-X_+}\Theta(-X_+) - \sqrt{-X_-}\Theta(-X_-)\right] - \frac{\varepsilon_0 + \omega_n(\mathbf{k})}{\sqrt{X_+}}\Theta(-X_+)\right\}, \quad (6)$$

where

$$\Theta(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x < 0 \end{cases}$$

We mention also that equations (5) and (6) were calculated under very general assumptions regarding $\omega_n(\mathbf{k})$. From (5) and (6) it follows that in the simplest case, when $k_z = 0$ and $k \to 0$,

$$\operatorname{Im}\omega \to 0, \quad \operatorname{Re}\Pi(\omega_n(\mathbf{k})) \to 0,$$
$$\operatorname{Im}\omega / \operatorname{Re}\omega \to 0.$$

For normal metals with film thickness greater than 100 Å, $\epsilon_F \gg \epsilon_1$ (the quasiclassical case). Using this fact, replacing the sum over ν in (5) by an integration, we shall calculate $\Pi(\omega_n(\mathbf{k}))$, using the estimate $g^2 \sim \theta\epsilon_F/p_0^3$ obtained by Kondratenko[7]. For thin films that satisfy the condition

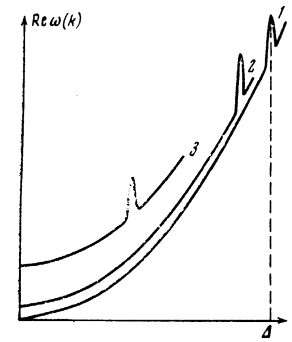
$$\varepsilon_1 \lesssim \theta, \quad (7)$$

and for k and $k_z \ll \Delta$ ($\Delta = p^+ - p^-$), the spin-wave spectrum is determined by the expression

$$\omega \approx [1 - 2(\theta/\varepsilon_F)^{1/2}]\omega_n(\mathbf{k}),$$

which agrees with the results obtained for an infinite crystal[8].

The interaction of magnons with s-electrons in a thin film leads to singularities in the magnon spectrum; these occur in consequence of the splitting of spin waves into a pair of Fermi excitations with a discrete value of the transverse momentum. Thus when condition (7) is satisfied, there are pulses on the curve of the spectrum (see Figure) at values $k \approx (\Delta^2 - k_z^2)^{1/2}$; in ordinary metals, these can attain values some tenths of ω. It can be shown that the width of these pulses is of the order $\Delta(\theta/\epsilon_F)^{1/2}$; consequently, in experimental investigation of the spin-wave spectrum in thin films, it is possible to separate with confidence up to ten peaks on the $\omega(k)$ curve. From formula (5) it is also clear that in the magnon spectrum, besides the singularities indicated above, which have logarithmic character, at $k_z > \Delta$ there are root singularities of the type $[(\epsilon_0 - \epsilon_1 n^2 - 2\epsilon_1 n)^2 - p_0^2 k^2/m^{*2}]^{1/2}$. When the film thickness is such that $\theta < \epsilon_1 < \epsilon_0$, there are only root singularities in the spectrum, and no logarithmic singularities. But for thicknesses such that $\epsilon_1 \approx \epsilon_0$, the attenuation becomes especially large for magnons with $n > 0$; this is due to many-particle processes, and apparently the spin wave is propagated predominantly in the film plane. The spectrum for $k \ll \Delta$ is given by the expression



Dependence of Re ω on k for n equal to 0 (curve 1), 1 (curve 2), and 2 (curve 3).

$$\omega \sim [1 - 2(\theta/\varepsilon_F)^{1/2}]\omega_0(\mathbf{k}).$$

At large magnon momenta, $k > \Delta$, there is also possible a splitting of the magnon into an electron and a hole with a discrete value of the momentum; and, it would appear, this effect should lead to oscillations and attenuation of magnons and to singularities in the spectrum, similar to what occurs in the phonon spectrum[9]. In actual fact this does not occur, because of the large separation of the Fermi surface. The width of the interval in which there occurs attenuation connected with the splitting of a spin wave into an electron and a hole is large in comparison with the distance between neighboring peaks in the attenuation; as a result, the total attenuation contains no oscillations of any kind—that is, the magnon spectrum in this case is practically the same as for a bulk specimen.

The figure gives the dependence of Re ω on k for films with a thickness L determined from the condition $\epsilon_1 \approx 0.2\Delta^2$; for metals with Curie point 300°K and $m^* = m_0$, this amounts to 400 Å.

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