

Electron Temperature of a Plasma Scattering Intense Light Beams

A. V. VINOGRADOV AND V. V. PUSTOVALOV

P. N. Lebedev Physics Institute, USSR Academy of Sciences

Submitted September 14, 1971

Zh. Eksp. Teor. Fiz. 62, 980-988 (March, 1972)

Stimulated Compton scattering of laser radiation by an electron plasma is considered. A general expression is obtained for the diffusion coefficient of electrons located in an isotropic radiation field of small spectral width. The heating rate of a Maxwellian plasma due to Compton scattering of the laser radiation and also a self-similar solution for the electron velocity distribution function are determined.

At the present time, the problem of attaining high temperatures by applying powerful laser pulses on solid targets has given rise to the question of the possible mechanisms whereby the energy of the light is absorbed in the plasma. The point is that the usual bremsstrahlung absorption, connected with the Coulomb collisions of the electrons with the ions, loses its efficacy at the presently attained laser-plasma parameters (electron temperature $T_e \sim 1$ keV and electron density $N_e \approx 10^{19} - 10^{22} \text{ cm}^{-3}$). Various linear and non-linear effects that lead to an increase of the absorption coefficient have been discussed in this connection, namely stimulated scattering of light by electrons^[1-3], parametric instability of the plasma in the field of the light wave^[4], and linear transformation of the light into damped Langmuir oscillations^[5]. We shall deal in the present article only with the first of these effects, namely stimulated (induced) scattering of light by electrons. The general theory of this phenomenon is a component part of the theory of weakly-turbulent plasma (see, for example,^[6-10]) and has been developed in sufficient detail. This theory is used here to study the singularities in the absorption of laser radiation by a plasma.

Most investigations of concrete manifestations of induced scattering of light in a plasma have been made, in one form or another, in connection with astrophysical problems (see, however,^[9]), in which, as a rule, a broad radiation spectrum is considered: $\Delta\omega \sim \omega_0$ (ω_0 and $\Delta\omega$ are the frequency and the width of the spectrum), and the characteristic velocity of the scattered particles satisfies the condition $v \ll c\Delta\omega/\omega_0$ (c is the speed of light). To the contrary, the laser radiation has a narrow spectrum ($\Delta\omega/\omega_0 \lesssim 10^{-2}$). Therefore in a high-temperature laser plasma there is realized the opposite limiting case $v \gg c\Delta\omega/\omega_0$, when the Doppler shift in the elementary scattering act exceeds the line width. As will be shown below, this circumstance changes noticeably the evolution of the distribution function of the electrons and the rate of radiation-energy input to the plasma.

In the first section we obtain an expression for the diffusion coefficient $D_{ij}(v)$ of free electrons situated in an isotropic radiation field. The radiation spectrum is assumed to be sufficiently narrow, $\Delta\omega \ll \omega_0$, but no limitations whatever are imposed on the parameter $v\omega_0/c\Delta\omega$. In the second section we present a general expression for the rate of heating of a Maxwellian

plasma as a function of the parameters of the heating radiation, and also of the temperature and density of the plasma. At high temperatures, the rate of heating decreases like $T_e^{-3/2}$, just as in the case of bremsstrahlung absorption, this being connected with the rapid decrease of the longitudinal diffusion coefficient $D^l(v) \sim v^{-3}$ when $v \gg c\Delta\omega/\omega_0$. In the case of radiation with a sufficiently broad spectrum, $v \ll c\Delta\omega/\omega_0$ and $vT_e \ll c\Delta\omega/\omega_0$ ($vT_e = (\kappa T_e/m)^{1/2}$ and m are the thermal velocity and mass of the electron and κ is Boltzmann's constant), the results agree with the conclusions of^[1-3]. In the third section we obtain a self-similar solution for the distribution function of the electrons that scatter high-power laser radiation. The rate of heating corresponding to the self-similar distribution function agrees qualitatively with the results obtained in Secs. 1 and 2 for a Maxwellian plasma.

1. DIFFUSION COEFFICIENT OF FREE ELECTRONS SITUATED IN AN ISOTROPIC RADIATION FIELD

Let us consider the interaction of radiation with a fully ionized plasma of such a low density that the most probable elementary process is Compton scattering of photons by electrons. The conditions under which we can neglect the collisions of the electrons with the ions will be clarified below. The equations for the distribution function of the electrons $f(v, t)$ and for the spectral density of the number of photons $N_k(t)$ with wave vector k are easiest to obtain by regarding the system as an aggregate of two gases—electrons and photons—the interaction between which is described by the usual collision term. If the momentum transfer in one collision is much smaller than the electron momentum $\hbar\omega_0/c \ll mv$, which is always the case in a laser plasma, then we can use the diffusion approximation in the kinetic equation for the electrons, so that the system of equations for $f(v, t)$ and $N_k(t)$ takes the form^[7, 11]:

$$\frac{\partial N_k}{\partial t} = 2 \frac{\hbar c^4}{m} N_k \int dk' dv N_{k'} \sigma_T(\mathbf{x}, \mathbf{x}') \frac{1}{\omega \omega'} \delta(\omega'' - k''v) k'' \frac{\partial f}{\partial v},$$

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v_i} D_{ij} \frac{\partial f}{\partial v_j}, \tag{1}$$

$$D_{ij}(v) = \frac{\hbar^2 c^4}{m^2} \int \frac{dk dk'}{(2\pi)^3} N_k N_{k'} \sigma_T(\mathbf{x}, \mathbf{x}') \frac{1}{\omega \omega'} k_i'' k_j'' \delta(\omega'' - k''v), \tag{2}$$

$$\sigma_T \equiv \frac{1}{2} \left(\frac{e^2}{mc^2} \right)^2 [1 + (\mathbf{x}\mathbf{x}')^2], \quad \omega'' \equiv \omega - \omega', \quad k'' = k - k' \tag{3}$$

where $\kappa = \mathbf{k}/k$, e is the electron charge, and σ_T is the Thomson cross section. We have discarded in (1)–(3) the terms describing the spontaneous scattering, since the radiation is assumed to be sufficiently strong.

Two simplified approaches corresponding to two different physical formulations of the problem are possible (see^[1,11]). In one of them the distribution function of the electrons is assumed known and the problem is to find the radiation spectrum $N_{\mathbf{k}}$ ^[12]. In the other the radiation spectrum is specified by the external source, and it is required to find the electron distribution function^[13]. We are interested mainly in the second formulation of the problem, when the radiation spectrum is specified by the laser parameters.

Before we proceed to investigate Eq. (2), we must calculate the diffusion coefficient $D_{ij}(\mathbf{v})$. We assume that the emission spectrum $N_{\mathbf{k}}$ is isotropic. Such an approximation agrees with experiments in strongly focused beams or with volume illumination of the target^[14]. The diffusion coefficient is then determined by two scalar functions

$$D_{ij}(\mathbf{v}) = D^l(v) \frac{v_i v_j}{v^2} + D^{tr}(v) \left(\delta_{ij} - \frac{v_i v_j}{v^2} \right). \quad (4)$$

It is easy to verify with the aid of (2) and (4) that the rate of heating is determined by the longitudinal diffusion coefficient $D^l(\mathbf{v})$:

$$\frac{\partial}{\partial t} \int d\mathbf{v} \frac{m v^2}{2} f(\mathbf{v}, t) = m \int d\mathbf{v} f(\mathbf{v}, t) \left\{ 3D^l(v) + v \frac{\partial D^l(v)}{\partial v} \right\}, \quad (5)$$

$$D^l(v) = \frac{1}{2(2\pi)^3} \left(\frac{e}{m} \right)^4 \hbar^2 \int \frac{d\mathbf{k} d\mathbf{k}'}{\omega \omega'} N_{\mathbf{k}} N_{\mathbf{k}'} [1 + (\kappa \kappa')^2] \frac{\omega''^2}{v^2} \delta(\omega'' - \mathbf{k}' \mathbf{v}). \quad (6)$$

The laser radiation has a narrow spectrum. Therefore, using the approximation $\Delta\omega \ll \omega_0$ in the calculation of (6), we put $\mathbf{k}'' \approx \omega_0(\boldsymbol{\kappa} - \boldsymbol{\kappa}')/c$ and take $\omega \approx \omega' \approx \omega_0$ outside the integral sign:

$$D^l(v) = \frac{1}{2(2\pi)^3} \left(\frac{e}{m} \right)^4 \frac{\hbar^2 \omega_0^2}{v^2 c^2} \int d\omega d\omega' N_{\omega} N_{\omega'} \times \int d\boldsymbol{\kappa} d\boldsymbol{\kappa}' [1 + (\boldsymbol{\kappa} \boldsymbol{\kappa}')^2] \delta \left[\omega'' - \frac{\omega_0}{c} (\boldsymbol{\kappa} - \boldsymbol{\kappa}', \mathbf{v}) \right].$$

Integration along the propagation direction of the scattered photons yields

$$D^l(v) = \frac{1}{15\pi} \left(\frac{e^2}{mc^2} \right)^2 \frac{\hbar^2 \omega_0}{m^2 c v^2} \int d\omega d\omega' N_{\omega} N_{\omega'} (\omega - \omega')^2 \times \left\{ 11 - 15 \frac{|\omega - \omega'|}{2\omega_0} \frac{c}{v} + 10 \left(\frac{|\omega - \omega'|}{2\omega_0} \frac{c}{v} \right)^3 - 6 \left(\frac{|\omega - \omega'|}{2\omega_0} \frac{c}{v} \right)^5 \right\} \quad (7)$$

This formula, which determines the diffusion coefficient at arbitrary velocity of the scattered electrons, is one of the main results of the present study.

If N_{ω} is a smooth spectrum with sufficiently rapidly decreasing wings, then, using the expansion

$$N_{\omega'} = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n N_{\omega}}{d\omega^n} (\omega' - \omega)^n,$$

we can integrate with respect to ω' in (7). The resultant expression is actually an expansion of the diffusion coefficient in powers of the parameter $2\omega_0 v / \Delta\omega c$:

$$D^l(v) = \frac{8}{\pi} \left(\frac{e^2}{mc^2} \right)^2 \frac{\hbar^2}{m^2} \left(\frac{\omega_0}{c} \right)^4 \times \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \frac{n^2 + 5n + 8}{(2n+3)(n+2)(n+3)(n+4)} \left(\frac{2\omega_0 v}{c} \right)^{2n} \int d\omega \left(\frac{d^n N_{\omega}}{d\omega^n} \right)^2 \quad (8)$$

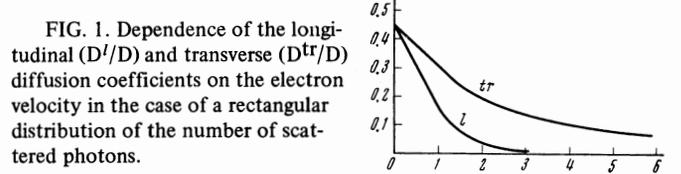


FIG. 1. Dependence of the longitudinal (D^l/D) and transverse (D^{tr}/D) diffusion coefficients on the electron velocity in the case of a rectangular distribution of the number of scattered photons.

In the opposite limiting case of large velocities, $v \geq c \Delta\omega / 2\omega_0$, formula (7) yields

$$D^l(v) = \frac{1}{15\pi} \left(\frac{e^2}{mc^2} \right)^2 \frac{\hbar^2 \omega_0}{m^2 c v^2} \int d\omega d\omega' N_{\omega} N_{\omega'} (\omega - \omega')^2 \times \left\{ 11 - 15 \frac{|\omega - \omega'|}{2\omega_0} \frac{c}{v} + 10 \left(\frac{|\omega - \omega'|}{2\omega_0} \frac{c}{v} \right)^3 - 6 \left(\frac{|\omega - \omega'|}{2\omega_0} \frac{c}{v} \right)^5 \right\} \quad (9)$$

Unlike in (7), the integration in (9) is over the entire spectrum.

A simple expression for the diffusion coefficient can be obtained with the aid of (7) by assuming that the spectrum of the heating radiation is rectangular, i.e., $N_{\omega} = N_0$ at $2|\omega - \omega_0| < \Delta\omega$ and $N_{\omega} = 0$ at $2|\omega - \omega_0| > \Delta\omega$:

$$D^l(v) = D \left(\frac{4}{9} - \frac{86}{315} \frac{v}{v_0} \right), \quad v \leq v_0; \\ D^l(v) = D \left\{ \frac{22}{45} \left(\frac{v_0}{v} \right)^3 - \frac{2}{5} \left(\frac{v_0}{v} \right)^4 + \frac{8}{63} \left(\frac{v_0}{v} \right)^6 - \frac{2}{45} \left(\frac{v_0}{v} \right)^8 \right\}, \quad v \geq v_0. \quad (10)$$

The scale of the diffusion coefficient D is determined here by the intensity I of the radiation

$$D = \frac{(2\pi)^3}{m^2} \left(\frac{e^2}{mc^2} \right)^2 \frac{I^2}{\omega_0^2 \Delta\omega}, \quad I = \frac{\hbar \omega_0^3 \Delta\omega N_0}{2\pi^2 c^2}, \quad v_0 = c \frac{\Delta\omega}{2\omega_0}. \quad (11)$$

An analogous relation can also be obtained for the transverse diffusion coefficient $D^{tr}(v)$ (see^[15]). Figure 1 shows plots of the longitudinal and transverse diffusion coefficients against the velocity for a rectangular spectrum. As seen from a comparison of expressions (8), (9) and (10), the rectangular and smooth spectra lead to identical values of the diffusion coefficient at $v = 0$ and to an identical decrease $D^l(v) \sim v^{-3}$ at $v \gg v_0$. Therefore, proceeding to discuss the rate of plasma heating and the temporal evolution of the electron velocity distribution function, we shall assume, for concreteness, that the radiation spectrum is rectangular, although the main results also remain valid for any smooth spectrum.

2. RATE OF HEATING OF MAXWELLIAN PLASMA

We obtain the rate of electron heating by substituting (10) in the right-hand side of (5).

$$\frac{\partial}{\partial t} \int d\mathbf{v} \frac{m v^2}{2} f(\mathbf{v}, t) = m D \int d\mathbf{n} \left\{ \frac{4}{3} \int_0^{v_0} v^2 dv f(\mathbf{v}, t) \left[1 - \frac{86}{105} \frac{v}{v_0} \right] + 2 \int_{v_0}^{\infty} v^2 dv f(\mathbf{v}, t) \left[\frac{1}{5} \left(\frac{v_0}{v} \right)^4 - \frac{4}{21} \left(\frac{v_0}{v} \right)^6 + \frac{1}{9} \left(\frac{v_0}{v} \right)^8 \right] \right\} \quad \mathbf{n} = \frac{\mathbf{v}}{v}. \quad (12)$$

The structure of the diffusion coefficient (10) is such that the heating rate (12) is positive at any distribution function $f(\mathbf{v})$. It follows therefore that the stimulated Compton scattering leads to a decrease of the radiation energy and to heating (and not cooling) of the plasma electrons. This circumstance is quite obvious in the case of an equilibrium (Maxwellian) electron distribu-

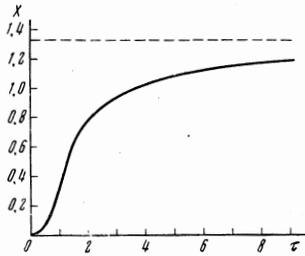


FIG. 2

tion function. Without an explicit calculation of the diffusion coefficient, it is in general impossible to draw a similar conclusion for a nonequilibrium electron velocity distribution.

To explain the dependence of the heating rate on the plasma parameters and the heating radiation, we consider in detail relation (12) in the case of an isotropic Maxwellian distribution

$$f(v) = \frac{N_e}{(2\pi)^{3/2} v_{Te}^3} \exp\left(-\frac{1}{2} \frac{v^2}{v_{Te}^2}\right).$$

It is convenient to represent the result in the form

$$\begin{aligned} \frac{d\kappa T_e}{dt} &= \frac{2}{3} mDX(\tau), \quad \tau = \frac{1}{2\sqrt{2}} \frac{c\Delta\omega}{v_{Te}\omega_0}, \\ X(\tau) &= 8\Phi(\tau) \left\{ \frac{1}{6} + \frac{1}{5}\tau^4 + \frac{8}{63}\tau^6 + \frac{4}{135}\tau^8 \right\} + \frac{8}{\sqrt{\pi}} e^{-\tau^2} \\ &\times \left\{ \frac{86}{315} \frac{1}{\tau} - \frac{19}{315}\tau + \frac{10}{63}\tau^3 + \frac{134}{945}\tau^5 + \frac{4}{135}\tau^7 \right\} - 8 \left\{ \frac{1}{5}\tau^4 + \frac{8}{63}\tau^6 \right. \\ &\left. + \frac{4}{135}\tau^8 + \frac{86}{315\sqrt{\pi}} \frac{1}{\tau} \right\}; \quad \Phi(\tau) \equiv \frac{2}{\sqrt{\pi}} \int_0^\tau dx e^{-x^2}. \end{aligned} \quad (13)$$

A plot of the function $X(\tau)$ is shown in Fig. 2. Using asymptotic expressions for $\Phi(\tau)$ at $\tau \gg 1$ and $\tau \ll 1$ ^[16], we obtain

$$X(\tau) \approx \frac{88}{45\sqrt{\pi}} \tau^3 - \frac{8}{5} \tau^4, \quad \tau \ll 1; \quad (14)$$

$$X(\tau) \approx \frac{4}{3} - \frac{688}{315\sqrt{\pi}} \tau^{-1}, \quad \tau \gg 1. \quad (15)$$

At low temperatures, when $vT_e \ll c\Delta\omega/\omega_0$, we have in accordance with (15) $X(\tau) = 4/3$ and formula (13) goes over into the expression used in^[1-3]. Substituting the value $\Delta\omega/\omega_0 \approx 3 \times 10^{-3}$ which is characteristic of lasers, we see that the opposite limiting case $\tau \ll 1$ is realized in a high-temperature laser plasma with $T_e \gg 1$ eV. In this case, using (13), (14), and (11), we obtain

$$\frac{d\kappa T_e}{dt} \approx \frac{11(2\pi)^{3/2}}{135} \left(\frac{e^2}{mc^2}\right)^2 \frac{I^2}{m\omega_0^3} \left(\frac{\Delta\omega}{\omega_0}\right)^2 \left(\frac{mc^2}{\kappa T_e}\right)^{3/2}. \quad (16)$$

It is interesting to determine the radiation flux intensity I_0 above which the rate of plasma heating by induced scattering by the electrons prevails over the rate of heating due to the bremsstrahlung absorption. If $\kappa T_e \gg \kappa T_0 = 1/8 mc^2 (\Delta\omega/\omega_0)^2$, then, as seen from (16), the rate of heating in stimulated Compton scattering has the same temperature dependence as the frequency of the Coulomb collisions. The value of I_0 is therefore determined only by the electron density N_e , and does not depend on the plasma temperature. Equating (16) to the rate of heating in bremsstrahlung absorption

$$\frac{d\kappa T_e}{dt} = \frac{8\pi}{3} \frac{e^2 v_{ei} I}{m\omega_0^2 c}, \quad v_{ei} = \frac{4(2\pi)^{1/2}}{3} \frac{e^2 N_e L}{(\kappa T_e)^{3/2} m^{1/2}}$$

(L is the Coulomb logarithm), we obtain

$$I_0 = \frac{30L}{11\pi^2} m\omega_0^3 \left(\frac{\omega_{Le}}{\Delta\omega}\right)^2.$$

Here $\omega_{Le} = (4\pi N_e e^2/m)^{1/2}$ is the electron Langmuir frequency. In particular, at a temperature $\kappa T_e \approx 10^3 - 10^4$ eV and an electron density $N_e \approx 10^{19}$ cm⁻³ ($L = 10$), the heating of the electrons is described by formula (16), if the intensity of the neodymium laser with frequency $\omega_0 \approx 1.8 \times 10^{15}$ sec⁻¹ and width $\Delta\omega/\omega_0 \approx 3 \times 10^{-3}$ exceeds the critical value $I_0 \approx 1.8 \times 10^{15}$ W/cm².

3. DISTRIBUTION FUNCTION OF ELECTRONS SCATTERING POWERFUL LASER RADIATION

In isotropic radiation, it is also natural to assume that the electron velocity distribution is isotropic. In this case the equation for the distribution function follows from (2) and (4):

$$\frac{\partial f}{\partial t} = \frac{1}{v^2} \frac{\partial}{\partial v} \left\{ D'(v) v^2 \frac{\partial f}{\partial v} \right\}. \quad (17)$$

If the initial distribution is Maxwellian with a temperature $T_e \ll T_0$, then the condition $v \ll v_0$ is satisfied at the initial instant for the majority of the electrons, and the diffusion coefficient can be regarded as constant in accordance with (8) and (10)¹⁾:

$$D'(v) = D''(v) = {}^4_0D.$$

Then, as follows from (17), the distribution remains Maxwellian, and the electron temperature increases linearly with time:

$$T_e(t) = T_e(0) + 8Dmt/9\kappa. \quad (18)$$

When the temperature reaches a value of $\sim T_0$, the diffusion coefficient begins to decrease like v^{-3} , and the distribution function becomes distorted. However, as noted above, the plasma heating will continue, since the average electron energy increases in induced scattering.

Let us indicate a self-similar solution for the distribution function in the case when the average energy already exceeds κT_0 appreciably, and consequently the diffusion coefficient is determined by the principal term in the expansions (9) and (10):

$$D'(v) \approx {}^{22}_{13}D(v_0/v)^3.$$

We introduce the dimensionless velocity $x = v/v_0$ and the dimensionless time $y = 22Dt/45v_0^2$. We then obtain for the dimensionless distribution function $F(x, y) = v_0^3 f(v, t)$

$$\frac{\partial F}{\partial y} = \frac{1}{x^2} \frac{\partial}{\partial x} \left\{ \frac{1}{x} \frac{\partial F(x, y)}{\partial x} \right\}. \quad (19)$$

We seek the solution of (19) in the form²⁾

$$F(x, y) = \Phi(u) / \psi^3(y), \quad u = x / \psi(y). \quad (20)$$

We see that the distribution function (20) satisfies the normalization condition at any $\psi(y)$

¹⁾The stationary electron temperature in the presence of losses to spontaneous scattering was obtained for this case by Zel'dovich and Levich^[13].

²⁾Equation (19) admits of an analytic solution for arbitrary initial conditions. We confine ourselves here to the self-similar solution.

$$\int dv f(v, t) = 4\pi \int_0^{\infty} u^2 du \Phi(u) = N_e. \quad (21)$$

Substituting (20) in (19) we get

$$-\frac{\Psi'(y)}{\Psi^4(y)} \left\{ 3\Phi(u) + u \frac{d\Phi(u)}{du} \right\} = \frac{1}{\Psi^3(y)} \frac{1}{u^2} \frac{d}{du} \left\{ \frac{1}{u} \frac{d\Phi(u)}{du} \right\}. \quad (22)$$

Obviously, a solution of the type (20) satisfies Eq. (22) if

$$\Psi'(y) \Psi^4(y) = 1, \quad 3\Phi + u \frac{d\Phi}{du} = -\frac{1}{u^2} \frac{d}{du} \left\{ \frac{1}{u} \frac{d\Phi}{du} \right\}. \quad (23)$$

Solving Eqs. (23) under the condition (21), we obtain

$$\Psi(y) = [5(y + y_0)]^{1/5}, \quad \Phi(u) = \frac{5^{5/3}}{4\pi\Gamma(3/5)} N_e \exp\left(-\frac{u^5}{5}\right), \quad (24)$$

$$F(x, y) = \frac{N_e}{4\pi 5^{1/3} \Gamma(3/5)} \frac{\exp\{-x^3/25(y + y_0)\}}{(y + y_0)^{3/5}},$$

where y_0 is the integration constant.

Returning to the initial variables, we obtain a final expression for the electron velocity distribution function:

$$f(v, t) = \frac{N_e}{4\pi 5^{1/3} \Gamma(3/5) v_0^3} \left[\frac{45v_0^2}{22D(t + t_0)} \right]^{3/5} \exp\left\{-\frac{9v^2}{110v_0^2 D(t + t_0)}\right\}. \quad (25)$$

From this we readily obtain the average electron energy:

$$\int dv \frac{mv^2}{2} f(v, t) = 2\pi N_e m v_0^2 \Psi^2(y) \int_0^{\infty} u^4 du \Phi(u) \quad (26)$$

$$= \frac{N_e m v_0^2}{2\Gamma(3/5)} \left[\frac{110}{9} \frac{D}{v_0^2} (t + t_0) \right]^{3/5}.$$

The fact that the average electron energy is proportional to $t^{2/5}$ follows already from the rougher estimate $dT_e/dt \sim T_e^{-3/2}$ obtained for a Maxwellian distribution (see (16)). However, the decrease of the distribution function (25) at large velocities is more abrupt: $f(v) \sim \exp(-\alpha v^5)$. This is due to the rapid ($\sim v^{-3}$) decrease of the diffusion coefficient at high velocities.

In the case of laser radiation with a narrow spectrum, the temperature T_0 corresponding to the transition from the linear growth of the temperature to the law $T_e \sim t^{2/5}$ is only a few electron volts. Therefore in a high-temperature plasma heated by stimulated Compton scattering of laser radiation with isotropic $N_{\mathbf{k}}$ the evolution of the distribution function of the electrons is described by formulas (25) and (26), and not by (18). It should be noted at the same time that in the case of anisotropic spectral density of the number of photons $N_{\mathbf{k}}$, for example for a laser beam with small aperture ($\sim \theta \ll 1$), the critical temperature T_0 increases ($\sim \theta^{-2}$) and the region of applicability of formula (18) broadens^[1,3]. The number of electrons whose energy increases in accordance with (18), however, decreases in this case, so that in the limit of very well-collimated laser beams one should speak not of plasma heating but of acceleration of a small fraction of its electrons. A detailed analysis of this question is outside the scope of the present paper.

We note in addition that the physical mechanism discussed here, which leads to the electron distribution function $f(v, t)$ obtained above and to the heating of the electron plasma, differs from the mechanism that comes into play in the quasilinear theory of a parametrically unstable plasma situated in a field of a

monochromatic wave^[17-19]. The difference between the physical processes becomes manifest, naturally, also in the final formulas of the theory of a parametrically unstable plasma, and the theory of a plasma that scatters light beams. For example, in a parametrically unstable plasma situated in a homogeneous alternating electric field of frequency $\omega_0 \approx \omega_{Le}$, the plasma electron velocities undergo a redistribution^[19] that becomes manifest in an increase of the number of fast electrons in a narrow velocity region $\sim \omega_0/k_m$. At such velocities $\sim \omega_0/k_m$, the distribution function differs strongly from Maxwellian (compare with the Maxwellian distribution with temperature (18)) and is similar to the distribution function of the electrons of a plasma with an electron beam. The appearance of fast electrons is due to the Cerenkov interaction with the parametrically unstable potential oscillations (with characteristic wave number k_m), the buildup of which can be completely neglected in our present approximation ($\omega_0 \gg \omega_{Le}$) of the theory of stimulated Compton scattering of light in a plasma. Intense longitudinal noise of a parametrically unstable plasma leads for the same reason not only to acceleration of the particles but also to the plasma heating predicted by Silin^[17]. The foregoing exposition shows that even a stable plasma without powerful longitudinal noise becomes heated by stimulated Compton scattering of laser radiation.

CONCLUSION

The main results obtained in the present paper are contained in formulas (7)–(11), (13), and (25). It must be emphasized that our analysis is limited to conditions of a sufficiently rarefied electron plasma, when the polarizability of the ionic and electronic components of the plasma can be neglected. Our results are realistically applicable to a laser plasma if the electron density is $N_e \lesssim 10^{19} \text{ cm}^{-3}$ and the electron temperature is $\kappa T_e \gtrsim 1 \text{ keV}$. Allowance for the plasma polarization at the beat frequency of the scattered light beams radically alters the physical picture of plasma heating by powerful light. The reason for this is that the cross section for light scattering by a dense plasma differs strongly from the Thomson cross section and exceeds considerably it in some cases (for example, when induced scattering of light by ions predominates). One of the manifestations of this circumstance is the effect of direct heating of the ions of a hot plasma by an electromagnetic wave, first observed by Kovrizhnykh^[20]. Thus, from the points of view of further development of the results obtained here and the prospects for laser heating, it is quite advantageous to generalize the theory developed here to include a denser plasma. This will extend the region of applicability of the theory up to densities $N_e = 10^{21} \text{ cm}^{-3}$, corresponding to equality of the electron Langmuir frequency to the frequency of a neodymium laser. The next step in this direction should apparently consist of combining the theory of induced scattering of light in a plasma with the theory of parametric resonance, i.e., of an account of the unstable potential oscillations excited in the plasma by the scattered light (see^[21]).

The authors are grateful to L. M. Kovrizhnykh, O. N. Krokhin, V. P. Silin, I. I. Sobel'man, and E. A.

Yukov for a discussion of the work and valuable remarks.

Note added in proof (17 January 1972). The evolution of the radiation spectrum and a few other effects occurring in stimulated scattering of photons by free electrons were considered by Ya. B. Zel'dovich, Ye. V. Levich, and R. A. Syunyaev (Zh. Eksp. Teor. Fiz. 62, No. 4 (1972)). We are most grateful to the authors for the opportunity to become acquainted with their work prior to publication.

¹J. Peyraud, J. Phys. (Paris) 29, 88 (1968); J. Phys. (Paris) 29, 306 (1968); J. Phys. (Paris) 29, 872 (1968).

²Ya. B. Zel'dovich and E. V. Levich, Zh. Eksp. Teor. Fiz., Pis'ma Red. 11, 497 (1970) [JETP Lett. 11, 339 (1970)].

³F. V. Bunkin and A. E. Kazakov, Zh. Eksp. Teor. Fiz. 59, 2233 (1970) [Sov. Phys. JETP 32, 1208 (1971)].

⁴V. V. Pustovalov and V. P. Silin, Zh. Eksp. Teor. Fiz. 59, 2215 (1970) [Sov. Phys. JETP 32, 1198 (1971)].

⁵A. V. Vinogradov and V. V. Pustovalov, Zh. Eksp. Teor. Fiz., Pis'ma Red. 13, 317 (1971) [JETP Lett. 13, 226 (1971)].

⁶L. M. Gorbunov, V. V. Pustovalov, and V. P. Silin, Zh. Eksp. Teor. Fiz. 47, 1437 (1964) [Sov. Phys. JETP 20, 967 (1965)].

⁷V. N. Tsytovich, Nelineinye éffekty v plazme (Nonlinear Effects in Plasma), Nauka, 1967.

⁸L. M. Kovrizhnykh, Zh. Eksp. Teor. Fiz. 48, 1114 (1965) [Sov. Phys. JETP 21, 744 (1965)].

⁹L. M. Kovrizhnykh, Dissertation, FIAN, 1967.

¹⁰L. M. Kovrizhnykh, FIAN Preprint No. 117, 1968.

¹¹H. Dreicer, Phys. Fluids 7, 735 (1964).

¹²A. S. Kompaneets, Zh. Eksp. Teor. Fiz. 31, 876 (1956) [Sov. Phys. JETP 4, 730 (1957)].

¹³Ya. B. Zel'dovich and E. V. Levich, Zh. Eksp. Teor. Fiz., Pis'ma Red. 11, 57 (1970) [JETP Lett. 11, 35 (1970)].

¹⁴N. G. Basov, O. N. Krokhin, G. V. Sklizkov, S. I. Fedotov, and A. S. Shikanov, Zh. Eksp. Teor. Fiz. 62, 203 (1972) [Sov. Phys. JETP 35 (1), (1972)].

¹⁵A. V. Vinogradov and V. V. Pustovalov, Kvantovaya élektronika (Quantum Electronics) No. 8, 3 (1972).

¹⁶I. S. Gradshtein and I. M. Ryzhik, Tablitsy integralov, summ, ryadov i proizvedenii (Tables of Integrals, Sums, Series, and Products), Moscow, 1962, p. 944.

¹⁷V. P. Silin, Zh. Eksp. Teor. Fiz. 57, 183 (1969) [Sov. Phys. JETP 30, 105 (1970)].

¹⁸A. Yu. Kiriñ, Zh. Eksp. Teor. Fiz. 60, 955 (1971) [Sov. Phys. JETP 33, 517 (1971)].

¹⁹V. V. Pustovalov and V. P. Silin, Zh. Eksp. Teor. Fiz., Pis'ma Red. 14, 439 (1971) [JETP Lett. 14, 299 (1971)].

²⁰L. M. Kovrizhnykh, Zh. Eksp. Teor. Fiz., Pis'ma Red. 2, 142 (1965) [JETP Lett. 2, 89 (1965)].

²¹A. G. Litvak and V. Yu. Trakhtengerts, Zh. Eksp. Teor. Fiz. 60, 1702 (1971) [Sov. Phys. JETP 33, 921 (1971)].

Translated by J. G. Adashko

116