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Possible Crystallization of Charge Carriers in Low-Density Inversion Layers

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In the limiting case of low density and temperature, the electrons or holes in an inversion layer should form a two-dimensional crystal lattice. The binding energies of such lattices, dia- and paramagnetism, and ultrasound absorption by the charge carriers are calculated. The absorption is within the accuracy of present-day experimental techniques and may serve as a criterion for crystallization.

In inversion layers produced near the surface of a semiconductor in a strong electric field, the carriers can fill (at low temperatures) only the first subband of transverse quantization (see[1]). The value of the parameter νa^{*2}, where ν is the surface concentration of the electrons or holes in the layer and a^* is the effective Bohr radius, determines the properties of the inversion-layer plasma. The case $\nu a^{*2} \gg 1$, which corresponds to a dense and therefore weakly nonideal plasma, was considered earlier[2]. The opposite inequality may, however, also occur. For example, if the effective carrier mass m is of the order of the mass of the free electron and the dielectric constant of the semiconductor is ~10, then we obtain $\nu a^{*2} \sim 10^{-4} \ll 1$ for $\nu \sim 10^{11}$ cm^{-2}. Under these conditions we are dealing with a two-dimensional Coulomb gas of low density. It is important that the neutralizing charge is localized on a metallic electrode in the situation under consideration and is separated from the semiconductor by a dielectric layer. Therefore the ground state of the system cannot be an aggregate of neutral hydrogen-like atoms, although the Bohr radius is much smaller than the distance between particles. In this case, as shown by Wigner[3], the ground state corresponds to a crystal lattice of electrons executing small oscillations about equilibrium positions. Thus, at sufficiently low temperature, the low-density inversion layer should form an electronic (or hole) crystal.

We recall that the existence of two-dimensional crystals is not forbidden in the sense that they possess stability with respect to fluctuation displacement of the particles (see[4]). This question will be discussed in greater detail below (after solving the vibrational problem).

Our purpose at present is to calculate the potential energy of the particle in a crystal lattice, and the matrix of the elastic constants that determine the oscillation frequencies. It is first necessary to find the potential produced by a point-like charge in a three-layer system in which region I ($-\infty < z < 0$) is occupied by a metal, region II ($0 < z < D$) by a dielectric with dielectric constant ϵ_2, and region III ($D < z < \infty$) by a semiconductor with dielectric constant ϵ_1. The z axis is perpendicular to the interfaces. The charge e lies in region III, at the point with coordinates $\rho = 0$, $z = z_0$; ρ is a two-dimensional vector.

The solution of the Poisson equation with boundary conditions

$$\varphi|_{z=0} = 0, \quad \varepsilon_2 \partial\varphi / \partial z|_{D-0} = \varepsilon_1 \partial\varphi / \partial z|_{D+0}$$

takes the form (in region III)

$$\varphi(\mathbf{q}; z, z_0) = \frac{2\pi e}{\varepsilon_1 q}\Big\{\exp(-q|z-z_0|) + \frac{\varepsilon_1 - \varepsilon_2 \operatorname{cth} qD}{\varepsilon_1 + \varepsilon_2 \operatorname{cth} qD}\exp[-q(z+z_0-2D)]\Big\}. \tag{1}$$

Here $\varphi(\mathbf{q}; z, z_0)$ is the Fourier component of the potential with respect to the coordinate ρ. Expression (1) should be averaged over the wave functions of the transverse motion, $\psi(z)$ and $\psi(z_0)$, corresponding to the first subband. The effective thickness of the inversion layer, previously obtained[1] from a solution of the self-consistent problem by a variational method, is of the order of $d \sim (a^*/\nu)^{1/3} \ll r_0$, where $r_0 = \nu^{-1/2}$ is the average distance between particles. An important role will be played subsequently by values of $|\rho|$ that are large compared with d. Equation (1) is therefore averaged under the conditions $q(z - D) \ll 1$ and $q(z_0 - D) \ll 1$, and as a result we obtain

$$\varphi(q) = 4\pi e / q(\varepsilon_1 + \varepsilon_2 \operatorname{cth} qD). \tag{2}$$

The potential energy of a lattice whose points are specified by a set of planar vectors $\mathbf{a}_i$ is equal to

$$W = \frac{1}{2}\sum_{i,j} W_{ij} = \frac{1}{2}\sum_{i\neq j}\int e\varphi(q)\exp[i\mathbf{q}(\mathbf{a}_i - \mathbf{a}_j)]\frac{d^2\mathbf{q}}{4\pi^2}. \tag{3}$$

As seen from (2) and (3), the interaction potential of a pair of particles located at the points $\mathbf{a}_i$ and $\mathbf{a}_j$ behaves differently depending on the relation between D and $|\mathbf{a}_i - \mathbf{a}_j|$. At distances $|\mathbf{a}_i - \mathbf{a}_j| \ll D$ we obtain Coulomb's law

$$W_{ij} = 2e^2 / (\varepsilon_1 + \varepsilon_2)|\mathbf{a}_i - \mathbf{a}_j|,$$

and in the case $|\mathbf{a}_i - \mathbf{a}_j| \gg D$ the interaction law corresponds to parallel dipoles with effective distance D/ϵ_2:

$$W_{ij} \sim e^2 D / \varepsilon_2 (\mathbf{a}_i - \mathbf{a}_j)^2$$

(The dipole is made up of the charge and of the electrostatic image in the metal).

We can now calculate the force matrix

$$T_{ij} \equiv T(\mathbf{a}_i - \mathbf{a}_j) = \frac{\partial^2 W}{\partial \mathbf{a}_i \partial \mathbf{a}_j}$$

and the elements of the characteristic determinant of the vibrational problem

$$\|\omega^2\delta_{\sigma\rho} - G_{\sigma\rho}(\mathbf{k})\| = 0, \quad \sigma, \rho = 1, 2,$$

$$G_{\sigma\rho}(\mathbf{k}) = \sum_{\mathbf{a}_n \neq 0} T(\mathbf{a}_n) e^{i\mathbf{k}\mathbf{a}_n}$$

$$= \frac{4\pi e^2}{m} \int \frac{q_\sigma q_\rho (2\pi)^{-2} d\mathbf{q}}{q(\varepsilon_1 + \varepsilon_2 \operatorname{cth} qD)} \sum_{\mathbf{a}_n \neq 0} (e^{i\mathbf{k}\mathbf{a}_n} - 1) e^{-i\mathbf{q}\mathbf{a}_n}. \tag{4}$$

Let us ascertain the form of the vibrational spectrum of the system in the long-wave limit $kr_0 \ll 1$. In the first approximation, the sum over a_n in (4) can be replaced by an integral, $\Sigma \to \nu \int da$ (continuous-medium approximation):

$$G_{\sigma\rho}(k) = \frac{4\pi e^2 \nu}{m} \frac{k_\sigma k_\rho}{k(\varepsilon_1 + \varepsilon_2 \operatorname{cth} kD)}. \tag{5}$$

From this we obtain for the frequencies

$$\omega_1(k) = \left[\frac{4\pi e^2 \nu k}{m(\varepsilon_1 + \varepsilon_2 \operatorname{cth} kD)}\right]^{1/2}, \qquad \omega_2(k) \equiv 0,$$

i.e., to find the dispersion law of the second vibrational branches it is necessary to calculate $G_{\sigma\rho}(k)$ with high accuracy.

To this end, we rewrite (4) in the form of a sum over the reciprocal-lattice vectors K_n, i.e., we use the formula

$$\sum_{\bar{a}_n \neq 0} e^{i\mathbf{k}\mathbf{a}_n} = 4\pi^2 \nu \sum_{\mathbf{K}_n} \delta(\mathbf{k} - \mathbf{K}_n) - 1.$$

Separating the term with $K_n = 0$, we obtain

$$G_{\sigma\rho} = \frac{4\pi e^2 \nu k_\sigma k_\rho}{mk(\varepsilon_1 + \varepsilon_2 \operatorname{cth} kD)} + \frac{4\pi e^2 \nu}{m} \sum_{\mathbf{K}_n \neq 0} \left[\frac{(k_\sigma + K_\sigma)(k_\rho + K_\rho)}{|\mathbf{k}+\mathbf{K}|(\varepsilon_1 + \varepsilon_2 \operatorname{cth} |\mathbf{k}+\mathbf{K}| D)} - \frac{K_\sigma K_\rho}{K(\varepsilon_1 + \varepsilon_2 \operatorname{cth} KD)} \right] - \frac{4\pi e^2}{m} \int \left[\frac{(k_\sigma + q_\sigma)(k_\rho + q_\rho)}{|\mathbf{k}+\mathbf{q}|(\varepsilon_1 + \varepsilon_2 |\mathbf{k}+\mathbf{q}| D)} - \frac{q_\sigma q_\rho}{q(\varepsilon_1 + \varepsilon_2 \operatorname{cth} qD)} \right] \frac{d^2\mathbf{q}}{4\pi^2}. \tag{6}$$

The first term corresponds here to the continuous-medium approximation; in the remaining terms it is necessary to expand in powers of k_σ and k_ρ, since the condition $kr_0 \ll 1$ denotes $k \ll K_n$. In addition, we can put $\coth K_n D = 1$. The point is that the thickness of the dielectric under typical conditions of experiments with inversion layers amounts to 10^{-5}–10^{-4} cm. This is much larger than the electron lattice constant r_0, which at the considered concentrations $\nu \sim 10^{11}$–10^{12} cm^{-2} is equal to $(1-3) \times 10^{-6}$ cm. Consequently $\min K_n D \sim 2\pi D/r_0 \gg 1$, i.e., $\coth K_n D = 1$. The linear terms of the expansion (6) in the components k vanish upon summation and integration, owing to the symmetry. The quadratic increment $G^{(2)}_{\sigma\rho}(\mathbf{k})$ to expression (5) takes the form

$$G^{(2)}_{\sigma\rho}(\mathbf{k}) = \frac{4\pi e^2 \nu}{\varepsilon_1 + \varepsilon_2} \sum_{\mathbf{K} \neq 0} \left(\frac{3K_\sigma K_\rho (\mathbf{k}\mathbf{K})^2}{2K^5} - \frac{k^2 K_\sigma K_\rho}{2K^3} \right) - \frac{4\pi e^2}{\varepsilon_1 + \varepsilon_2} \int \left(\frac{3q_\sigma q_\rho (\mathbf{k}\mathbf{q})^2}{2q^5} - \frac{k^2 q_\sigma q_\rho}{2q^3} \right) \frac{d^2\mathbf{q}}{4\pi^2}. \tag{7}$$

The calculation can be continued by specifying a concrete type of the lattice. In the simplest case of a quadratic lattice, the components of the vector K are equal to $2\pi n/r_0$ and $2\pi m/r_0$, where n and m are integers. The only nonvanishing terms in the sum and in the integral of (7) are those containing even powers of the components of K, and the following obvious relations are satisfied

$$\sum \frac{K_1^2}{K^3} = \sum \frac{K_2^2}{K^3} = \frac{1}{2} \sum K^{-1}, \qquad \sum \frac{K_1^4}{K^5} = \sum \frac{K_2^4}{K^5} = \frac{1}{2} \sum K^{-1} - \sum \frac{K_1^2 K_2^2}{K^5}.$$

We note that the sum and the integral in (7) diverge separately, but their difference has a finite limit. For example, it is easy to show that

$$\lim_{N\to\infty} \left[\sum_{n^2+m^2 \neq 0}^{N} (n^2 + m^2)^{-1/2} - \int_0^N (u^2 + v^2)^{-1/2} du\, dv \right] = 2(C - \ln 2\pi) + \sum_{m=1}^{\infty} \int_1^{\infty} \frac{4\pi m \ln[x + (x^2 - 1)^{1/2}]}{\operatorname{sh}^2 m\pi x}.$$

Here C is Euler's constant; the last term converges rapidly independently of the sequence of summation and integration. Thus, we obtain

$$G^{(2)}_{\sigma\rho}(\mathbf{k}) = \frac{e^2}{mr_0(\varepsilon_1 + \varepsilon_2)} (\Lambda_1 k^2 \delta_{\sigma\rho} + \Lambda_2 k_\sigma k_\rho),$$

where $\Lambda_{1,2}$ are certain numerical factors. This leads to the dispersion law of the second branch

$$\omega_2^2(\mathbf{k}) = s_{\sigma\rho}^2 k_\sigma k_\rho,$$

where $s^2 \sim e^2/(\epsilon_1 + \epsilon_2) mr_0$; in order of magnitude; at $\nu \sim 10^{11}$ cm^{-2} we have $s \sim 5 \times 10^6$ cm/sec. In the considered long-wave region $\omega_2^2/\omega_1^2 \sim kr_0 \ll 1$.

The first branch of the vibrational spectrum $\omega_1(k)$ has in the region $D^{-1} \ll k \ll r_0^{-1}$ the form

$$[4\pi e^2 \nu k / m(\varepsilon_1 + \varepsilon_2)]^{1/2}$$

and corresponds to longitudinal plasma oscillations in the two-dimensional case (see[2]). At still smaller k ($kD \ll 1$), the dispersion law becomes linear, and the speed of sound is equal to $(4\pi e^2 D/m\epsilon_2 r_0^2)^{1/2} \gg s$ (to avoid misunderstandings, we note that in the region $k \lesssim e^2/\bar{\epsilon} r_0^2 mc^2$, where c is the speed of light and $\bar{\epsilon} = (\epsilon_1 + \epsilon_2/2$, the dispersion law $\omega_1(k)$ can be shown to take the form $ck/\bar{\epsilon}^{1/2}$. The groups velocity, of course, is smaller than c everywhere, but the indicated region of values of k is quite negligible and will henceforth be neglected).

Let us estimate the parameters of the vibrational spectrum. The Debye frequency is $\omega_0 \sim (e^2/\bar{\epsilon} mr_0^3)^{1/2}$, the average oscillation amplitude is $u_0 \sim (a^* r_0^3)^{1/4}$, and the frequency of motion along the coordinate z is of the order of $\hbar/md^2 \sim \hbar(a^* r_0^2)^{-2/3}/m \sim \omega_0(r_0/a^*)^{1/6}$. Thus, the dimension of the region of motion along z is much smaller than the period of the two-dimensional lattice and the average oscillation amplitude, and the frequency of motion perpendicular to the surface is larger than the characteristic frequency of the oscillations along it. The value of ω_0 for $\nu \sim 10^{11}$ cm^{-2} and $\bar{\epsilon} \sim 10$ is approximately equal to 2×10^{12} sec^{-1}, corresponding to ~15°K.

Having determined the form of the vibrational spectra of the system, let us discuss in greater detail the question of the stability of the crystalline state. The already mentioned result of Landau and Peierls[4] pertains to bodies in which the density of the particles is constant along the third coordinate. The case under consideration is different because the dimension of the layer along the z axis is smaller than the oscillation amplitude in the (x, y) plane, although their ratio is proportional to the small parameter raised to a very small power: $d/u_0 \sim (a^*/r_0)^{1/12}$. However, the change of motion along the z axis is connected with the relatively large excitation energy (the energy gap is on the order of $\hbar^2/md^2$). It is therefore necessary to investigate the stability of the lattice with respect to oscillations in the (x, y) plane. The mean-squared particle displacement in the oscillations is (see[5]):

$$\langle u^2\rangle = \frac{\hbar r_0^2}{4\pi^2 m}\sum_i \int \left[n_i(\mathbf{k}) + \frac{1}{2}\right]\frac{d\mathbf{k}}{\omega_i(\mathbf{k})}, \qquad i = 1, 2,$$

where $n_i(k)$ are the Bose occupation numbers. We see therefore that only the second branch $\omega_2 = sk$ is dangerous in the sense of the divergence of $\langle u^2 \rangle$, and the contribution of the zero-point oscillations is finite for both branches. Therefore, separating the principal contribution to $\langle u^2 \rangle$, we replace $n_2(k)$ by $T/\hbar sk$ and obtain

$$\frac{\langle u^2\rangle}{r_0^2} \sim \frac{T}{2\pi m s^2}\ln\frac{\pi L}{r_0},$$

where L is the linear dimension of the system in the (x, y) plane. This result signifies formally that the crystalline state is unstable at infinitely large lattice dimensions (see[6]). However, the coefficient preceding the logarithm, at the indicated values of the characteristic parameters of the problem, is approximately 10^{-3} T (where T is in degrees K). Consequently, below the Debye temperature ($T < \hbar\omega_0/k_B \approx 14$°K), the obtained divergence is of practically no significance at any reasonable system dimension. (An analogous situation obtains for the two-dimensional lattice of Abrikosov vortex filaments in type-II superconductors. The fluctuations of the ordering parameters increase logarithmically with increasing sample dimensions, but become significant only for astronomical objects[7].) Thus, the thermal vibrations of the particles do not destroy the crystalline state, at least in the region $T < \hbar\omega_0/k_B$.

The existence of a second branch $\omega_2(k)$ is an attribute of the crystalline structure of the inversion-layer plasma. To assist the possibility of experimentally observing such a crystal, let us consider its behavior in a magnetic field perpendicular to the lattice plane. The vibrational problem reduces in the harmonic approximation to a solution of the particle equations of motion:

$$m\ddot{\mathbf{u}}_i = -\sum_j \mathbf{A}_{ij}\mathbf{u}_j + \frac{e}{c}[\dot{\mathbf{u}}_i\mathbf{H}], \tag{8}^*$$

where $\mathbf{u}_i$ are the displacement vectors, $\mathbf{A}_{ij}$ the tensor of the force constants, **H** is the magnetic field, and the summation is carried out over the lattice sites. Using the standard procedure, we obtain a dispersion equation corresponding to the system (8). Its solutions $\bar{\omega}_{1,2}(k)$ can be expressed in terms of the frequencies in the absence of a magnetic field $\omega_{1,2}$:

$$\tilde{\omega}_{1,2}^2 = {}^1/_2(\omega_1^2 + \omega_2^2 + \Omega^2) \pm {}^1/_2[(\omega_1^2 + \omega_2^2 + \Omega^2)^2 - 4\omega_1^2\omega_2^2]^{1/2}, \tag{9}$$

$\Omega \equiv eH/mc$. As seen from (9), the magnetic field lifts the degeneracy of the vibrational spectrum at the point k = 0. As $kr_0 \to 0$ we have

$$\tilde{\omega}_1(k) \approx \Omega + \frac{\alpha k + s^2k^2}{2\Omega}, \qquad \alpha \equiv \frac{2\pi e^2\nu}{\varepsilon m}, \tag{10a}$$

$$\tilde{\omega}_2(k) \approx \alpha^{1/2} s k^{3/2}/\Omega. \tag{10b}$$

After calculating the vibrational branches $\tilde{\omega}_1(k)$ and $\tilde{\omega}_2(k)$ and the free energy F, we can obtain the magnetic moment of the electron lattice:

$$F = F(0) + F(T) = \frac{1}{2}\sum_{\mathbf{k},\sigma}\tilde{\omega}_\sigma(k) + T\sum_{\mathbf{k},\sigma}\ln\left[1 - \exp\left(-\frac{\tilde{\omega}_\sigma(k)}{T}\right)\right], \tag{11}$$

$\sigma = 1, 2$. The diamagnetic susceptibility at T = 0 is determined by the first term in (11); using (9), we obtain

$$\chi(0) = -\frac{\partial^2 F}{\partial H^2}\bigg|_{H=0} = -\frac{\hbar e^2}{m^2c^2}\sum_{\mathbf{k}}\frac{1}{\omega_1(k) + \omega_2(k)}. \tag{12}$$

If we approximate $\omega_1(k)$ by a square-root law and $\omega_2(k)$ by a linear law, and introduce the limiting value of the wave vector $k_0 = 2(\pi\nu)^{1/2}$, so as to obtain the correct number of degrees of freedom, then we get for the susceptibility per unit surface

$$\chi(0) = -\frac{\hbar e^2\alpha}{\pi m^2c^2s^3}\left[\ln(1+\eta) - \eta + \frac{1}{2}\eta^2\right], \qquad \eta \equiv s(\pi\nu)^{-1/4}(\varepsilon m/e^2)^{1/2} \sim 1. \tag{13}$$

In order of magnitude, $\chi(0) \sim (e^2/mc^2)(a^{*2}\nu)^{1/4}$.

In the low-temperature limit $T \ll \hbar\Omega$, the T-dependent part of the magnetic moment is connected with the "crystalline" branch $\tilde{\omega}_2$, whereas the first branch gives an exponentially small contribution:

$$M(T) = -\frac{\partial F}{\partial H} = \frac{\Gamma(^7/_3)\zeta(^7/_3)}{3\pi}T^{7/3}\left(\frac{e}{mc}\right)^{4/3}H^{1/3}(\alpha s^2)^{-2/3}. \tag{14}$$

The contribution to the magnetization by the electron spins is described by the well-known Brillouin formula

$$M_s = (e\hbar\nu/2mc)\ \mathrm{th}\ (\hbar\Omega/T),$$

(M_S and M(T) are the magnetic moments per unit surface). The specific dependence of the magnetization on T, H, and ν, were it possible to observe it experimentally, could serve as a criterion of the crystalline character of the inversion layer. However, owing to the relatively small number of surface electrons in the sample, the magnetization connected with these electrons is apparently difficult to observe in experiment.

We can point to another effect in which the crystalline structure of the inversion layers also plays a decisive role and which, according to estimates, is perfectly accessible to experimentation. We have in mind the absorption of ultrasound, in a magnetic field, by the electrons or holes of the inversion layer. As seen from (10), the acoustic wave interacts effectively only with the second branch $\omega_2(k)$. Indeed, in semiconductors the speed of sound v does not exceed 10^6 cm/sec. It is easy to show that for $\nu > 10^9$ cm^{-2} the sound dispersion vk intersects, within the limits of the Brillouin zone, only the branch $\omega_2(k)$. This results in

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*$[\mathbf{u}_i\mathbf{H}] \equiv \mathbf{u}_i \times \mathbf{H}$.

a narrow absorption line at the frequency[1)]

$$\omega^* = \frac{\Omega^2 v^3}{a s^2} \sim \frac{(m\varepsilon\Omega)^2 v^3}{e^4 \nu^{3/2}}. \qquad (15)$$

For longitudinal sound in silicon, $v = 9 \times 10^5$ cm/sec; taking the value $H = 10^4$ G and $\nu = 2 \times 10^{11}$ cm^{-2}, we obtain $\omega^* = 4.5 \times 10^8$ sec^{-1}. The corresponding value $k = \omega^*/\nu$ is much smaller than r_0^{-1}, so that the asymptotic formula (10b) is perfectly legitimate. The absorption line width γ is obviously determined by the decay of the phonon from the branch $\tilde{\omega}_2(k)$ into two other phonons of the same branch, as a result of the anharmonicity of the oscillations. Calculation in accordance with the well-known procedure (see, for example[8]) leads to the estimate $\gamma \sim \omega^* \epsilon T^3 |b|^2 s^2 / \times \nu^{3/2}(e\hbar\nu)^2$, where B is the anharmonicity constant ($b \sim 1/s$); using the same numbers as in the estimate of ω^*, we obtain $\gamma/\omega^* \sim 3 \times 10^{-2}$ at $T = 2°$K.

Let us estimate finally the absorption intensity. We describe the electron-phonon interaction in the approximation of the deformation potential. Without dwelling on the obvious calculations, we present an expression for the probability of conversion of an ultrasonic phonon into a phonon of the electron lattice:

$$W_\omega = \frac{\pi}{2} \frac{E^2 \omega^{*2} \nu \cos^2\theta}{m v^4 \rho_0 L} N(\omega^*) [n(\omega^*) + 1] \delta(\omega - \omega^*). \qquad (16)$$

Here E is the constant of the deformation potential ρ_0 is the density of the semiconductor, L is the thickness of the sample, θ is the angle between the direction of the ultrasonic wave and the polarization vector of the $\tilde{\omega}_2(k)$ oscillation, and $N(\omega^*)$ and $n(\omega^*)$ are the Bose occupation numbers of the ultrasonic phonons and of the electron-lattice phonons, respectively. Knowing W, we can set up kinetic equations for the quantities $n(\omega)$ and $N(\omega)$ and obtain the attenuation of the ultrasound. Unlike the absorption by the free electrons, in this case the absorption coefficient has a resonant form with a maximum at $\omega = \omega^*$ and with a width on the order of γ. It is therefore meaningful to estimate the integral absorption of an ultrasonic pulse with a broad spectrum. If it is assumed that the acoustic energy is uniformly distributed over a frequency interval of width δ, then the relative energy loss per unit path of the ultrasonic wave is equal to

$$\frac{\pi}{2} \frac{E^2 \Omega^4 v}{m \rho_0 L \nu^2 \delta} \left(\frac{\varepsilon m^4}{e^2}\right).$$

For $E = 3$ eV, $L = 0.1$ cm, $\delta = 0.1\,\omega^*$, $\rho_0 = 4$ g/cm^3, $H = 10^4$ G, and $\nu = 2 \times 10^{11}$ cm^{-2}, the absorption amounts to 4.3×10^{-4} dB/cm. Such an absorption is relatively easy to measure in experiment. The lattice absorption of the ultrasound, which at $T \approx 3°$K is comparable with the described electron absorption (for an estimate see, for example,[9]), can easily be separated, since it does not depend on the magnetic field or on the carrier density in the inversion layer. In addition, the effect of interest to us can be significantly enhanced if the absorption is not by a volume acoustic wave but by a Rayleigh one. Then the denominator of the expression for the absorption intensity contains in place of the quantity L (the thickness of the sample) the much smaller depth of penetration of the surface wave.

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[1)] This process is analogous to the interaction of light with optical phonons in ionic crystals. The vicinity of the resonance frequency ω^* corresponds to the region of nontransparency of the ionic crystal.

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