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Theory of Sound and Quantum Waves in Strong Magnetic Fields

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The dispersion law of transverse acoustic and quantum waves moving along a magnetic field is studied. A most complete dispersion equation for coupled oscillations is derived which takes into account inductive as well as deformation interaction between the sound and electrons located in a quantized magnetic field. Dispersion curves for strongly interacting waves are found in various regions of the (ω,q) plane which are free of Landau damping. It is shown that owing to resonance interaction some new possibilities appear regarding an experimental investigation of the dispersion dependences of right- and left-hand polarized quantum waves in metals and semimetals.

1. INTRODUCTION

In metals at low temperatures and with sufficiently large electron mean free paths, the main features of the propagation of different excitations, including sound, are determined by the direct interaction of the excitations with the conduction electrons. This phenomenon is of particular interest in a multicomponent system, such as an electron gas in a quantizing magnetic field, under conditions when there are many transparency windows in the regions of collisionless Landau damping, where there exist branches of the spectrum of electromagnetic oscillations.

The existence of several types of low-frequency waves was predicted[1-6] in recent theoretical papers devoted to the study of collective excitations in a degenerate electron gas situated in a quantizing magnetic field. These excitations, called quantum waves, are connected with definite resonant transitions between the Landau level n.[1)] Each single-particle resonance with definite $\Delta n_i (i = 1, \ldots, N - 1)$ corresponds to one Bose branch of the collective excitations. If the wave propagates along the magnetic field, then transitions with $\Delta n = 0$ lead to the existence of a series of arbitrary quantum waves[1-3]. Transitions with $\Delta n = +1$ form a spectrum of left-polarized waves[5], and those with $\Delta n = -1$ a spectrum of right-polarized quantum waves[6]. In the case when the excitations propagate at an angle to the magnetic field or in the case when the Fermi surface is anisotropic[4], the polarization of the quantum waves changes, and transitions with arbitrary Δn transform the spectrum[8]. The Landau-damping regions corresponding to these excitations were found in[5,6,8,9].

In the description of the longitudinal and transverse quantum waves, as a rule, account was taken of only the electronic subsystem, whose oscillations occur against the background of the smeared positive charge of the immobile lattice. Such an approach becomes incorrect once we start to consider the low-frequency region of the excitation spectrum.

At low frequencies, in the case when the velocity of sound coincides with the lowest electron velocity at the Landau levels, giant oscillations of the absorption coefficient occur[10], and in the short-wave region of the spectrum there exists a set of logarithmic singularities of the phase velocity of the longitudinal acoustic oscillation[11]. In the high-frequency region, when the longitudinal wave propagates along the magnetic field, the quantum wave interacts strongly with the optical phonons[9]. If the sound propagates at an angle to the magnetic field, then when the direction is varied resonant interaction of the longitudinal quantum and sound waves occur in the angle $\pi/2$[12].

Skobov and Kaner have shown[13] that inductive interaction between a left-polarized long-wave electromagnetic wave (helicon) and sound leads to the appearance of coupled waves at $qR \ll 1$ (R is the Larmor radius and q the wave vector). If $qR \gtrsim 1$, then the energy of the transverse sound wave is sufficient to realize an electronic transition with $\Delta n = \pm 1$. Such absorption of sound was considered in[14] where, assuming without sufficient justification that the inductive interaction predominates, the study was devoted mainly to threshold effects and damping.

The intersection of the dispersion curves that exist near the thresholds of electromagnetic transverse quantum waves[5,6] and sound shows that a strong renormalization of the spectrum of the transverse quantum and sound waves should take place and that coupled oscillations should appear just as at $qR \ll 1$. The latter circumstance indicates one more possibility of experimentally investigating the quantum-wave spectrum. The present paper is devoted to a study of these problems.

The self-consistent system of equations describing the propagation of transverse acoustic and quantum waves is solved in Sec. 2, and a dispersion equation for the coupled oscillations is obtained. The inductive and deformation interactions of the lattice vibrations with the electrons are taken into account in most complete form. The study of the interacting waves is carried out in a coordinate system connected with the lattice. In the third section, solutions of the dispersion equation are obtained for different limiting cases. It is shown that for left-polarized waves in each transparency window there exists a frequency interval where there are no solutions of the dispersion equation. The quantum waves and the sound in semi-metals, the

[1)]Quantum waves in a Fermi liquid have been considered in[7].

anisotropy of the Fermi surface, and the requirements to be satisfied in the experiments are discussed in the fourth section.

2. FUNDAMENTAL EQUATIONS

The system of equations describing the propagation of coupled acoustic and quantum waves consists of the lattice-vibration equation and of Maxwell's equations:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \lambda_{ijkm} \frac{\partial^2 u_m}{\partial x_j \partial x_k} + f_i, \quad (2.1)$$

$$\operatorname{rot}\operatorname{rot}\mathbf{E} = -\frac{4\pi}{c^2}\frac{\partial \mathbf{j}}{\partial t}, \quad (2.2)$$

where ρ is the density, u_i are the components of the lattice displacement vector, λ_{ijkm} is the tensor of the elastic moduli, f is the volume force exerted on the lattice by the electrons, **E** is the electric field intensity, c is the speed of light, and **j** is the current density. In (2.2), the displacement current is neglected, because div **j** = 0. This equation, which is satisfied for transverse waves identically, is equivalent to the condition of the electron neutrality of the crystal.

An expression for the volume force in the quasiclassical case was obtained by Kontorovich[15] and by Skobov and Kaner[13]:

$$f_i = c^{-1}[\mathbf{jH}]_i + f_i^g, \quad (2.3)^*$$

where the first term describes the inductive interaction and the second is the deformation force, while **H** is the intensity of the constant magnetic field. We emphasize once more that the analysis is carried out in a coordinate system moving together with the lattice[2)]. In (2.3) the Stewart-Tolman force, which analysis shows to make a small contribution in metals, is neglected.

In the essentially quantum region of our problem, where $kT \ll \hbar\Omega$ and $\nu \ll \Omega, \omega$ (k is Boltzmann's constant, T the temperature, Ω the cyclotron frequency, ν the collision frequency, and ω the wave frequency), the deformation force is given by

$$f_i^g = 2\frac{\partial}{\partial x_j}\left[\frac{1}{2}\{\Lambda_{ij}(\hat{\mathbf{P}}) + \Lambda_{ij}(\hat{\mathbf{P}}')\}G^{(1)}(x, x')\Big|_{\substack{\mathbf{r}'\to\mathbf{r},\\ t'\to t+0}}\right]. \quad (2.4)$$

The coefficient 2 takes the spin degeneracy into account, $x = (\mathbf{r}, t)$, $\Lambda_{ik}(\mathbf{P})$ is a symmetrical tensor of the deformation potential, which depends on the operator

$$\hat{\mathbf{P}} = \hat{\mathbf{P}}_0 - \frac{e}{c}\mathbf{A}' = \frac{\hbar}{i}\nabla_r - \frac{e}{c}(\mathbf{A}_0 + \mathbf{A}') \quad (2.5)$$

and yields zero when averaged over the Fermi surface, $\mathbf{A}_0 = (0, Hx, 0)$ is the vector potential of the constant magnetic field, $\mathbf{A}'$ is the vector potential of the wave in the lattice frame and determines the electric field intensity

$$\mathbf{E}' = \mathbf{E} + c^{-1}[\dot{\mathbf{u}}\mathbf{H}]; \quad (2.6)$$

and $G^{(1)}(x, x')$ is the linear term in the expansion of the complete Green's function $G(x, x')$ in terms of **u** and $\mathbf{A}'$. Expression (2.4) has a simple physical meaning. It defines the force acting on the lattice as the gradient of the energy of the electron excess $\delta n(\mathbf{r}) = 2G^{(1)}(x, x')_{x'\to x}$ produced by the wave. The current density in (2.3) is obtained in terms of the Green's function from the well known relation[17]

$$\mathbf{j}(x) = \frac{e}{m}(\hat{\mathbf{P}}_0 + \hat{\mathbf{P}}_0'^*)G(x, x')\Big|_{\substack{\mathbf{r}'\to\mathbf{r},\\ t'\to t+0}} - \frac{2e^2}{m}\mathbf{A}'G(x, x')\Big|_{\substack{\mathbf{r}'\to\mathbf{r}\\ t'\to t+0}} \quad (2.7)$$

where e is the absolute value of the electron charge and m is its effective mass.

Thus, in order to make the system of equations closed, it is necessary to find $G^{(1)}(x, x')$. We write down the equations of motion for the Green's function

$$\left\{i\hbar\frac{\partial}{\partial t} - \frac{\hat{P}^2}{2m} - \hat{W} - \mu\right\}G(x, x') = \delta(x - x'), \quad (2.8)$$

where $\hat{W} = (1/2)[u_{ik}\Lambda_{ik}(\hat{\mathbf{P}}) + \Lambda_{ik}(\hat{\mathbf{P}})u_{ik}]$, μ is the Fermi energy, u_{ik} is the strain tensor, and the gauge for the potential was chosen with $\varphi = 0$. Representing the complete Green's function in the form of a sum

$$G = G^{(0)} + G^{(1)} \quad (2.9)$$

and taking the Fourier transform with respect to time, we obtain

$$G^{(1)}(\mathbf{r}, \mathbf{r}', \omega) = \sum_{\alpha,\beta}\frac{f_0(\varepsilon_\alpha) - f_0(\varepsilon_\beta)}{\varepsilon_\alpha - \varepsilon_\beta + \hbar\omega + i\delta\operatorname{sign}\omega}\Big\{\langle\alpha|\hat{W}|\beta\rangle - \frac{e}{2c}\langle\alpha|\hat{\mathbf{V}}\mathbf{A}' + \mathbf{A}'\hat{\mathbf{V}}|\beta\rangle\Big\}\psi_\alpha(\mathbf{r})\psi_\beta^*(\mathbf{r}') \quad (2.10)$$

In the derivation of (2.10), account is taken of the fact that

$$G^{(0)}(\mathbf{r}, \mathbf{r}', \omega) = \sum_\alpha \frac{\psi_\alpha(\mathbf{r})\psi_\alpha^*(\mathbf{r}')}{\hbar\omega - \varepsilon_\alpha + \mu + i\delta\operatorname{sign}(\varepsilon_\alpha - \mu)} \quad (2.11)$$

and the notation

$$\varepsilon_\alpha \equiv \varepsilon_n(k_z) = \hbar\Omega(n + 1/2) + \hbar^2 k_z^2/2m,$$
$$\psi_\alpha(\mathbf{r}) \equiv |n, k_z, k_y\rangle = (L_y L_z l)^{-1/2}\exp[i(k_y y + k_z z)]\Phi_n(x + l^2 k_y) \quad (2.12)$$

was introduced for the spectrum and the wave function of the electron in the magnetic field. It is assumed that in the absence of a field the electron spectrum is quadratic and isotropic (the anisotropy will be discussed below.) Φ_n is the normalized wave function of the oscillator, $f_0(\epsilon_\alpha)$ is the Fermi function, $\delta \to +0$, sign ω is the sign function, L_y and L_z are the dimensions of the crystal along the y and z axes, $l = (c\hbar/eH)^{1/2}$ is the magnetic length, $\hat{\Lambda}_{ik}$ in (2.10) depends on the operator $\hat{\mathbf{P}}_0$, and $\hat{\mathbf{V}} = \hat{\mathbf{P}}_0/m$, where $\hat{\mathbf{P}}_0$

*[jH] ≡ j × H.

2)The question of distinguishing between inductive and deformation interactions between sound and electrons, together with the relation between them in different reference frames, was investigated in[16].

$= (\hbar/i)\nabla_r - (e/c)\mathbf{A}_0$.

For a transverse monochromatic plane wave in which **u** and **E** are proportional to $\exp[i(qz - \omega t)]$ and which propagates along **H**, we obtain after substituting (2.10) and (2.11) in (2.4) and (2.7)

$$(s_0^2q^2 - \omega^2)u_k = -\frac{1}{\rho c}[\mathbf{jH}]_k + \frac{1}{\rho}f_k^g(\omega, q), \tag{2.13}$$

$$q^2E_k = 4\pi i\omega c^{-2}j_k, \tag{2.14}$$

$$j_k = \sigma_{km}(E_m + G_m) + J_{km}u_m \tag{2.15}$$

For simplicity we neglect the anisotropic lattice, assuming that the sound propagates along a high-symmetry axis. The deformation force which enters in (2.13) is given by

$$f_k^g = D_{km}u_m + F_{km}(E_m + G_m), \tag{2.16}$$

$$D_{km} = q^2\sum_{\alpha,\beta}I(\alpha, \beta, \omega, q)\,|\langle\alpha|\hat{w}_k(q)|\beta\rangle\langle\beta|\hat{w}_m(-q)|\alpha\rangle, \tag{2.17}$$

$$F_{km} = \frac{eq}{\omega}\sum_{\alpha,\beta}I(\alpha, \beta, \omega, q)\langle\alpha|\hat{v}_m(-q)|\beta\rangle\langle\beta|\hat{w}_k(q)|\alpha\rangle, \tag{2.18}$$

$$\hat{\mathbf{v}}(q) = \frac{1}{2}[e^{-iqz}\hat{\mathbf{V}} + \hat{\mathbf{V}}e^{-iqz}], \qquad \hat{w_k}(q) = \frac{1}{2}[e^{-iqz}\hat{\Lambda}_{kz} + \hat{\Lambda}_{kz}e^{-iqz}]. \tag{2.19}$$

When the first and second terms in the brackets of (2.10) are substituted in (2.7), we obtain an expression for the tensor J_{km} and for the conductivity tensor in a quantizing magnetic field[3)]

$$J_{km} = -ieq\sum_{\alpha,\beta}I(\alpha, \beta, \omega, q)\langle\beta|\hat{v}_k(q)|\alpha\rangle\langle\alpha|\hat{w}_m(-q)|\beta\rangle, \tag{2.20}$$

$$\sigma_{km} = \frac{i\omega_p^2}{4\pi\omega}\delta_{km} - \frac{ie^2}{\omega}\sum_{\alpha,\beta}I(\alpha, \beta, \omega, q)\langle\beta|\hat{v}_m(-q)|\alpha\rangle\langle\alpha|\hat{v}_k(q)|\beta\rangle, \tag{2.21}$$

$$J_{km}(\omega, q) = -i\omega F_{mk}(-\omega, -q), \tag{2.22}$$

where ω_p is the plasma frequency. In (2.15) and (2.16) the inductive field G_m is written our explicitly[13,16]

$$G_m = \omega[\mathbf{uH}]_m / ic, \tag{2.23}$$

$$I(\alpha, \beta, \omega, q) = \frac{f_0(\varepsilon_\alpha) - f_0(\varepsilon_\beta)}{\varepsilon_\alpha - \varepsilon_\beta + \hbar\omega + i\delta\operatorname{sign}\omega}. \tag{2.24}$$

To find the dispersion equation, we change over in the formulas to circularly-polarized components $u_\pm = u_x \pm iu_y$ and $E_\pm = E_x \pm iE_y$, after first calculating the following matrix elements:

$$\langle\alpha|\hat{v}_x \pm i\hat{v}_y|\beta\rangle = \pm i\left(\frac{2\hbar\Omega}{m}\right)^{1/2}\left(n_\alpha + \frac{1}{2} \pm \frac{1}{2}\right)^{1/2} \times \delta(k_z^\alpha - k_z^\beta + q)\delta(k_y^\alpha - k_y^\beta)\delta(n_\beta, n_\alpha \pm 1), \tag{2.25}$$

$$\langle\alpha|\hat{w}_x \pm i\hat{w}_y|\beta\rangle = \pm i\sqrt{2}\Lambda\left(n_\alpha + \frac{1}{2} \pm \frac{1}{2}\right)^{1/2} \times \delta(k_z^\alpha - k_z^\beta + q)\delta(k_y^\alpha - k_y^\beta)\delta(n_\beta, n_\alpha \pm 1), \tag{2.26}$$

The plus and minus signs in these components correspond to left-hand and right-hand circular polarization, and $\mathbf{q} \parallel \mathbf{H}$. It is obvious that the approximations (2.25) and (2.26) correspond to a sufficiently high symmetry of the Fermi surface, either a sphere (alkali metals) or an ellipsoidal revolution with axis parallel to the magnetic field. The coefficient $\sqrt{2}$ in (2.26) has been introduced for convenience.

Taking (2.25) and (2.26) into account and summing in (2.17)–(2.21) we obtained after eliminating the circular components $E_\pm$ and $U_\pm$ from (2.13)–(2.15) the dispersion equation for the right- and left-hand polarized coupled waves:

$$s^2q^2 - \omega^2 = \frac{-v_a^2q^2[1 + a(lq)^{-1}\Sigma_\pm] \pm a(\delta l^{-1}v_\Lambda q)^2\Sigma_\pm}{1 + \delta^2q^2 + a(lq)^{-1}\Sigma_\pm} - a\tilde{N}(ql)^{-1}v_\Lambda^2q^2\Sigma_\pm + a(v_\Lambda l^{-1})^2\Sigma_\pm\left\{\frac{a\tilde{N}\Sigma_\pm \pm 1}{1 + \delta^2q^2 + a(lq)^{-1}\Sigma_\pm}\right\}, \tag{2.27}$$

where s is the speed of the transverse sound. It differs from s_0 in (2.13) in that account is taken of the renormalization that is due to the interaction with the electrons and does not depend on ω and H; $v_a = H/\sqrt{4\pi\rho}$ is the Alfven velocity, $\delta = c/\omega_p$, $a = (4\pi^2l^3n_0)^{-1}$, n_0 is the electron concentration, $v_\Lambda^2 = \Lambda/M$, M is the mass of the atom, and $\tilde{N} = \Lambda/\hbar\Omega$. Since $\Lambda \sim \mu$, the order of magnitude of N coincides with the number of filled levels. In the calculation of (2.17)–(2.21) it was assumed that T = 0 and $\nu = 0$. In addition

$$\Sigma_\pm = \sum_{n=0}^{N}\ln\left\{\left|\frac{v_nq - \hbar q^2/2m - (\Omega \pm \omega)}{v_nq + \hbar q^2/2m + (\Omega \pm \omega)}\right|^{n+1} \times \left|\frac{v_nq - \hbar q^2/2m + (\Omega \pm \omega)}{v_nq + \hbar q^2/2m - (\Omega \pm \omega)}\right|^{n}\right\}. \tag{2.28}$$

Here N is the number of filled levels, $v_n = \sqrt{2[\mu - (n + 1/2)\hbar\Omega]/m}$ is the maximum velocity at the n-th Landau level, and the upper and lower signs in (2.27) and (2.28) correspond to right- and left-hand circular polarization.

Before we proceed to the solution, let us explain how the expressions in the right-hand side of (2.27) come about. The last two terms are due to the deformation force f_k^g, and the existence of the second term in the numerator of the first term is due to the deformation current. Equating the denominator to zero, we obtain an expression for left-polarized[5] and right-polarized[6] quantum waves. Thus, the expressions in the numerators characterize the coupling between the acoustic and quantum waves as a result of the inductive (v_a) and deformation (v_Λ) interactions. Equation (2.27) is the most complete equation describing the propagation of transverse quantum and acoustic waves in a degenerate electron gas.

3)We note that similar expressions were obtained in[18] for a current situation.

3. SPECTRUM OF INTERACTING WAVES

We solve (2.27) with respect to $\Sigma_\pm$. As a result we obtain

$$\sum_{n=0}^{N}\ln\left\{\left|\frac{v_nq-\hbar q^2/2m-(\Omega\pm\omega)}{v_nq+\hbar q^2/2m+(\Omega\pm\omega)}\right|^{n+1}\times\left|\frac{v_nq-\hbar q^2/2m+(\Omega\pm\omega)}{v_nq+\hbar q^2/2m-(\Omega\pm\omega)}\right|^{n}\right\}=-\frac{lq}{a}(1+\delta^2q^2)\frac{\omega^2-s_1^2q^2}{\omega^2-s_\pm^2q^2}; \quad (3.1)$$

$$s_1^2=s^2+v_a^2(1+\delta^2q^2)^{-1}, \quad (3.2)$$

$$s_\pm^2=s^2+v_a^2+g_\pm v_\Lambda^2. \quad (3.3)$$

Here $g_\pm = (ql)^{-1}(1+\delta^2q^2)(\tilde{N}ql \mp 1)$. We note that by setting v_a and v_Λ equal to zero ($\rho\to\infty$) we obtain the equation for transverse quantum waves[5,6].

We investigate first the spectrum of left-polarized waves. To this end, we analyze the singularities of the right and left parts of (3.1). The quantity Σ_- has logarithmic singularities on the two boundaries, which are of interest to us now, of the region of collisionless damping:

$$\omega_1=\Omega-\hbar q^2/2m-v_nq,\quad \omega_2=\Omega+\hbar q^2/2m-v_nq. \quad (3.4)$$

As $\omega\to\omega_2$ we get $\Sigma_-\to-\infty$ and as $\omega\to\omega_1$ we have $\Sigma_-\to+\infty$. An analysis of the expressions in the right-hand side of (3.1) shows that it always has a singularity at $\omega\to\omega_3^{-1}=s_-q$, and the function itself is positive when $s_1q<\omega<\omega_3^-$ and negative when $\omega>\omega_3^-$ and $\omega<s_1q$. Consequently, the solutions (3.1) should be exponentially close to ω_1 as $\omega-\omega_3^{-1}\to-0$ and to $\omega=\omega_2\to0$ as $q-(k_{n-1}-k_n)\to-0$.

In the absence of interaction we have an acoustic wave $\omega=sq$ and a quantum wave near ω_2 (dashed lines in Fig. 1). Allowance for the interaction causes the quantum wave existing near ω_2 to become an acoustic wave at $\omega=s_1q$, and become again the quantum wave when ω_1 is approached. In addition, the interaction splits the spectrum of the quantum waves, and the distinction between the acoustic and quantum waves become meaningless near the resonance $q=q_\pm$, where $\hbar q_\pm/m = \pm[(v_n+s_1)-\sqrt{(v_n+s_1)^2\pm2v_N^2}]$, where $v_N^2=\hbar\Omega/m$.

If $\omega-\omega_3^-\to+0$, then the spectrum comes exponentially close to ω_2 and terminates at $\omega=\Omega$ and $q=0$. Such solutions give the upper branch of the spectrum of left-polarized quantum waves that interact strongly with sound. The derived formulas make it easy to determine the frequency interval in which there are no solutions (3.1) in each transparency window:

$$\Delta\omega_n=\frac{v_N^2s_-}{(v_n+s_-)^3}\Omega \quad (3.5)$$

Let us find the analytic form of the solutions. To this end, we separate the resonant term in the left-hand side of (3.1), and estimate the contribution of the remaining ones by approximate summation. As a result, we obtain for $\omega\ll s_1q$ and $qR\geq1$

$$\omega_-(n,q)=v_n\tilde{q}-(v_{n-1}-v_n)\tilde{q}\exp\left\{-\frac{1}{n}[\mathcal{F}_-(q)-d]\right\} \quad (3.6)$$

$$\mathcal{F}_\pm=\frac{s_1^2}{as_\pm^2}(1+\delta^2q^2)lq,\qquad \tilde{q}=k_{n-1}-k_n-q. \quad (3.7)$$

The function d takes into account the contribution of the nonresonant logarithms. In the most interesting case $n\approx N$ and $q\approx l^{-1}$ (wide transparency window) we have $d\approx a^{-1}$. The analytic expression near the other singular point can be obtained in perfect analogy. The dispersion curves are shown schematically in Fig. 1.

Let us proceed to investigate the spectrum of right-polarized waves. We separate in Σ_+ the singularities on the following curves in the (ω, q) plane: $\omega_4=-\omega_1$ and $\omega_5=-\omega_2$, where $\Sigma_+\to+\infty$ as $\omega\to\omega_4$ and $\Sigma_+\to-\infty$ as $\omega\to\omega_5$. From this and from the character of the behavior of the function in the right-hand side of (3.1) it follows that the solutions inside of each transparency window begin with $\omega=\omega_5=0$ as $q-(k_{n-1}-k_n)\to+0$, go through zero in the middle ($\omega=s_1q$), and then pass exponentially close to ω_4 with $\omega-\omega_3^+\to-0$. At $\omega=s_1q$ these solutions form the spectrum of right-hand polarized acoustic waves which go over into quantum waves near ω_4 and ω_5.

The region $\omega>\omega_3^+$, $qR>1$ contains the upper branches of the quantum-wave spectrum, which start out from $\omega_3^+=\omega_5$ and terminate at the points $\omega=\Omega$ and $q=2k_n$. A feature of the spectrum of the right-polarized oscillations is the absence of a frequency interval analogous to (3.5). For $\omega\ll s_1q$ and $qR\geq1$ we obtain

$$\omega_+(n,q)=-v_n\tilde{q}+(v_{n-1}-v_n)\tilde{q}\exp\left\{-\frac{1}{n}[\mathcal{F}_+(q)-d]\right\} \quad (3.8)$$

We call attention to the fact that when $qR<N^{-1/2}$ we have $g_+<0$ ($\tilde{N}\approx N$). Since $g_\pm v_\Lambda^2>s^2+v_a^2$, it follows that the "pole" in the right-hand side of (3.2) disappears. The sound wave goes over into a quantum wave at the line $\omega=v_0[k_0-k_1-q]$ near $q=k_0-k_1\approx R^{-1}$. The spectrum of the right-polarized waves is shown schematically in Fig. 2.

So far we have considered the solutions of (3.1) only in the region $qR\gtrsim1$. If $qR\ll1$, then, expanding the logarithms, we obtain

$$\frac{\Omega}{\Omega\pm\omega}+\frac{\Omega^3\beta_Nl^2q^2}{(\Omega\pm\omega)^3}=(1+\delta^2q^2)\frac{\omega^2-s_1^2q^2}{\omega^2-s_\pm^2q^2}, \quad (3.9)$$

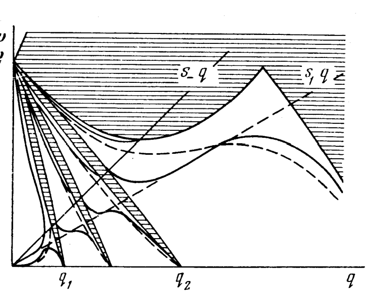

FIG. 1. Dispersion curves of interacting left-polarized acoustic and quantum waves in a strong magnetic field (schematic form). The Landau damping regions are shown shaded. The case when four magnetic levels are occupied is demonstrated. The dashed lines show the spectrum of the non-interacting oscillations. $q_1\approx R^{-1}$, $q_2\approx l^{-1}$.

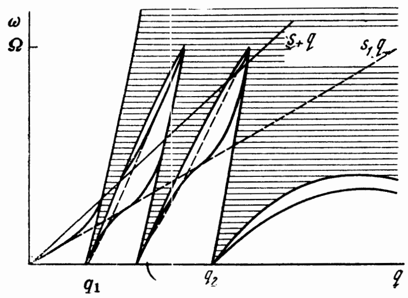

FIG. 2. Spectrum of interacting right-polarized acoustic and quantum waves in a strong magnetic field (schematic form). The case when four Landau levels are occupied is shown. The damping regions are shaded. The dashed lines show the dispersion curves of the non-interacting oscillations. $q_1 \approx R^{-1}$, $q_2 \approx l^{-1}$.

where

$$\beta_N = (4\pi^2 n_0)^{-1} \sum_{n=0}^{N} k_n^3$$

(see[9] for the calculation of the sum that enters in β_N). Equation (9) describes the de Haas-Van Alphen oscillations of the velocity of right-polarized acoustic oscillations, and also the spectrum and analogous oscillations of resonantly interacting left-polarized waves. The dispersion curves of these excitations in the region $qR \approx 1$ are shown in Figs. 1 and 2.

4. DISCUSSION

In order to visualize the region in the (ω, q) plane where branches of the collective-excitation spectrum exist, let us estimate the coefficient $g_\pm$ in (3.3) for two cases, a metal and a semi-metal. In a metal with concentration $n_0 \sim 10^{22}\ \mathrm{cm}^{-3}$, $\mu \sim 1$ eV and in a magnetic field $H \sim 10^5$ Oe, $\tilde{N} \sim 10^4$, $\delta^2 \sim 10^{-11}\ \mathrm{cm}^2$, and also for q from the interval from $R^{-1} \sim 10^4\ \mathrm{cm}^{-1}$ to $l^{-1} \approx \sqrt{N}R^{-1} \sim 10^6\ \mathrm{cm}^{-1}$, we obtain $s_\pm \sim 10^7$ cm/sec. This means that in a metal, owing to the interaction with the sound, only a relatively small number of windows (~ 100) in the Landau damping region ($R^{-1} \le q \le l^{-1}$) contain upper dispersion curves of the left-polarized quantum waves. The spectrum of the upper branches of the left-polarized quantum waves approaches the Doppler-shifted cyclotron resonance $\omega = \Omega - v_n q$, where the best resolved branch is located at $\omega_3^- < \omega < \Omega + \hbar q^2/2m - v_0 q$. It constitutes parts of the former spectrum of the helicon interacting with the acoustic oscillations, goes over into the latter, and then is transformed into a quantum wave at $qR \lesssim 1$ (see Fig. 1). In the remaining sections of the (ω, q) plane there exist branches of the acoustic-wave spectrum, which go over near the boundaries of the Landau damping region into quantum waves.

In a semi-metal, say in bismuth, $n_0 \sim 10^{17}\ \mathrm{cm}^{-3}$ and $\omega_p^2 \sim 10^{27}\ \mathrm{sec}^{-1}$; if $H \sim 10^5$ Oe, then $N \sim 4$, $l^{-1} \sim 10^6\ \mathrm{cm}^{-1}$, $R^{-1} \approx 5 \times 10^5\ \mathrm{cm}^{-1}$, and $\delta^2 q^2 \sim (10^5-10^6)$.

Consequently, just as in a metal, $s_\pm \sim 10^7$ cm/sec. The difference lies in the fact that in the region $1 < qR < \sqrt{N}$, since the Larmor radius is small, there are no solutions of (3.1) corresponding to the upper branches of the right-polarized quantum waves. The spectrum of the renormalized right-polarized acoustic oscillations and of the low-frequency quantum waves lies in these "windows." There are no changes in the behavior of the dispersion curves of the left-polarized waves in comparison with the case of a metal.

In spite of the narrowing down of the region of the existence of the quantum waves, semi-metals are worthy of attention, since it is precisely in them that the most favorable conditions are realized for an experimental study of the singularities in the propagation of sound and quantum waves. Indeed, we have assumed so far that $T = 0$ and $\nu = 0$. When the nonzero temperature smearing of the distribution function and the nonzero electron collision frequency are taken into account the logarithmic singularities on the boundaries of the Landau damping region becomes finite and smeared out by the collisions. Recognizing that the temperature-induced electron-velocity scatter is $\Delta v_n \approx kT/mv_n \ll \Delta v$, and the collision frequency should satisfy the inequality $\nu \ll q\Delta v$, we write out the necessary conditions for neglecting the deviations from ideal for the two polarizations of the quantum and acoustic waves. Here Δv is the interval where the quantum wave exist. In the case of a left-polarized wave ($\Delta v = \hbar q/m$, $\omega > s_- q$) we have

$$kT \ll v_n \hbar q \approx \hbar\Omega q R v_n / v_F, \qquad \nu \ll \hbar q^2/m \approx \frac{1}{N} q^2 R^2 \Omega, \tag{4.1}$$

where $v_F \approx v_0$ is the Fermi velocity.

In the case of right-polarized and left-polarized ($\omega < s_- q$) quantum waves ($\Delta v \approx \hbar\Omega/mv_F$, $n \approx 1$; $\Delta v = \hbar l^{-1}/m$ if $n \approx N$) we have

$$kT \ll \hbar\Omega v_n / v_F, \ \nu \ll \Omega / \sqrt{N}. \tag{4.2}$$

Obviously, these conditions turn out to be sufficient in the resonant region.

The inequalities show that in all cases the object most suitable for the experimental investigations is a semi-metal. The transitions with $\Delta n = 0$ should be suppressed in this case, and this is done by letting the sound propagate along sufficiently symmetrical directions. It is also easy to take into account the spin splitting of the levels and the simultaneous transitions with $\Delta n = \pm 1$[8,9]. The role of the spin splitting reduces to a decrease of the existence region of the collective excitations, and the transitions with $\Delta n = \pm 1$ lead to new singularities in the spectrum near $q = 2\Omega(v_n + v_m)^{-1}$.

The resonant interaction of the quantum and acoustic oscillations investigated in the present paper, can apparently become manifest in direct electromagnetic excitation of coherent acoustic waves[19], which have the polarization of the external electric field, and also, in experiments on the propagation of acoustic oscillations, which is of particular importance for right-polarized waves in a semi-metal.

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43