Buildup of Low-Frequency Potential Oscillations in a Cold Magnetoactive Plasma by the Field of an Electromagnetic Wave

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Solutions of the dispersion equation for potential oscillations of a magnetoactive plasma located in the field of a plane high frequency electromagnetic wave are investigated for frequencies less than, or comparable to, the ion gyroscopic frequency. The maximal increments and minimal threshold fields determined by the collision frequency between charged and neutral particles are found in the case of weak coupling between the wave and plasma perturbations. The directions of propagation of unstable potential oscillations for which the increments are maximal are determined. In the case of strong coupling the instability is aperiodic.

THE theory of parametric resonance[1-4] predicts the occurrence of a number of instabilities in a plasma subjected to the action of a homogeneous high-frequency electric field. Such parametric instabilities lead to a radical change in the plasma properties. This change of the properties becomes manifest, in particular, in the strong heating of a plasma without collisions by an external high-frequency field, as indicated by the theory[2]. The conclusions of the theory do not contradict the experimental data[5-8]. The coefficient of absorption, by strong electromagnetic waves, of isotropic plasma without collisions, as a function of the electric field intensity of the wave, was measured in[5-6]. The theory[9] yields for the "anomalous" absorption coefficient an explicit expression that agrees qualitatively with these measurements. Measurements in a magnetoactive plasma[7,8,10] call for a detailed development of the theory of parametric resonance of a plasma situated in a constant and homogeneous magnetic field. The principles of such a theory are contained in[3].

If the intensity of the external homogeneous high-frequency electric field is not too high (the amplitude of the electron oscillations in such a field is small compared with the plasma oscillation wavelength) then, as a rule, buildup of the oscillations occurs only when the field frequency is close to one of the natural frequencies of the plasma. At the same time, growth of the plasma perturbations is possible also in an equally weak electromagnetic-wave field at a frequency much larger than all the characteristic frequencies of the plasma, if the length of the external wave is comparable with the perturbation wavelength[11]. For an isotropic plasma, the finite length of the external wave was taken into account in[11,12]. In the present paper we show how the region of paramagnetic instability of the magnetoactive plasma is broadened as a result of the finite wavelength of the pump field.

1. We consider a cold plasma situated in a constant and homogeneous magnetic field **B**. In such a plasma there exist potential low-frequency oscillations (see Fig. 1) with frequency close to the ion-gyroscopic frequency (at $\cos^2\theta > m/M$):

$$\omega^2 = \Omega_i^2\left(1 - \frac{m}{M}\operatorname{tg}^2\theta\right). \qquad (1)$$

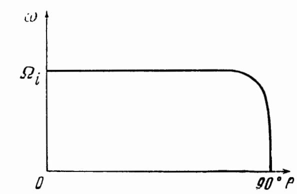


FIG. 1. Dependence of the frequency of low-frequency oscillations of a cold magnetoactive plasma on the angle θ between the wave vector **k** and the constant magnetic field **B** (see [13]).

Here $\Omega_i \equiv e_iB/Mc$ is the gyroscopic frequency of rotation of the ions in the constant magnetic field, e_i and M are the charge and mass of the ion, c is the velocity of light in vacuum, m is the electron mass, and θ is the angle between the wave vector **k** of the oscillations and the constant magnetic field **B**. If the angle θ is close to $\pi/2$ ($\cos^2\theta < m/M$), then the spectrum is determined by the expression

$$\omega^2 = \omega_{Le}^2\left(1 + \frac{\omega_{Li}^2}{\Omega_i^2}\sin^2\theta\right)^{-1}\cos^2\theta, \qquad (2)$$

in which $\omega_{Le} \equiv (4\pi N_e e^2/m)^{1/2}$, $\omega_{Li} \equiv (4\pi N_i e_i^2/M)^{1/2}$ are the electron and ion Langmuir frequencies, and N_e and N_i are respectively the numbers of electrons and ions per cm^3.

Let such a plasma be subjected to the action of the field of a plane monochromatic linearly polarized transverse electromagnetic wave with electric field

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0\cos(\omega_0 t - \mathbf{k}_0\mathbf{r}) \qquad (3)$$

and with a frequency $\omega_0 = ck_0$ greatly exceeding the electron gyroscope frequency Ω_e and the electron Langmuir frequency ω_{Le}: $\omega_0 \gg \Omega_e$, $\omega_0 \gg \omega_{Le}$. The field (3) is limited, i.e., the amplitude of the electron oscillations in the field (3) does not exceed the wavelength of the low-frequency potential oscillations (1) and (2): $\mathbf{k}\cdot\mathbf{r}_E \ll 1$. We assume furthermore that the oscillations (1) and (2) and the external field (3) have wavelengths of the same order: $k_0 \sim k$. Under these conditions, we obtain[1)] the following dispersion equation

[1)]Equation (4) can be obtained, for example, within the framework of two-fluid magnetohydrodynamics. In those cases when the plasma-perturbation energy dissipation is connected with damping due to the inverse Cerenkov effect, the dispersion equation (12) is also obtained by solving the kinetic equation in a magnetoactive plasma.

relating the frequencies and wave numbers of the plasma potential oscillations:

$$1+\delta\varepsilon_e(\omega,\mathbf{k})+\delta\varepsilon_i(\omega,\mathbf{k})+\frac{c^2k^4}{2}\delta\varepsilon_e(\omega,\mathbf{k})\,[1+\delta\varepsilon_i(\omega,\mathbf{k})]\times\left[v_E^2-\frac{1}{k_0^2}(\mathbf{k}\mathbf{v}_E)^2\right]\left\{2\omega\omega_0-c^2(k^2+2\mathbf{k}\mathbf{k}_0)+i\frac{\nu_e\omega_{Le}^2}{\omega_0}\right\}^{-1}\times\left\{2\omega\omega_0+c^2(k^2-2\mathbf{k}\mathbf{k}_0)+i\frac{\nu_e\omega_{Le}^2}{\omega_0}\right\}^{-1}=0. \quad (4)$$

Here $\delta\epsilon_{e,i}(\omega, \mathbf{k})$ is the electronic (ionic) polarizability of the magnetoactive plasma, $\mathbf{v}_E \equiv e\mathbf{E}_0/m\omega_0$ is the velocity of the electron oscillations in the external field (3), and ν_e is the electron collision frequency. Equation (4) has the same appearance as Eq. (3.1) (or (1.18)) obtained in[11] for an isotropic nonisothermal plasma. In our case, however, Eq. (4) contains a constant magnetic field, on which the partial polarizabilities $\delta\epsilon_{e,i}(\omega, \mathbf{k})$ at low frequencies $\omega \ll \omega_0$ are strongly dependent (see, for example[14], p. 140).

Allowance for the external wave (3) in Eq. (4) causes the plasma oscillations, with frequency given by expressions (1) and (2), to become increasing with time, starting with a certain threshold value E_{thr} of the electric field intensity of the external wave (3). The mechanism producing the instability is analogous (in the case of weak coupling) to the mechanism that generates sound in stimulated Mandel-shtam-Brillouin scattering (SMBS). It must, of course, be borne in mind that in the "stimulated scattering" considered by us it is not acoustic waves that build up (see[11]), but low-frequency potential oscillations (1) and (2), the very existence of which in a cold plasma is due to the presence of a constant and homogeneous external magnetic field **B**. This is accompanied by an increase, with increasing time[2)], of the amplitude of the scattered electromagnetic wave (Stokes component), the condition for the excitation of which is

$$2\omega\omega_0+c^2(k^2-2\mathbf{k}\mathbf{k}_0)=0.$$

It turns out that when $k^2 \approx 2\mathbf{k}\cdot\mathbf{k}_0$ and in the case of dissipation due to the collisions, the potential oscillations that build up most intensely are those having wave vectors **k** parallel to the propagation direction $\mathbf{k}_0$ of the external wave. We shall consider just this case, since it is of greatest interest. The scattered electromagnetic wave (Stokes component) propagates in a direction opposite to the direction of propagation of the external wave (3) and the plasma oscillations (1) and (2)—"backward scattering."

Let us trace the solutions (see footnote 2) of the dispersion equation (4) as the electric field E_0 of the external wave (3) is increased. If the field intensity E_0 of the external wave is low ($E_0 \ll E_{thr}$), then the low-frequency (with frequency $\omega \lesssim \Omega_i$) solution of (4) takes the form (1) and (2). Then the plasma oscillations (1) and (2) attenuate weakly, with eecrements $\nu_i + (1/2)\nu_e(m/M)\tan^2\theta$ and $\nu_e/2$ (we assume that the dissipation is due to the collisions of the electrons (ν_e) and of the ions (ν_i) with the neutral particles). With increasing intensity E_0 of the external field (3), the damping decrements of the potential oscillations decrease and vanish at $E_0 = E_{thr}$.

2. If the field intensity $E_0 \gtrsim E_{thr}$, then the potential oscillations with frequencies (1) and (2) no longer attenuate, but increase with time with increment γ. For example, at an angle $0 \le \theta < \pi/2$ and a frequency $\omega \approx \Omega_i$ (see (1)), the increment γ obtained from the solution of the dispersion equation (4) is given by the expression

$$\gamma=\frac{1}{16}\frac{\omega_{Li}^2}{\omega_0\Omega_i}k^2(v_E^2-v_{E\,\mathrm{thr}}^2)\left\{\nu_i+\frac{1}{2}\nu_e\left(\frac{\omega_{Le}^2}{\omega_0^2}+\frac{m}{M}\,\mathrm{tg}^2\,\theta\right)\right\}^{-1}\sin^2\theta. \quad (5)$$

We see (see Fig. 2) that the increment (5) increases with increasing angle θ ($\backsim \sin^2\theta$) from zero (at $\theta = \theta_1$) to a maximum value

$$\gamma_{max}=\frac{1}{4}\frac{v_E^2}{c^2}\frac{\omega_0}{\Omega_i}\omega_{Li}^2\left\{\sqrt{\nu_i}+\sqrt{\nu_e\frac{m}{2M}}\right\}^{-2}-\frac{1}{2}\nu_e\frac{\omega_{Le}^2}{\omega_0^2}, \quad (6)$$

which is attained at an angle $\theta = \theta_0$ such that $\tan\theta_0 = (2M\nu_i/m\nu_e)^{1/4}$. With further increase of the angle θ, the increment decreases ($\backsim \cos^2\theta$), and vanishes at an angle θ_2. The values of the angles $\theta_{1,2}$ at which the increment γ is equal to zero are obtained from the equation

$$\frac{1}{2}\frac{v_E^2}{c^2}\frac{\omega_{Li}^2}{\omega_{Le}^2}\frac{\omega_0^2}{\Omega_i\nu_e}\sin^4\theta-\frac{1}{\omega_0}\left(\nu_i+\nu_e\frac{m}{2M}\right)\sin^2\theta+\frac{1}{2}\frac{\nu_e}{\omega_0}\frac{\omega_{Li}^2}{\omega_{Le}^2}=0.$$

When the propagation is at the angle $\theta = \theta_0$, the threshold value of intensity E_0 of the external field (3) is minimal (compared with formula (10) of[12]):

$$E_{min}^2=8\pi N_e mc^2\frac{\nu_e\Omega_i}{\omega_0\omega_{Li}^2}\cdot\left\{\sqrt{\nu_i}+\sqrt{\nu_e\frac{m}{2M}}\right\}^2. \quad (7)$$

The angular dependence of the threshold field at $\theta < \pi/2$ is determined by the expression

$$E_{\mathrm{thr}}^2=\frac{16\pi}{\sin^2\theta}N_e mc^2\frac{\nu_e\Omega_i}{\omega_0\omega_{Li}^2}\cdot\left(\nu_i+\frac{1}{2}\nu_e\frac{m}{M}\mathrm{tg}^2\,\theta\right). \quad (8)$$

Relations (8) and (5) determine the threshold intensity and the increment for plasma oscillations with frequency $\omega \approx \Omega_i$. At the same frequencies, expressions (6) and (7) hold for the maximum increment and the minimum threshold field E_{min}, provided that the frequency ν_i of the collisions between the ions and the neutral particles is bounded ($\nu_i \ll \nu_e M/m$). In the opposite case, when the collision frequency ν_i is large, $\nu_i > \nu_e M/m$, the extremal angle θ_0 is close to $\pi/2$. Then relations (6) and (7) yield the maximum increment and the minimum threshold intensity E_{min} of the external-wave electric field for oscillations with a frequency (2) much lower than the ion gyroscopic fre-

2)The dispersion equation (4) is solved here in a formulation corresponding to the initial problem, i.e., to the thermal evolution of the plasma perturbation. We obtain here complex solutions for the frequency ω at a specified real wave vector **k**. To the contrary, if the frequency ω is assumed to be specified and real, then we can obtain from (4) a complex expression for the wave vector **k**, corresponding to amplification of the potential oscillations of the cold magnetoactive plasma in space (boundary-value problem).

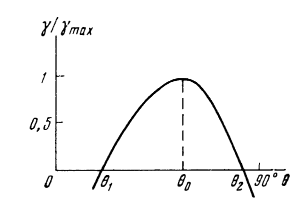


FIG. 2. Growth increment (5) of low-frequency potential oscillations in the near-threshold region $E_0 \approx 2.24\, E_{min}$; $\nu_1 = 0.9\,\nu_e m/M$, $\theta_0 \approx 50°$, $\theta_1 \approx 20°$, $\theta_2 \approx 82°$, $\gamma_{max} = 0.62\,\nu_e(\omega_{Le}/\omega_0)^2$.

quency. If $\pi/2 \approx \theta > \theta_0$, then the threshold field intensity of the external wave increases in proportion to $|\cos|^{-1/2}$:

$$E_{\text{thr}}^2 = 4\pi N_e mc^2 \frac{\nu_e^2}{\omega_0\omega_{Le}}\left\{\cos^2\theta + \frac{\omega_{Li}^2}{\Omega_i^2}\sin^2\theta\cos^2\theta\right\}^{-1/2}. \qquad (9)$$

The increment then decreases ($\sim \cos\theta$):

$$\gamma = \frac{1}{8}\frac{\omega_{Le}}{\omega_0\nu_e}k^2(v_E^2 - v_{E\text{thr}}^2)\left(1 + \frac{\omega_{Li}^2}{\Omega_i^2}\sin^2\theta\right)^{1/2}|\cos\theta|. \qquad (10)$$

3. Let the intensity E_0 of the electric field of the external wave (3) greatly exceed the threshold value E_{thr}, $E_0 \gg E_{\text{thr}}$ (see (7), (8), (9)). In the weak-coupling approximation, when E_0 is smaller than the critical value E_{cr} (at a field value equal to the critical, the increment is comparable with the oscillation frequency), the solutions of the dispersion equation (4) for the frequency ω ($\omega \lesssim \Omega_i$) have the form (1) and (2) as before. The increments, unlike the preceding case $E_0 \gtrsim E_{\text{thr}}$ (see, for example, (6) and (10)), are determined entirely by the field intensity E_0 of the external wave. In the region of frequencies ω smaller than or comparable with the ion gyroscopic frequency, $\omega \lesssim \Omega_i$, or, which is the same, at an angle θ not close to $\pi/2$ (see Fig. 1), the increment increases with increasing angle ($\sim |\sin\theta|$):

$$\gamma = {}^1/_4 k|v_E\sin\theta|\,\omega_{Li}(\omega_0\Omega_i)^{-1/2}. \qquad (11)$$

In this case the critical electric field intensity of the external wave is given by

$$E_{\text{cr}}^2 = \frac{16\pi}{\sin^2\theta}N_e mc^2\frac{\omega_0\Omega_i^3}{\omega_{Le}^2\,\omega_{Li}^2}.$$

At frequencies ω lower than the ion gyrofrequency ($\omega \ll \Omega_i$) the increment decreases in proportion to $|\cos\theta|^{1/2}$:

$$\gamma = \frac{1}{4}k|v_E|\left\{\frac{\omega_{Le}^2}{\omega_0^2}\left(1 + \frac{\omega_{Li}^2}{\Omega_i^2}\sin^2\theta\right)\cos^2\theta\right\}^{1/4}. \qquad (12)$$

The critical field then decreases in the same manner as the increment (12):

$$E_{\text{cr}}^2 = 16\pi N_e mc^2\frac{\omega_0}{\omega_{Le}}\left(1 + \frac{\omega_{Li}^2}{\Omega_i^2}\sin^2\theta\right)^{-3/2}|\cos\theta|.$$

If the plasma is sufficiently dense, $\omega_{\text{Li}}^2 \gg \Omega_i^2$, i.e., the Alfven velocity $V_A = c(\Omega_i/\omega_{\text{Li}})$ is much lower than the velocity of light, then we can use for the increment a simple expression valid for all angles θ:

$$\gamma = \frac{1}{4}k|v_E\sin\theta|\left(\frac{c}{v_A}\frac{\omega_{Li}}{\omega_0}\right)^{1/2}\cdot\left(1 + \frac{m}{M}\text{tg}^2\theta\right)^{-1/4}. \qquad (13)$$

This expression leads to relations (11) and (12), which are simplified when $c \gg v_A$. It is obvious from (13) that the increment γ reaches a maximum at an angle θ_0 (see Fig. 3) such that $\cos\theta_0 = (m/2M)^{1/4}$ (see formula (9) of [12]):

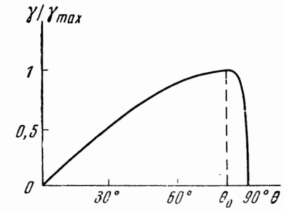
$$\gamma_{max} = {}^1/_2(\omega_{Li}\omega_0 v_E^2/cv_A)^{1/2}. \qquad (14)$$

In this case the critical value of the field intensity of the external wave is equal to

$$E_{\text{cr}}^2 = 16\pi N_e mc^2\Omega_i^3\omega_0/\omega_{Li}^2\omega_{Le}^2.$$

4. On the other hand, if the electric field intensity E_0 of the external wave (3) exceeds the critical value ($E_0 > E_{\text{cr}} \gg E_{\text{thr}}$), then the buildup increment becomes

FIG. 3. The increment (13) far above the threshold ($E_0 \gg E_{\text{thr}}$) in the case of weak coupling. The maximum (14) in the hydrogen plasma is reached at an angle $\theta_0 \approx 82°$.



larger than the oscillation frequency. In this case, neglecting the dissipation in the dispersion equation (4), and assuming that k^4 is close enough to $4(\mathbf{k}\cdot\mathbf{k}_0)^2$, but not exactly equal to it (see[11]), we obtain an aperiodic instability

$$\omega^2 = -\frac{1}{2}\frac{\omega_{Le}^2}{c^2}\left[v_E^2 - \frac{1}{k_0^2}(\mathbf{k}\mathbf{v}_E)^2\right]\frac{k^4\cos^2\theta}{k^4 - 4(\mathbf{k}\mathbf{k}_0)^2}. \qquad (15)$$

At the resonance point $k^2 = 4(\mathbf{k}\cdot\mathbf{k}_0)^2$ itself, the solution of (4) is given by (at $\mathbf{k} \parallel \mathbf{k}_0$ and $E_0 > E_{\text{cr}}$)

$$\omega = 2^{-4/3}(1 + i\sqrt{3})(\omega_0\omega_{Le}^2 v_E^2 c^{-2}\cos^2\theta)^{1/3}. \qquad (16)$$

We note that expressions (15) and (16) contain not the ion Langmuir frequency ω_{Li}, as in (4.3) and (4.7) of[11], but the quantity $\omega_{\text{Le}}\cos\theta$. This difference is due to the fact that formulas (15) and (16) were derived under the assumption that the frequency is low compared with the ion gyrofrequency, $\omega < \Omega_i$, i.e., with allowance for a strong external magnetic field **B**.

Summarizing, we emphasize that a consistent allowance for the finite wavelength of the pump field (3) leads to a broadening of the region of parametric instability of the plasma. Namely, the plasma perturbations that grow most rapidly are those whose wavelength is equal to half the wavelength of the external field. These instabilities do not appear in the plasma if the pump field is homogeneous. The presence of a constant magnetic field **B** causes the cold plasma to be unstable already in the weak-coupling approximation (at relatively weak fields E_0). The frequencies of the oscillations excited thereby are determined by the constant magnetic field, and the increments by the electric field of the external wave. In a magnetoactive plasma, unlike an isotropic one, there is a clearly pronounced anisotropy of the direction of the maximum buildup of the potential oscillations. For example, in a cold hydrogen magnetoactive plasma of sufficient density ($c > v_A$), the most intense buildup of low-frequency waves (14) (B is in gauss and E_0 in V/cm)

$$\gamma_{max} \approx 3.7\cdot 10^5 E_0(N_e/\omega_0 B)^{1/2}\ \ [\text{sec}^{-1}]$$

occurs at an angle $\theta_0 = 82°$ (see Fig. 3) to the constant magnetic field **B**. The intensity E_0 of the electric field of the external electromagnetic wave (3) greatly exceeds in this case the threshold value (7) (ν_i, ν_e, and ω_0 are expressed in sec^{-1})

$$E_{min} = 0{,}1\{\sqrt{\nu_i} + \sqrt{\nu_e m/2M}\}(B/\omega_0)^{1/2}\ \ [\text{V/cm}].$$

In the ionosphere at an altitude of 200 km and an external-wave frequency $\omega_0 \approx 10^9\ \text{sec}^{-1}$ (the meter band), the minimum threshold intensity $E_{\text{min}} \approx 7\times 10^{-6}$ V/cm corresponds to a flux of $6.5\times 10^{-10}\ \text{W/m}^2$. The maximum increment $\gamma_{\text{max}} \approx 1\ \text{sec}^{-1}$ is reached when the threshold is exceeded tenfold, $E_0 \approx 10E_{\text{min}}$.

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