Induced Wave Scattering in a Magnetoactive Plasma

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Some features of stimulated scattering of monochromatic electromagnetic waves in a magnetoactive plasma are considered. Expressions are obtained for the scattering increments in the hydrodynamic stage. The possibility of employing the processes considered for diagnostics and heating of a plasma is discussed.

1. Nonlinear wave interaction in a magnetoactive plasma has been considered many times in a number of papers. In particular, induced scattering processes were investigated in[1,2]. As a rule, an analysis of nonlinear interactions in a magnetoactive plasma is made difficult by the cumbersome mathematical manipulations connected with the determination of the nonlinear current, and the general formulas are difficult to interpret.

In the present article we extend the method developed in[3] for considering nonlinear interactions in an isotropic plasma to the case of a magnetoactive plasma. Simple general relations are obtained for the description of induced wave scattering. The hydrodynamic stage of the induced scattering is considered in detail. It is shown that induced scattering by electrons increases sharply when the wave vector of the averaged high-frequency potential $\boldsymbol{\kappa} = \mathbf{k}_1 - \mathbf{k}_2$ is almost perpendicular to the constant magnetic field $\mathbf{H}_0$. The decrements of the nonlinear gyroresonance absorption of high-frequency waves are determined, and induced scattering in systems with helical beams of charged particles is investigated. Estimates are presented illustrating the possibility of observing these effects in laboratory and in outer-space plasma.

2. The method used to calculate the effects of induced scattering in a magnetoactive plasma is based on a calculation of the work performed on the plasma particles by the averaged high-frequency force produced upon scattering. This method is described in greater detail in our earlier paper[3]. We present here only the final expression, which will be used subsequently, for the work Q performed on the particles in the field of two quasimonochromatic waves:

$$Q^{(\alpha)} = -|P^{(\alpha)}|^2 \operatorname{Re} \sigma_{zz}^{(\alpha)} \qquad (\alpha = e, i), \tag{1}$$

where

$$|P^{(e)}|^2 = \left(\frac{\varkappa\Phi_e}{e}\right)^2 \left|\frac{1+\varepsilon_i}{1+\varepsilon_e+\varepsilon_i}\right|^2, \quad |P^i|^2 = \left(\frac{\varkappa\Phi_e}{e}\right)^2 \left|\frac{\varepsilon_e}{1+\varepsilon_e+\varepsilon_i}\right|^2 \tag{2}$$

The high-frequency potential Φ_α is of the following form:

$$\Phi_\alpha = \frac{e_\alpha^2 E_1 E_2}{2m_\alpha}\chi, \quad \chi^2 = \left\{\frac{\omega_2}{\omega_1}\frac{a_{x1}a_{x2}^* + a_{y1}a_{y2}^*}{\omega_{H\alpha}^2 - \omega_2^2} - \frac{\omega_{H\alpha}}{\omega_1}\frac{a_{x1}a_{y2}^* - a_{y1}a_{x2}^*}{\omega_{H\alpha}^2 - \omega_2^2} - \frac{a_{z1}a_{z2}^*}{\omega_1\omega_2}\right\}^2. \tag{3}$$

Here

$$\varepsilon_{e,i} = \frac{4\pi i}{\Omega}\sigma_{zz}^{(e,i)}, \quad \varepsilon_e + \varepsilon_i + 1 = \varepsilon_{zz}, \quad \boldsymbol{\varkappa} = \mathbf{k}_1 - \mathbf{k}_2, \quad \Omega = \omega_1 - \omega_2,$$

$\omega_{1,2}$ and $\mathbf{k}_{1,2}$ are the frequencies and wave vectors of the interacting waves, $\mathbf{E}_{1,2}$ are their amplitudes, a_μ are the components of the polarization vector, $\omega_{H\alpha}$ is the gyrofrequency of the particles of species α, ϵ_{zz} is the component of the linear dielectric tensor of the magnetoactive plasma in a coordinate system with z axis along $\boldsymbol{\kappa}$: $\epsilon_{zz} = \epsilon_1 \sin^2\theta + \eta\cos^2\theta + \xi\sin 2\theta$, where θ is the angle between $\boldsymbol{\kappa}$ and $\mathbf{H}_0$; the expressions for ϵ, η, ξ and a_μ are well known and are given, for example, in[4]. Formula (12) describes scattering via a quasilongitudinal virtual wave, which is predominant in a nonrelativistic plasma under the condition $\Omega \ll \omega_{1,2}$.

Using, in analogy with[3], the energy–momentum conservation laws for the particle plus field system, we can easily obtain with the aid of (1) and (2) expressions for the nonlinear interaction coefficients:

$$\frac{dw_1}{dt} = -\frac{\omega_1}{\Omega}Q = -\alpha_1 w_1 w_2,$$
$$\frac{dw_2}{dt} = \frac{\omega_2}{\Omega}Q = \alpha_2 w_1 w_2, \tag{4}$$

where $w_{1,2}$ is the energy density of the interacting waves.

Generalization to the case of an interaction between packets of waves with random phase yields

$$\frac{dw_{k1}}{dt} = w_{k1}\int G(\mathbf{k}_1, \mathbf{k}_2)\, w_{k2}\, d\mathbf{k}_2\, d\omega_2, \tag{5}$$

where

$$G(\mathbf{k}_1, \mathbf{k}_2) = \frac{16\pi^2(\mathbf{k}_1 - \mathbf{k}_2)^2\omega_2 e^2}{m^2\Omega}\chi^2\left\{\left|\frac{1+\varepsilon_i}{1+\varepsilon_e+\varepsilon_i}\right|^2 \operatorname{Re}\sigma_l^e + \left|\frac{\varepsilon_e}{1+\varepsilon_e+\varepsilon_i}\right|^2 \operatorname{Re}\sigma_l^i\right\} \left|\frac{\partial(\omega^2\varepsilon_{\mu\nu})a_\mu a_\nu^*}{\omega\, d\omega}\right|^{-1}_{\omega=\omega_1} \left|\frac{\partial(\omega^2\varepsilon_{\mu\nu})a_\mu a_\nu^*}{\omega\,\partial\omega}\right|^{-1}_{\omega=\omega_2}. \tag{6}$$

3. Let us discuss certain general consequences resulting from the relations given above.

In the case $\Omega/\omega_{H\alpha} \sim \kappa_z v_{T\alpha}/\omega_{H\alpha} \ll 1$ the scattering via a longitudinal virtual wave in a magnetoactive plasma is described by the formulas for an isotropic plasma, in which κ is replaced by κ_z, and using the corresponding expression (3) for the high-frequency potential, except for the small region of angles $\tan^2\theta < \Omega^2/\omega_{H\alpha}^2$ (θ is the angle between $\boldsymbol{\kappa}$ and $\mathbf{H}_0$). The character of the interaction changes significantly in the case of quasitransverse propagation of the average force ($\theta \sim \pi/2$) and in cyclotron resonance ($\Omega = \omega_{H\alpha}$). We shall discuss these cases in greater detail.

If the initial energy densities of the interacting waves satisfy the relation $w_{10} \gg w_{20}$, then Eqs. (4) can be linearized by assuming w_{10} = const, and the scattering processes can be characterized with the aid of the growth increments γ of the weak wave with frequency ω_2. Then, just as in an isotropic plasma, it is necessary to distinguish between two limiting stages (with respect to the increment) of the induced scattering, kinetic and hydrodynamic. If the scattering occurs at angles for which[1)] $\theta \approx \pi/2$, then the criterion for attaining the hydrodynamic stage becomes less stringent (for waves with a narrow spectrum $\Delta\omega \ll \kappa v_{T\alpha}$), namely $\gamma > \kappa v_{T\alpha} \cos\theta, \Delta\omega$, and the effects of compensation for induced scattering by electrons vanish in a wide range of parameters (if $\kappa^2 r_d^2 \ll 1$, where r_d is the Debye radius). There is no compensation if

$$\varepsilon_e \leqslant \max(1, \varepsilon_i), \tag{7}$$

and when $\Omega \gg \kappa_z v_{T\alpha}$ we have

$$\varepsilon_e \approx \frac{\omega_{0e}^2}{\Omega^2 - \omega_{He}^2}\sin^2\theta + \frac{\omega_{0e}^2}{\Omega^2}\cos^2\theta.$$

In an isotropic plasma, the condition (7) reduces to the inequality[3] $\Omega > \omega_{0e}$, and in a plasma with a strong magnetic field (for $\omega_{He} \gg \omega_{0e}$) it takes the form

$$\Omega > \omega_{0ez} = \omega_{0e}\cos\theta. \tag{8}$$

The increment of the induced scattering by electrons is determined in this case by the expression

$$\gamma^3 = (\varkappa v_\sim/\omega_1)^2\,\omega_{0ez}^2\,\omega_1, \tag{9}$$

where $v_\sim = eE_{10}\chi/\sqrt{2m\omega_1}$ is the velocity of the electron oscillations in the field of the pump wave. Thus, at a low pump-wave amplitude the scattering by electrons proceeds predominantly at an angle $\theta \approx \pi/2$ (at $\cos\theta < (\kappa v_\sim/\omega_1)^2\omega_1/\omega_{0e}$).

In a dense plasma ($\omega_{0e} \gg \omega_{He}$) the dependence of γ on the amplitude of the incident wave is more complicated. If $\omega_{Hi} < \gamma < (\omega_{He}\omega_{Hi})^{1/2}$, then $\epsilon_e \sim \epsilon_i \gg 1$, there is no compensation of the scattering by the electrons, and expression (9) holds for the increment. With further increase of the pump amplitude ($\gamma^2 > \omega_{He}\omega_{Hi}$), compensation effects appear and scattering by ions begins to predominate, with an increment

$$\gamma^3 = (\varkappa v_\sim/\omega_1)^2\,\omega_{0i}^2\,\omega_1, \tag{10}$$

which has the same form as the analogous expression for an isotropic plasma[3]. In this case the increment for scattering by electrons is given by

$$\gamma^7 \approx (\varkappa v_\sim/\omega_1)^2(\omega_{He}\omega_{Hi})^2\,\omega_{0ez}^2\,\omega_1. \tag{11}$$

5. In the case of scattering in a magnetoactive plasma, additional resonances arise, and in particular, nonlinear gyroresonant scattering can occur, in which the frequencies of the interacting waves satisfy the condition $\omega_1 - \omega_2 \approx s\omega_{H\alpha}$ ($s = 1, 2, \ldots$). Just as in the case of an isotropic plasma, the analogy with the theory of two-stream instability of plasma oscillations is very useful here. Thus, the increments of the induced scattering coincide with the increments of the two-stream instabilities if the beam density is replaced by the effective density

$$N_{\text{eff}} = (\varkappa v_\sim/\omega_1)^2 N_0. \tag{12}$$

Let us consider in somewhat greater detail nonlinear gyroresonant absorption of high-frequency waves of frequency $\omega_1 \gg \omega_{He}$ in a plasma in a strong magnetic field ($\omega_{He} \gg \omega_{0e}$). This absorption mechanism is due to the interaction between the plasma and the averaged high-frequency force at the frequency $\Omega \approx \omega_{He}$. In the simplest case of transverse propagation of the pump wave ($\mathbf{k}_1 \perp \mathbf{H}_0$), there is no compensation at $\gamma > \omega_{0e}^2/2\omega_{He}$, and the increment of the induced scattering by the electrons is determined by the relation

$$\gamma = \frac{v_\sim}{v_{\text{ph}}}\left(\frac{\omega_1}{\omega_{He}}\right)^{1/2}\omega_{0e}. \tag{13}$$

Since the quantum frequency changes immediately by $\omega_{He} \gg \gamma$ in gyroresonance scattering, the relative fraction of the pump-wave energy absorbed by the plasma is larger than in ordinary induced scattering at Cerenkov resonance $\Omega = \boldsymbol{\kappa}\cdot\mathbf{v}$. In addition, in this case the hydrodynamic stage of scattering should last longer, since the interaction gives rise to narrow spectral lines with $\Delta\omega \lesssim \gamma_H$ separated by $\omega_{He} \gg \Delta\Omega$.

As an illustration of the possibility of using the effects under consideration for plasma heating, we present certain numerical estimates. The expression for the maximum increment (13) can be represented in the form

$$\gamma \approx 3\cdot 10^6\,\tilde{P}^{1/2}\frac{\omega_{0e}}{(\omega_1\omega_{He})^{1/2}}, \tag{14}$$

where $\tilde{P}$[W/cm²] is the energy flux density of the coherent radiation. For example, in the case of a pump wave of frequency $\omega_1 = 3\times 10^{11}$ sec^{-1} and $\tilde{P} = 10^4$ W/cm² at $\omega_{He} = 10^{11}$ sec^{-1} and $N_e = 3\times 10^{11}$ cm^{-3}, the increment is equal to $\gamma = 6\times 10^7$ sec^{-1}. Analogously, in optical pumping with $\omega_1 = 2\times 10^{15}$ and $\tilde{P} = 10^{12}$ W/cm², we have at the same plasma parameters $\gamma = 5\times 10^9$ sec^{-1}. Of course, to draw final conclusions it is necessary to consider the nonlinear stage of the process with allowance for effects of saturation of the induced scattering.

6. Let us stop to discuss further the effects of induced scattering in a system with a helical electron beam[2)]. In such a system, the induced scattering can be appreciable for two reasons: 1) as a rule, the beam density used in electronics is low ($\omega_{0e} \ll \omega_{He}$), so that the scattering compensation is easily lifted; 2) a helical beam excited electromagnetic waves with $\omega = s\omega_{He}$, and the average high-frequency force leading to the scattering is strongly increased thereby.

For simplicity we consider waves excited by a beam almost along the magnetic field. The induced-scattering increment can readily be determined by using (1)–(4):

$$\gamma = \frac{2\varkappa^2\pi\omega_{0e}^2E_{10}^2}{mN}\left|\frac{1}{\varepsilon_e+1}\right|^2\frac{\omega_{0s}^2}{\omega_{He}(\omega_{He}-\omega)^2}\frac{\operatorname{Re}\sigma_{zz}^{(s)}}{\operatorname{Re}\Omega}, \tag{15}$$

where $\omega_{0s}^2 = 4\pi e^2N_s/m$ is the Langmuir frequency of the

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[1)]From the condition $\theta\approx\pi/2$ it follows that at a small frequency shift, the angle between the propagation of the scattered wave and the magnetic field $\mathbf{H}_0$ is close to specular relative to the direction of the propagation of the initial wave ("specular scattering"): $\sphericalangle\, \mathbf{k}_1\mathbf{H}_0 = -\sphericalangle(\mathbf{k}_2\mathbf{H}_2)$.

[2)]Such systems are extensively used in microwave electronics. In particular, they include masers operating at cyclotron resonance[5].

beam. As already discussed above, compensation vanishes at $\gamma > \omega_{0sz}$. Taking into account the expression for $\sigma_{zz}^{(e)} = (\Omega/4\pi i)\omega_{0s}^2/(\Omega - \kappa_z v_0)^2$ and assuming $\gamma_1 > \omega_{0sz}$, we obtain

$$\gamma^3 = \frac{2w_E}{mNv_{ph}^2}\frac{\omega_{0s}^2\,\omega_{He}\,\omega_{0sz}^2}{(\omega_{He}-\omega)^2}, \quad w_E = \frac{E_{10}^2}{8\pi}. \tag{16}$$

The minimum detuning is determined, in order of magnitude, by the value of the linear instability increment γ_L, which is equal to[5]

$$\gamma_L = \tfrac{1}{2}\sqrt{3}(\omega_{He}\omega_{0s}^2\beta_{0\perp}^2)^{1/3}, \quad \beta_{\perp 0} = v_{\perp 0}/c. \tag{17}$$

Assuming $\Delta\omega \sim \gamma_L$, we get

$$\left(\frac{\gamma_H}{\gamma_L}\right)^3 \approx b^2\frac{\omega_{0sz}^2}{\gamma_L^2}, \quad b^2 = \frac{c^2}{v_{ph}^2}\frac{w_E}{w_s}, \quad w_s = \frac{mN_s v_0^2}{2}. \tag{18}$$

As follows from (18), induced scattering may turn out to be significant already at the initial stage of excitation of waves by a beam, provided the scattering is accompanied by transformation into sufficiently slow (longitudinal) waves. In particular, systems with helical electron beams can be used for an experimental investigation of induced-scattering processes.

7. In addition to different laboratory applications, induced hydrodynamic scattering can play an important role in the investigation of outer-space plasma (in the ionosphere and magnetosphere). Estimates show that effects of scattering by ions and nonlinear gyroresonance heating could have appeared in the recently performed experiments on heating of the ionosphere by electromagnetic radiation in the shortwave band[6], namely at the experimentally employed energy flux density $\tilde{P} = 50\ \mu W/m^2$, $\omega_1 = 10^8\ sec^{-1}$, and at the characteristic ionospheric F-layer parameters $N_e \simeq 5 \times 10^5\ cm^{-3}$ the scattering increment is $\gamma \approx 10^3\ sec^{-1}$.

New possibilities are also uncovered for the diagnostics of the magnetosphere when acoustic waves are scattered (waves of the whistler type, 1—50 kHz), for which the scattering increment is given by

$$\gamma \approx (h/H)^{2/3}(\omega_{Hi}\omega_1^2)^{1/3}, \tag{19}$$

where H is the geomagnetic field and h is the amplitude of the magnetic field in the wave. An analysis of the possible models of whistler propagation in the magnetosphere shows that when account is taken of the dependences of N_e, H and h on the distance, the increment (19) varies little with altitude. Estimates for a transmitter with nondirectional antenna and a pulse power $P = 10^6$ W at a frequency f = 15 kHz yield a value $\gamma = 10^2\ sec^{-1}$ (at a coefficient of power transmission through the ionosphere $T = 10^{-2}$). The necessary condition $\gamma > \kappa v_{Ti} \approx 10$ is satisfied here. We note that induced scattering can even now be used to explain many effects observed in the operation of low-frequency navigation stations (the Doppler frequency shift, the decrease of the pulse duration, the anomalous radio-echo delay times)[7].

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